Responses to referee 2:

We would like to thank the referee for the useful comments and constructive suggestions. In the following, we address the referee's comments and describe the corresponding changes we have made to the manuscript. The referee's comments are listed in *italics*, followed by our response in blue. New/modified text in the manuscript is in **bold**.

General comments.

The authors present the detectability of column-integrated N_2O *mixing ratio* (X_{N_2O}) *using a joint SWIR-TIR setting. In the conventional approach for space-based* N_2O *observation, TIR signal is only used for retrieval. In contrast, SWIR signal is weak for* N_2O *retrieval and challenging to improve the accuracy and precision. Thus, the authors developed a joint SWIR-TIR retrieval framework and applied to both airborne and spaceborne instruments to evaluate the influence of platform characteristics on retrieval performance. This approach is quite attractive and innovate to retrieve* X_{N_2O}*, precisely. I understand that this activity is important to assess, improve and extend the* X_{N_2O} *observation for the future remote sensing based observatory. However, some description and assumption are unclear or missing in the text.*

We thank the reviewer for the careful reading of the manuscript and for recognizing the novelty and potential of proposed joint SWIR–TIR retrieval framework for X_{N_2O} detectability. We appreciate the reviewer's positive assessment and agree that several assumptions underlying the instrument design and observational framework were not sufficiently explicit in the original manuscript. In the revised manuscript, we have clarified these assumptions and strengthened the justifications.

The authors describe the stronger N_2O *features are found in 4.4 or 7.8* μ*m spectral region. However, the* N_2O *absorption lines in 7.8* μ*m spectral regions are only considered for a joint SWIR-TIR retrieval. The retrieval precision and accuracy of* X_{N_2O} *is also affected by the spectral resolution, spectral coverage and spectral windows. However, it is not clear how to identify the designed instrument (spectral coverage, spectral resolution and spectral windows). I understand the designed instrument is based on the current technology, however, at least, spectral resolution and selected windows (combined spectral region between SWIR and TIR) have to be assessed.*

We thank the reviewer for this important comment. Although N_2O has absorption features near both 4.4 and 7.8 μm, the present study focuses on the 7.8 μm region for the TIR component of the joint SWIR–TIR framework. The 7.8 μm region lies firmly within the thermal infrared regime, where the observed radiance is dominated by Earth-emitted thermal radiation and can therefore be treated as a cleaner thermal emission retrieval problem. In contrast, the 4.4 μm region lies near the transition between the solar-reflected shortwave and thermally emitted longwave regimes, resulting in mixed radiance contributions from reflected solar and thermal emission components. Retrievals in this spectral region therefore require more complex treatment of solar reflectance, thermal emission, atmospheric scattering, and potential non-local thermodynamic equilibrium (NLTE) effects (Barnet et al., 2023). We have revised line 89-90 to add the following explanation of using 7.8 μm instead of 4.4 μm for TIR band.

"The TIR N_2O band at 7.8 μm is selected to provide crucial observational constraints to X_{N_2O}, additional to the weak SWIR band. Although N_2O also exhibits strong absorption features near 4.4 μm, the 7.8 μm region was selected as it avoids mixed solar-thermal radiance contributions and potential non-Local Thermodynamic Equilibrium (non-LTE) effects (Barnet et al., 2023)."

The key instrument design parameters, including spectral coverage and spectral resolution, were selected through iterative assessment of N_2O absorption strength and instrument performance using linear sensitivity analysis in concert with the current technology and industry standards, for both airborne and spaceborne instruments. Different spectral sampling and spectral coverage configurations were evaluated to identify combinations that provide the required performance while remaining achievable within current infrared spectrometer technology. The adopted spectral resolutions are also broadly consistent with existing instruments. In addition, a preliminary optical design assessment has been performed for the proposed TIR instrument, including the grating requirements and optical train. The grating is the component most strongly affected by the high spectral sampling, and a manufacturability assessment indicates high confidence that such a grating can be produced. The following is added to line 228 of the original manuscript:

