Sandy beaches' chaos: shoreline-sandbar coupling inferred from observational time series

- Supplementary Material -

Torrey Pines

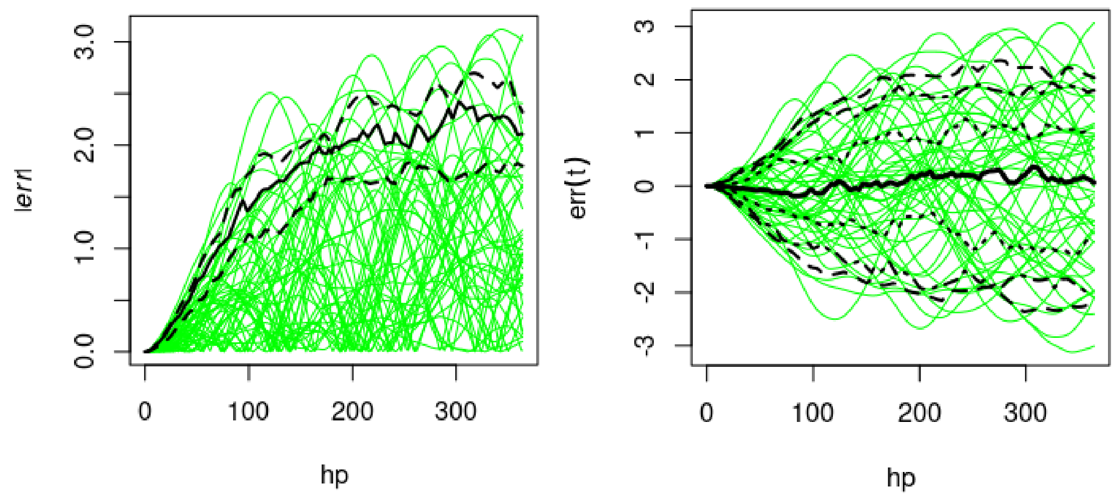

***Figure S1**. Prediction error growth for Torrey Pines's model over a 365-day forecast horizon using the GPoM predictability framework. **Left:** absolute error magnitude ∣err∣ for 50 prediction runs initialized from different points along the observed trajectory. **Right:** raw error err(t) showing divergence patterns across runs.Thin green lines correspond to individual trajectories, while dashed black lines indicate the overall spread (minimum and maximum errors across all runs. The growth of the error envelope illustrates the nonlinear predictability limit and sensitivity to initial conditions.*

