

Post-Review Adjustment

Title: Modeling microbial regulatory feedback in organic matter decomposition identifies copiotrophic traits as key drivers of positive priming

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Requested adjustments to Eqs. (4) – (6)

- Current version:

$$\frac{dE_i}{dt} = \alpha_{E,i}X_i + u_{E,i}Y_{E,i}r_{S,i}X_i - \beta_{E,i}E_i; \quad i = 1, 2 \quad (4)$$

$$\frac{de_i}{dt} = \alpha_{e,i} + u_{e,i}Y_{e,i}r_{S,i} - \beta_{e,i}e_i; \quad i = 1, 2 \quad (5)$$

$$r_{S,i} = k_{S,i} \frac{(S/K_{S,i})}{1 + (S/K_{S,i})} \frac{(e_i/K_{e,i})}{1 + (e_i/K_{e,i})}; \quad i = 1, 2 \quad (6)$$

- Corrected version:

$$\frac{dE_i}{dt} = \alpha_{E,i}X_i + u_{E,i}Y_{E,i}r_{S,i}^{kin}X_i - \beta_{E,i}E_i; \quad i = 1, 2 \quad (4)$$

$$\frac{de_i}{dt} = \alpha_{e,i} + u_{e,i}Y_{e,i}r_{S,i}^{kin} - \beta_{e,i}e_i; \quad i = 1, 2 \quad (5)$$

$$r_{S,i} = \underbrace{k_{S,i} \frac{(S/K_{S,i})}{1 + (S/K_{S,i})}}_{\equiv r_{S,i}^{kin}} \frac{(e_i/K_{e,i})}{1 + (e_i/K_{e,i})}; \quad i = 1, 2 \quad (6)$$

- Reason for the change:

During the final proofreading of the typeset article, we identified an inconsistency in Eqs. (4) – (6), as highlighted above. The proposed corrections are intended solely to ensure the correctness of the model equations and improve their clarity for readers. The term $r_{S,i}$ in Eqs. (4) and (5) represents the unregulated substrate uptake rate, which is given by Eq. (6) without the enzyme term. Including the enzyme term in Eqs. (4) and (5) would be incorrect, as it would introduce a cyclic dependency in the equations. We initially overlooked this issue because it was apparent to us from the model formulation that the enzyme term should be omitted. However, we subsequently recognized that this distinction may not be readily

apparent to general readers. To address this potential ambiguity, we now denote the unregulated substrate uptake rate in Eqs. (4) and (5) as $r_{S,i}^{kin}$, thereby distinguishing it from the regulated uptake rate that includes the enzyme term. We also explicitly clarify that $r_{S,i}^{kin}$ is defined in Eq. (6) without the enzyme term. Therefore, the proposed corrections are necessary to ensure the equations are presented correctly and unambiguously. These changes are purely corrective and do not alter the underlying results or conclusions of the paper.