"The key instrument design parameters, including spectral coverage and spectral resolution, are selected through iterative assessment of N_2O absorption strength and instrument performance using linear sensitivity analysis in concert with the current technology and industry standards for both airborne and spaceborne instruments. A Gaussian instrument spectral response function (ISRF) is assumed with a full width at half maximum (FWHM) spanning three spectral sampling intervals ($d\lambda$). This corresponds to spectral resolutions of approximately 0.1725 nm (0.33 cm^{-1}) in the SWIR band for both airborne and spaceborne instruments. In the TIR band, the spectral resolutions are 0.90 nm (0.14 cm^{-1}) and 0.75 nm (0.12 cm^{-1}) for the airborne and spaceborne instruments, respectively. These spectral resolutions are within the range of existing and proposed trace-gas remote sensing instruments operating in the SWIR and TIR spectral regions (Bowman et al., 2006; Ricaud et al., 2021; Nakajima et al., 2012). In addition, a preliminary optical design assessment has been performed for the proposed TIR instrument, including the grating requirements and optical train. The grating is the component most strongly affected by the high spectral sampling, and a manufacturability assessment indicates high confidence that such a grating can be produced."

In addition, I understand the observation geolocation between CrIS soundings (Fig.1, observed 23 Aug. 2023) and MAIZE campaign (Fig. 2, May.2022) have different geolocation *and observation time. It is unclear why do the authors not extract and/or merge CrIS data from the same time and location with MAIZE. It is also unclear that the land type is equal or not between CrIS soundings region (Fig.1) and MAIZE campaign region (Fig.2). The authors have to explain the acceptance to combine the different observation condition between MAIZE and CrIS data. For these reasons, I recommend this paper for publication with minor changes to the technical content.*

We thank the reviewer for bringing this up. The CrIS and MAIZE datasets are not intended to represent coincident observations, but rather provide complementary information representative of summertime agricultural conditions over the US Midwest. The CrIS soundings provide atmospheric and surface states for the radiative transfer simulations and linear sensitivity analysis, including profiles of temperature, H_2O, CH_4, and N_2O, along with surface temperature, emissivity, and thermal contrast. In contrast, the MAIZE campaign is used *independently* to characterize empirical boundary-layer N_2O spatial variability through semivariogram analysis. Although the CrIS and MAIZE observations differ in observation time and exact location, both datasets sample agricultural regions of the US Midwest dominated primarily by corn and soybean croplands during summertime conditions favorable for elevated N_2O emissions. The objective of this study is therefore not direct scene-by-scene comparison, but rather the development of a sensitivity and detectability framework constrained by realistic environmental and observational variability representative of Midwest agricultural conditions. To implement the response, we have added the following to line 114:

"Although the CrIS and MAIZE datasets are not colocated in time and space, both are representative of summertime agricultural conditions over the US Midwest dominated primarily by corn and soybean croplands. These datasets are used as complementary constraints for realistic environmental variability rather than for direct scene-by-scene comparison."

Specific comments.

Abstract

1. Page 1, line 5: Spell out "SPLAT-VLIDORT".

Revised as suggested.

2. Data

2. Page 4, line 109: I understand the standard definition of SWIR is covering between 900nm to 2500 nm. The spectral coverage of CrIS is 3.9 to 15.4 µm. Then, the wording of SWIR is not preferred. Mid infrared (MIR) is preferred instead of SWIR.

In this study, CrIS is not used as a SWIR observing system, nor do we use the radiance from CrIS directly. Rather, CrIS is used only to provide realistic atmospheric and surface state quantities (e.g., temperature, H_2O, CH_4, and N_2O profiles, surface temperature, and emissivity) that constrain the radiative transfer simulations and prior covariance assumptions for H_2O and temperature. These quantities are associated with the thermal infrared and are used to the construct realistic environmental and prior-state conditions for the linear sensitivity analysis. To clarify this point, we have revised line 108-111 to the following:

"First, satellite-retrieved atmospheric state data from the Cross-track Infrared Sounder (CrIS) provide surface temperature, profiles of key absorbers (N_2O, CH_4, and H_2O) and temperature, along with the surface emissivity used for the TIR band simulations. For SWIR, a representative grass albedo from the Terra Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) climatology is assumed, with values ranging from 0.146 to 0.171 over the selected SWIR spectral window. These quantities define the atmospheric and surface conditions for forward radiative transfer modeling (Section 3.1.4) in the SWIR and TIR spectral regions and allow the estimation of prior error structures (Section 3.1.2)."