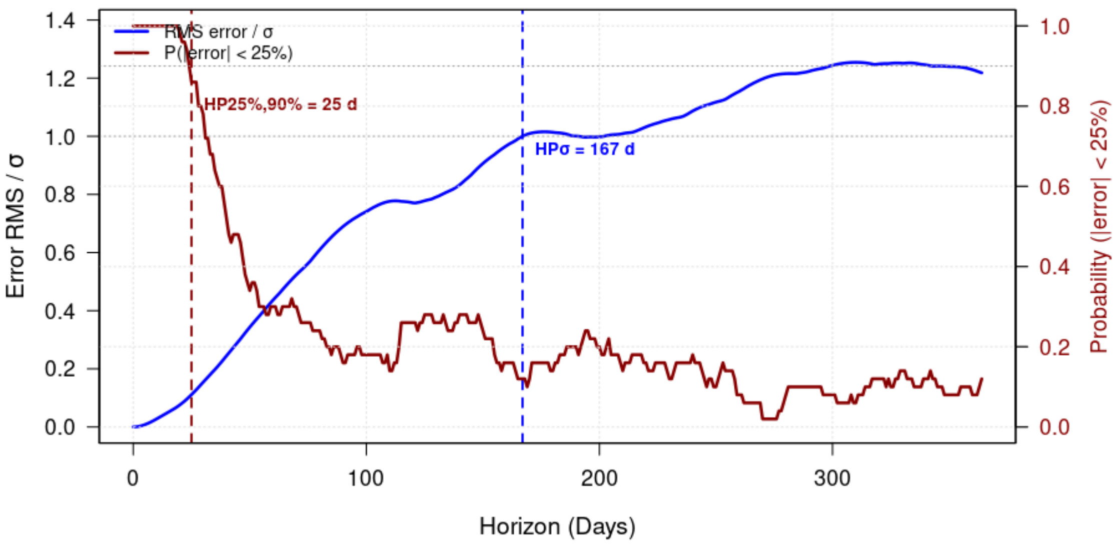

***Figure S2.** Comparison of two predictability metrics of Torrey Pines' model. The blue curve shows the normalized root-mean-square (RMS) error relative to the signal standard deviation (σ). The red curve shows the probability P(|err|<25%) that predictions remain within a 25% relative error. Vertical dashed lines indicate the predictability horizon HP_{σ} (RMS/σ = 1) and the probabilistic horizon $HP_{25\%,90\%}$ defined as the time when 90% of runs exceed 25% relative error. The short horizon $HP_{25\%,90\%}$ = 25*

days marks the loss of trajectory memory under stochastic forcing, while the longer HP_σ = 160 days shows that the model still captures the global structure and quasi-periodic shoreline–bar dynamics.
*In physical terms, the system keeps the **shape of the cycle** (structural memory) but rapidly loses the **exact phase** (trajectory memory).*

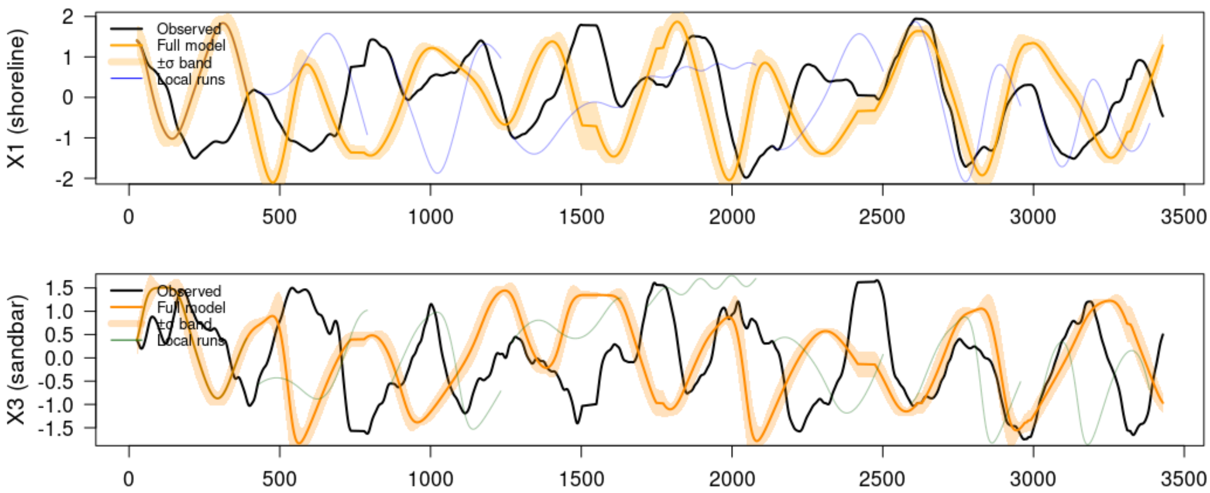

Figure S3. *Comparison between the observed time series (black) and model trajectories for X1 (shoreline, top) and X3 (sandbar, bottom).The orange line shows the full model integration over the entire time span, with the shaded ±σ band representing the local variability (computed over a 90-day moving window). Semi-transparent blue and green lines represent short model runs (365 days) initiated every 400 days from different observed conditions.*
The ensemble divergence highlights the local predictability range and temporal variability of model skill across the coupled shoreline–sandbar dynamics.

