Responses to Referee 1 (Second Round):

We would like to thank the referee for the careful evaluation of our revised manuscript. We have carefully considered all the second round comments and revised the manuscript accordingly. The referee's comments are listed in *italics*, followed by our responses in blue. New or modified text in the manuscript is shown in **bold**.

The authors have carefully dealt with each of the two reviewer's comments, and the manuscript has been sufficiently improved. I recommend it to be published as a scientific paper after considering some minor comments for modification.

We sincerely thank the referee for thorough evaluation. We are encouraged that the referee considers the manuscript suitable for publication. Below, we provide point-by-point responses and detail the corresponding changes made to the manuscript.

Minor comments

1) Introduction, page 4, lines 90-94. *Although you have clearly explained the reason why you selected 7.8 μm absorption band is to avoid mixed solar-thermal radiance contributions in the revised manuscript, 4.4 μm band has stronger N_2O absorption than 7.8 μm band. It may be better to mention as like: "Although N_2O exhibits much stronger absorption features near 4.4 μm, the 7.8 μm region was selected to abode mixed solar-thermal radiance contributions ..."*

We have implemented the suggested change and revised the lines 91-93 as follows:

"Although N_2O exhibits much stronger absorption features near 4.4 μm, the 7.8 μm region was selected to avoid mixed solar–thermal radiance contributions and potential non-Local Thermodynamic Equilibrium (non-LTE) effects (Barnet et al., 2023)."

2) 3. Measurement Methodology *The authors use the term "measurement error" for uncertainties of both measurement spectra and retrieval quantities. It may be better to use "measurement error" for spectral noises, and uncertainties of state vectors through retrieval processes should be referred to as "retrieval errors" or "retrieval uncertainties".*

We thank the referee for this important comment. In this study, the reported measurement error represents the component of the retrieval uncertainty that is propagated from spectral noise through the gain matrix, rather than the total posterior retrieval error. Therefore, referring to this quantity simply as retrieval error could imply that all components of posterior uncertainty are included. To remove ambiguity, we have revised the manuscript to use the term "spectral error" when referring directly to radiance or detector noise. The term "measurement error" is retained for the propagated measurement-error component of X_{N_2O}. Specifically, the following changes have been made in the manuscript:

Section 3.1.1, line 182-183:

"Here, y is the expected values of a spectrum observed by a remote sensing instrument with Gaussian spectral error covariance matrix $\mathbf{S_o}$, ..."

Section 3.1.1, line 190:

"The spectral error covariance matrix $\mathbf{S_o}$ is constructed based on the instrument specification given in section 3.1.3."

Section 3.1.3, line 280-282

"The spectral error covariance matrix ($\mathbf{S_o}$) is assumed to be diagonal; in other words, the noise of individual detector pixels follows independent Gaussian distributions. The standard deviation of spectral error is calculated by multiplying each single-channel radiance r with SNR"

3) 3. Measurement Methodology, page 10, lines 235-238 *Assuming realistic N_2O variabilities as a priori leads to stronger constrain to the a priori and lower averaging kernel sensitivities. This is quite reasonable, and for this reason, detecting such smaller variabilities in N_2O concentrations by satellite-borne sensors is challenging, which requires observations with much higher signal-to-noise rations. The authors should mention this.*

We thank the referee for this valuable suggestion. We have added the a sentence to implement the suggestion. Line 237-238 is revised as follows now.

"This also implies that resolving the relatively small N_2O atmospheric variability from spaceborne instrument would require observations with high signal-to-noise ratio (SNR). Section 4.1 further evaluates the tradeoff between the a priori constraint and observational information by continuously scaling the prior strength."

References

Barnet, C. D., Smith, N., Ide, K., Garrett, K., & Jones, E. (2023). Evaluating the value of cris shortwave-infrared channels in atmospheric-sounding retrievals. *Remote Sensing*, *15*(3). Retrieved from `https://www.mdpi.com/2072-4292/15/3/547` doi: 10.3390/rs15030547