2.1 Realistic geophysical quantities from CrIS Level 2 product

3. Page 5, line 120: Why the authors select 23 August 2023? In addition, how about the variation of surface albedo at 2.3 μm and land use type for interesting region in Fig.1?

We thank the reviewer for this important question. The CrIS soundings on 23 August 2023 were not selected to represent a specific event, but rather to provide a realistic ensemble of summertime atmospheric and surface states over the US Midwest for the sensitivity analysis. The data for this date were readily accessible and provide a large number of clear-sky CrIS soundings with a wide range of thermal contrast values (2–12 K), which is important for TIR retrieval sensitivity. We have added the following in line 120 to emphasize this point.

"The selected date provides a large number of clear-sky soundings spanning a wide range of thermal contrast (2–12 K), providing a representative sample of US Midwest summertime conditions for evaluating retrieval sensitivity."

For the SWIR simulations, we assume a representative grass surface albedo from ASTER. The reflectance over the selected SWIR spectral window ranges from 0.146 to 0.171. In addition, the selected region in Fig. 1 is dominated by agricultural land cover, primarily corn and soybean croplands, making the assumed vegetated surface reflectance representative of the study domain. We have already given the surface albedo values in response to specific comment 2. For land use type, we have added the following in line 122 of the manuscript.

"The selected region in Fig. 1 is dominated by agricultural land cover, primarily corn and soybean croplands."

3.1 Theory

4. Page 9, line 206: How many layers do you treat in the radiative transfer calculation? The simulation condition is unclear. The authors have to explain the simulation conditions for radiative transfer

Thanks for bringing this up. All forward radiative transfer simulations are performed on a 19-layer pressure grid extending from the surface to the top of atmosphere, throughout the linear sensitivity analysis. Elements included in the state vector for radiative transfer simulations are surface temperature and profiles of H_2O, CH_4, N_2O, and temperature. The solar zenith angle is kept at 30° and viewing zenith angle at 0°. For SWIR, band grass albedo is used from ASTER and for TIR band surface emissivity is taken from CrIS L2 product. To provide information about simulation conditions we have added the following to line 283 of the manuscript.

"All radiative transfer simulations are performed on a 19-layer pressure grid extending from the surface to the top of atmosphere. The simulations use a solar zenith angle of 30°, viewing zenith angle of 0° and assume clear-sky conditions using the SPLAT–VLIDORT radiative transfer model and HITRAN2020 spectroscopy. Surface emissivity from CrIS L2 product is used for TIR band and representative grass surface albedo from ASTER is used for SWIR band. The state vector include profiles of N_2O, CH_4, H_2O, and temperature, along with surface temperature."

3.1.3 Instrument design parameters and observational constraints

5. Page 12, line 256: What condition do you select as a typical sounding? Generally, solar zenith angle, surface albedo, aerosol condition and surface temperature are key parameters for signal intensity both SWIR and TIR. In addition, the spectral resolution of Fig.5 is not clear. Then, clear explanation for calculation condition is needed.

The "typical sounding" used in Fig. 5 corresponds to a representative daytime clear-sky summertime atmospheric state selected from the 100 CrIS soundings over the US Midwest. The simulations use a solar zenith angle of 30°, nadir viewing geometry (VZA = 0°), representative SWIR grass surface albedo of 0.159 (mean), TIR band surface emissivity of 0.98 from CrIS, surface temperature of 307.3 K and a thermal contrast of 4.6 K. Aerosols are not included in the scope of current work.

The SWIR simulations use a spectral sampling interval of 0.0575 nm with a 3-sample Gaussian ISRF, corresponding to an effective spectral resolution of 0.1725 nm (0.33 cm^{-1}). For spaceborne instrument the TIR simulations use a spectral sampling interval of 0.25 nm with a 3-sample Gaussian ISRF, corresponding to an effective spectral resolution of 0.75 nm (0.12 cm^{-1}). We have revised line 256-257 of the manuscript to following to mention these simulation conditions.