Gold Coast

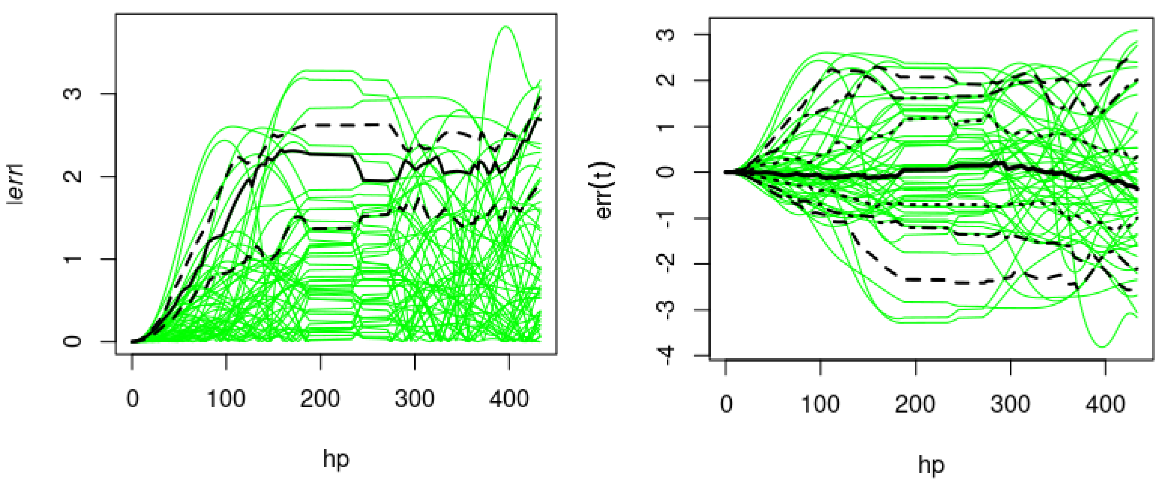

Figure S4*. Prediction error growth for Gold Coast's model over a 365-day forecast horizon using the GPoM predictability framework.* ***Left:*** *absolute error magnitude ∣err∣ for 50 prediction runs initialized from different points along the observed trajectory.* ***Right:*** *raw error err(t) showing divergence patterns across runs.Thin green lines correspond to individual trajectories, while dashed black lines indicate the overall spread (minimum and maximum errors across all runs. The growth of the error envelope illustrates the nonlinear predictability limit and sensitivity to initial conditions.*

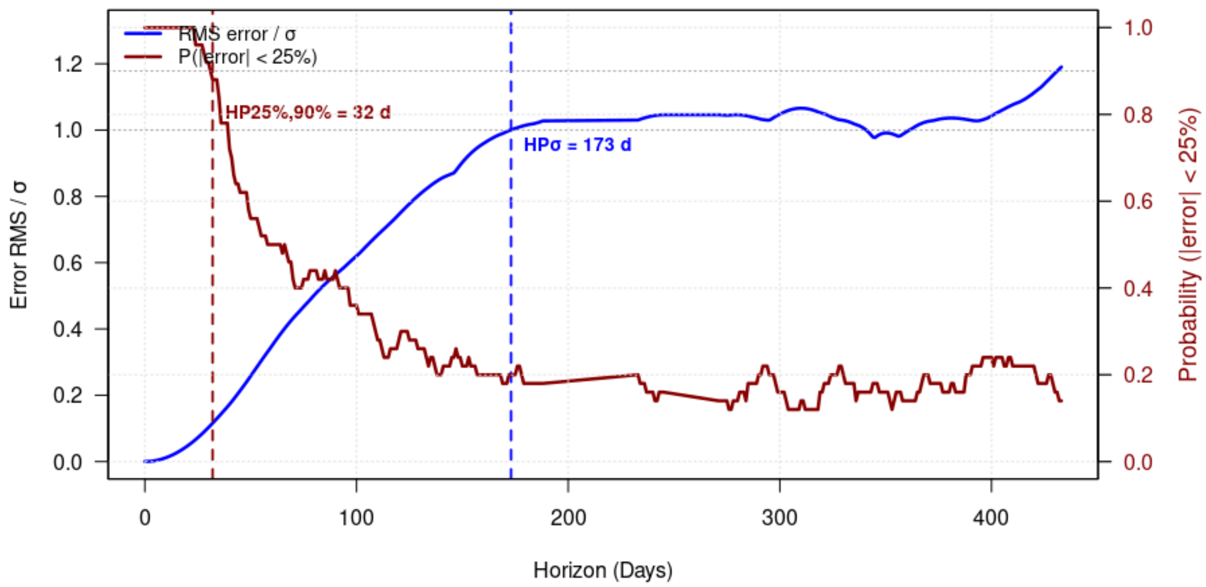

Figure S5. *Comparison of two predictability metrics of Gold Coast's model. The blue curve shows the normalized root-mean-square (RMS) error relative to the signal standard deviation (σ). The red curve shows the probability P(|err|<25%) that predictions remain within a 25% relative error. Vertical dashed lines indicate the predictability horizon* HP_{σ} *(RMS/σ = 1) and the probabilistic horizon* $HP_{25\%,90\%}$ *defined as the time when 90% of runs exceed 25% relative error. The short horizon* $HP_{25\%,90\%}$ *= 32 days marks the loss of trajectory memory under stochastic forcing, while the longer* HP_{σ} *= 173 days shows that the model still captures the global structure and quasi-periodic shoreline–bar dynamics.*

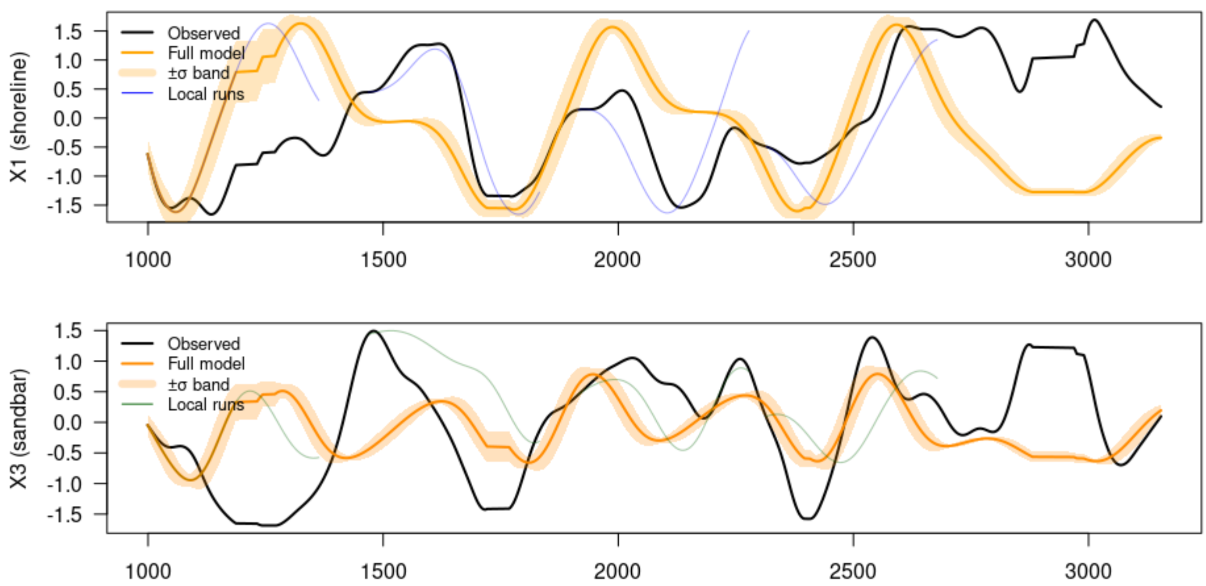

Figure S6. *Comparison between the observed time series (black) and model trajectories for X1 (shoreline, top) and X3 (sandbar, bottom).The orange line shows the full model integration over the entire time span, with the shaded ±σ band representing the local variability (computed over a 90-day moving window). Semi-transparent blue and green lines represent short model runs (365 days) initiated every 400 days from different observed conditions.*
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Ensenada