"Figure 5 (top row) shows the simulated radiance spectra of a daytime clear-sky summertime sounding selected from the CrIS ensemble over the US Midwest, at the spectral ranges and resolutions of the spaceborne instrument, with and without N_2O, using the radiative transfer model detailed in Section 3.1.4. The simulation assumes a a solar zenith angle (SZA) of 30°, nadir viewing geometry, representative SWIR grass surface albedo of 0.159, TIR surface emissivity of 0.98, surface temperature of 307.3 K and thermal contrast of 4.6 K from the selected CrIS sounding. The spectral resolutions of the simulated spectra are 0.1725 nm for the SWIR band and 0.75 nm for the TIR band, corresponding to three times the spectral sampling intervals."

We also added the following line to Figure 5 caption to mention the spectral resolution values.

“The simulated SWIR and TIR spectra correspond to spectral resolutions of approximately 0.1725 and 0.75 nm, respectively.”

3.1.4 Radiative transfer simulation

6. Page 13, line 285: It is unclear that “a typical Midwest summer environmental conditions”. The authors have to add clear explanation.

We thank the reviewer for these comments. The phrase “typical Midwest summer environmental conditions” refers to clear-sky summertime atmospheric and surface conditions representative of the US Midwest agricultural region, derived from the selected CrIS soundings. These conditions are described in response to comment 5 and they are consistent for the Fig. 6. We have revised the lines 284-285 of manuscript to the following:

“Figure 6 shows the Jacobians of the SWIR (left column) and TIR bands (right column) with respect to atmospheric N_2O, CH_4, H_2O, and temperature profiles simulated by SPLAT-VLIDORT using profiles from a representative CrIS sounding and HITRAN2020 spectroscopy. The simulation conditions are same as described for Fig. 5 in Section 3.1.3”

7. Page 14, Figure 6: How many layers consider in this calculation?

The Jacobian calculations shown in Fig. 6 are performed using the same 19-layer atmospheric pressure grid adopted throughout the radiative transfer simulations and linear sensitivity analysis. To mention this, we have revised the line 286 to the following:

“These Jacobians are calculated using the 19-layer atmospheric pressure grid, convolved with the ISRF and sampled at the spectral intervals of the spaceborne instrument specified in Table 1.”

4 Results

8. Page 17, Figure 7: “near-surface layers” is unclear. I understand the original output from calculation is “pressure height”. Then, the describing with pressure height is needed. For the measurement error estimation, I understand solar zenith angle, surface albedo, and surface temperature for each sounding are considered. At least, the mean values for those parameters have to address in the text as well.

The near-surface layers sensitivity shown in Fig. 7 refers to the mean column averaging kernel value of the lowest two atmospheric layers. These lowest two layers correspond approximately to pressure levels of 962 and 901 hPa, representing the lowest ~ 1 km of the atmosphere within the planetary boundary layer. We have added following in line 355 to define near-surface sensitivity.

“The near-surface sensitivity here is defined as the mean averaging kernel value of the lowest two atmospheric layers, representing the lowest $\sim$ 1 km of the

atmosphere (corresponding pressure values are 962 and 901 hPa).”

For the measurement error calculations shown in Fig. 7, the environmental conditions for each CrIS sounding are individually considered, including surface temperature, thermal contrast, and TIR surface emissivity, while a fixed SZA of 30° and representative SWIR grass surface albedo ranges from 0.146 to 0.171 over the selected spectral window. The SZA, VZA and albedo have already been mentioned earlier in response to comment 4 and 5. Across the 100 selected CrIS soundings, the surface temperature ranges from 302.8 to 318.8 K with a mean value of 308.8 K, while the thermal contrast ranges from 2.8 to 12.3 K with a mean value of 6.6 K. Information about surface temperature and thermal contrast of 100 soundings has been added to line 359 as following:

“Across the 100 selected CrIS soundings, the surface temperature ranges from 302.8 to 318.8 K (mean: 308.8 K), while the thermal contrast ranges from 2.8 to 12.3 K (mean: 6.6 K).”

References
9. Page 31, line 692: change the order of the Spurr et al’ s list. Now, Spurr et al, 2006 is last the list.

Thanks for noting this. It has been revised.

References

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