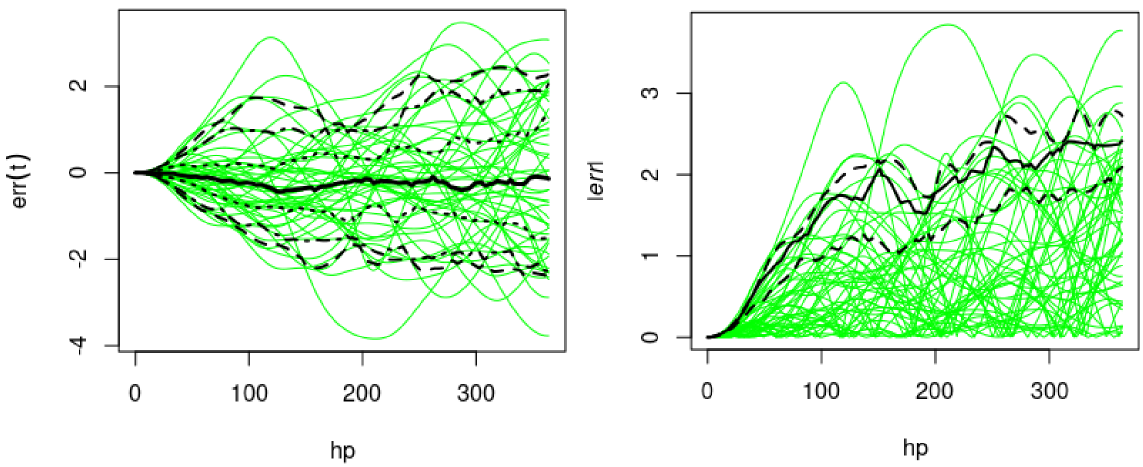

Figure S7. *Prediction error growth for Ensenada's model over a 365-day forecast horizon using the GPoM predictability framework.* ***Left:*** *absolute error magnitude |err| for 50 prediction runs initialized from different points along the observed trajectory.* ***Right:*** *raw error err(t) showing divergence patterns across runs.Thin green lines correspond to individual trajectories, while dashed black lines indicate the overall spread (minimum and maximum errors across all runs. The growth of the error envelope illustrates the nonlinear predictability limit and sensitivity to initial conditions.*

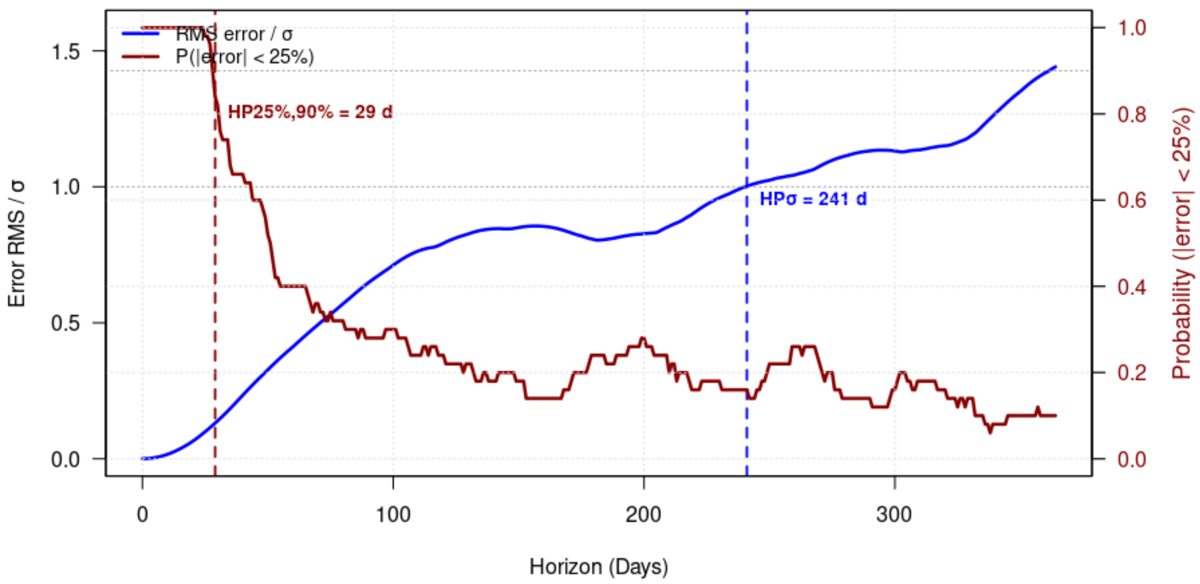

Figure S8. *Comparison of two predictability metrics of Ensenada's model. The blue curve shows the normalized root-mean-square (RMS) error relative to the signal standard deviation (σ). The red curve shows the probability P(|err|<25%) that predictions remain within a 25% relative error. Vertical dashed lines indicate the predictability horizon* HP_{σ} *(RMS/σ = 1) and the probabilistic horizon* $HP_{25\%,90\%}$ *defined as the time when 90% of runs exceed 25% relative error. The short horizon* $HP_{25\%,90\%}$ = *29 days marks the loss of trajectory memory under stochastic forcing, while the longer* HP_{σ} = *241 days shows that the model still captures the global structure and quasi-periodic shoreline–bar dynamics.*

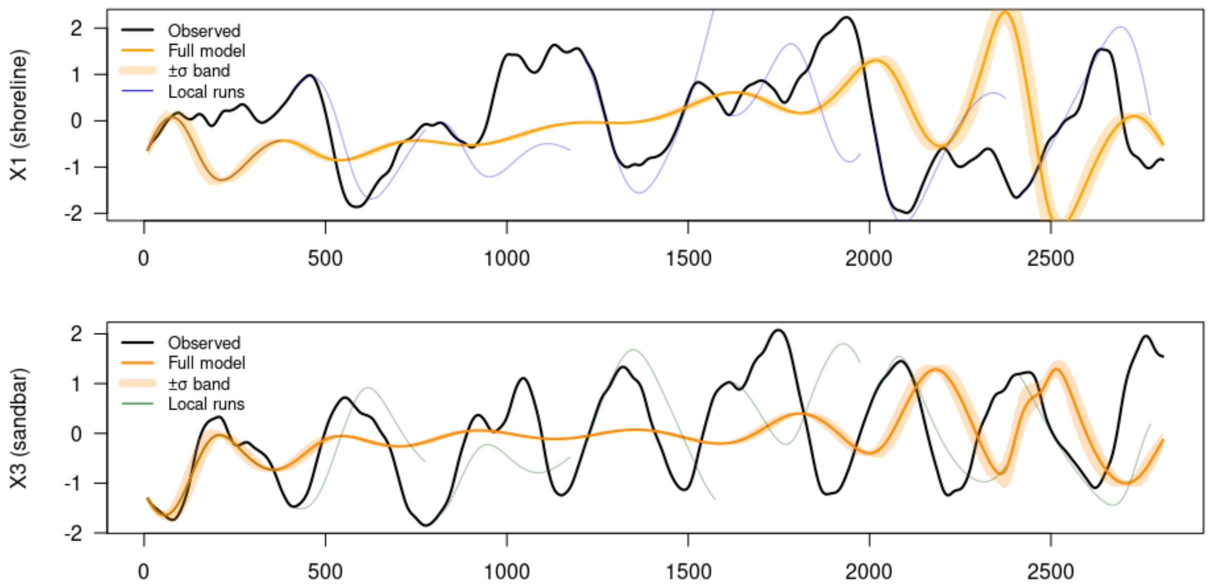

Figure S9. *Comparison between the observed time series (black) and model trajectories for X1 (shoreline, top) and X3 (sandbar, bottom).The orange line shows the full model integration over the entire time span, with the shaded ±σ band representing the local variability (computed over a 90-day moving window). Semi-transparent blue and green lines represent short model runs (365 days) initiated every 400 days from different observed conditions.*
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Duck

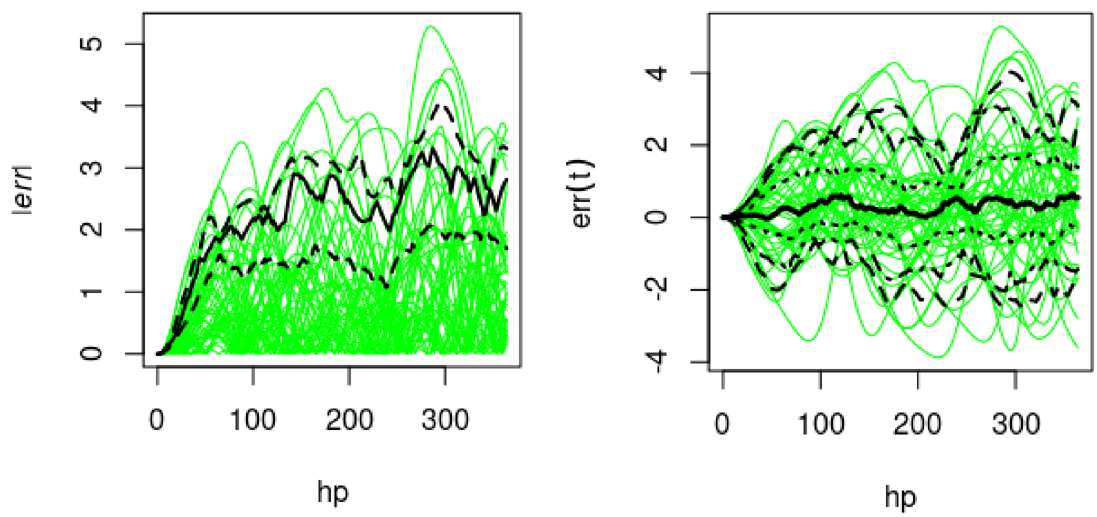

Figure S10. *Prediction error growth for Duck's model over a 365-day forecast horizon using the GPoM predictability framework.* ***Left:*** *absolute error magnitude ∣err∣ for 50 prediction runs initialized from different points along the observed trajectory.* ***Right:*** *raw error err(t) showing divergence patterns across runs.Thin green lines correspond to individual trajectories, while dashed black lines indicate the overall spread (minimum and maximum errors across all runs. The growth of the error envelope illustrates the nonlinear predictability limit and sensitivity to initial conditions.*

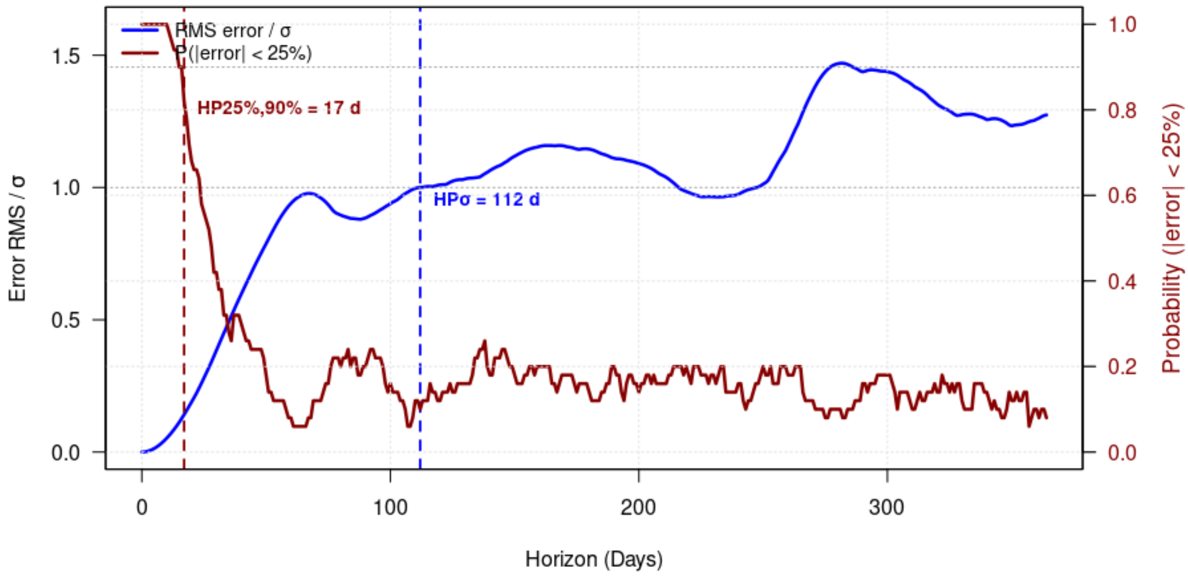

Figure S11. *Comparison of two predictability metrics of Duck's model. The blue curve shows the normalized root-mean-square (RMS) error relative to the signal standard deviation (σ). The red curve shows the probability P(|err|<25%) that predictions remain within a 25% relative error. Vertical dashed lines indicate the predictability horizon* HP_{σ} *(RMS/σ = 1) and the probabilistic horizon* $HP_{25\%,90\%}$ *defined as the time when 90% of runs exceed 25% relative error. The short horizon* $HP_{25\%,90\%}$ *= 17 days marks the loss of trajectory memory under stochastic forcing, while the longer* HP_{σ} *= 112 days shows that the model still captures the global structure and quasi-periodic shoreline–bar dynamics.*

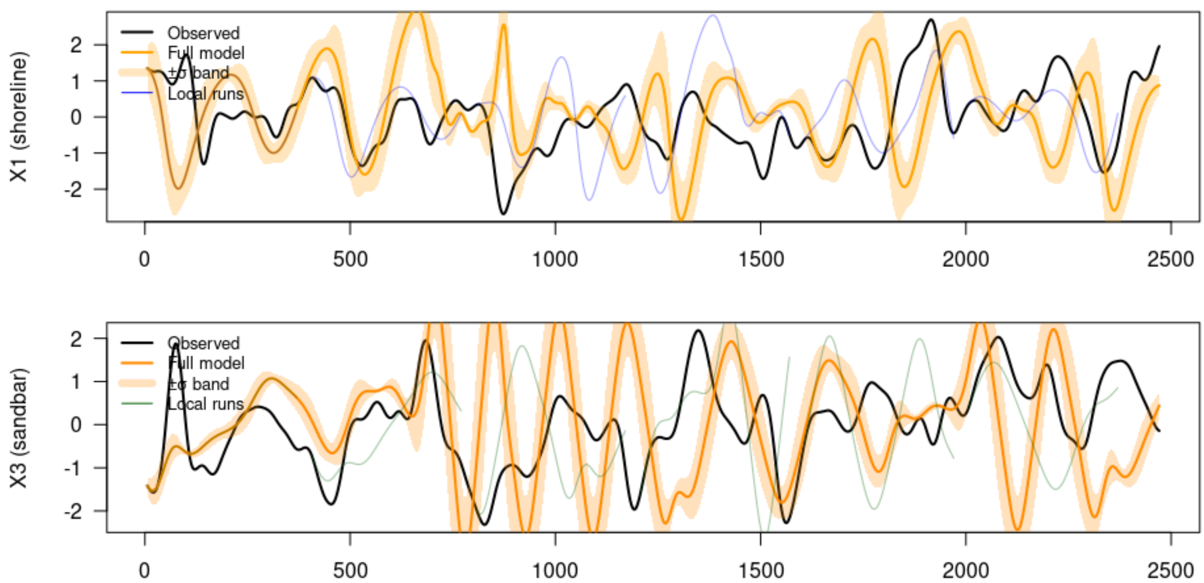

Figure S12. *Comparison between the observed time series (black) and model trajectories for X1 (shoreline, top) and X3 (sandbar, bottom).The orange line shows the full model integration over the entire time span, with the shaded ±σ band representing the local variability (computed over a 90-day moving window). Semi-transparent blue and green lines represent short model runs (365 days) initiated every 400 days from different observed conditions.*
The ensemble divergence highlights the local predictability range and temporal variability of model skill across the coupled shoreline–sandbar dynamics.