## Review Comments to the Authors

The manuscript proposes a multi-objective fuzzy seismic inversion framework for acoustic impedance estimation from post-stack seismic data. The method is designed to integrate conventional inversion constraints with clustering information derived from well logs or geological prior knowledge. The study is relevant and potentially valuable. In particular, the generation of membership sections may provide an additional interpretation attribute if its geological meaning can be validated.

Nevertheless, the manuscript requires substantial improvement before publication. The main issues concern the clarity of novelty, mathematical consistency, parameter selection, and validation. In particular, the synthetic model used in the current manuscript is too simple to convincingly demonstrate the claimed robustness and interpretability of the method. A more complex benchmark test is needed.

### Major comments

#### 1. Novelty should be more clearly defined

The idea of using fuzzy clustering or petrophysical clustering constraints in geophysical/seismic inversion has been reported in previous studies. The authors should clearly explain what is new in the present work compared with existing fuzzy clustering constrained inversion and guided fuzzy clustering inversion methods. The novelty should not be stated only as "using fuzzy clustering"; instead, the authors should specify whether the main contribution lies in the multi-constraint objective function, the windowed multi-trace implementation, the membership sections, or the proposed weighting strategy.

The literature review would also benefit from a more balanced discussion of related sparse, envelope-based, basis-pursuit, and broadband seismic inversion methods. For example, the authors may consider discussing Yang et al. (2024), Yin et al. (2015), Yang et al. (2022), and Zong et al. (2018), which are relevant to sparse regularization, basis pursuit inversion, low-frequency recovery, and broadband seismic inversion. This would help position the proposed multi-constraint inversion framework more clearly within existing seismic inversion research.

#### 2. Some key equations and definitions appear inconsistent

The derivation from acoustic impedance to reflectivity and log-impedance needs to be checked carefully. In Eq. (2), if x is defined as log(z), then the factor of 1/2 must be retained in the reflectivity expression. If x is defined as 0.5 log(z), this should be stated explicitly. Eq. (3a) also appears problematic because a local difference is equated to a cumulative summation of reflectivity. The authors should rewrite Eqs. (2)-(4) with consistent definitions of z, x, r, D, and H.

The smoothness terms also require clarification. Some formulations combine vertical and horizontal derivatives directly, but these derivatives have different sampling intervals and physical units. The role and scaling of the parameter alpha should therefore be explained.

#### 3. The objective-function weighting and L2-L1 formulation need a more rigorous explanation

The normalization factors and weights in Eq. (14) are not fully justified. Setting the weights to sum to one does not automatically solve the regularization-parameter selection problem. The authors should provide sensitivity tests or objective criteria for choosing the weights. In addition, the sparsity term appears to have no independent weighting factor, which makes its relative contribution unclear.

The transformation from Eq. (14) to Eq. (15) should be derived more explicitly. If weighted squared norms are stacked into a single least-squares system, the matrix coefficients should normally involve the square roots of

the weights and normalization factors. The authors should verify whether the current formulation uses w or sqrt(w), and whether the dimensions of all matrices are consistent.

### 4. The inversion-clustering coupling is nonlinear and should be described as an alternating optimization

The memberships and centroids depend on the current impedance model, while the impedance model is estimated using the clustering term. Therefore, the problem is not a simple fixed L2-L1 inversion unless the memberships and centroids are held constant within each iteration. The authors should clearly describe the alternating optimization process, convergence criteria, stopping conditions, and computational cost.

### 6. Synthetic validation should be strengthened using a more complex benchmark model

The synthetic model used in the manuscript is relatively simple and contains limited structural and stratigraphic complexity. Such a simple model is useful for illustrating the workflow, but it is not sufficient to support strong claims about robustness, high resolution, reduced dependence on the initial model or wavelet, and the interpretability of membership sections. I suggest that the authors add a more challenging synthetic test based on a well-known complex benchmark model, such as the Marmousi or Marmousi2 model.

A Marmousi-type test would provide a more convincing evaluation because it contains stronger lateral heterogeneity, complex structures, and more realistic impedance contrasts. The authors could compare the proposed method with conventional model-based inversion or sparse inversion, and report impedance error maps, trace-by-trace comparisons, sensitivity to noise/wavelet errors, and the behavior of the membership sections under complex geological conditions.

## Minor comments

- Please correct typographical and grammatical errors, for example “Department op” should be “Department of”, “try and error” should be “trial and error”, and “Euclidian” should be “Euclidean”.
- Please check all equation references. For example, the text appears to refer to Eq. (7b) as a smoothness term, but Eq. (7) is the K-means objective function.
- Please define all symbols before use and keep notation consistent, especially u, v, O, C, M, H, Dv, and Dh.
- Please provide more details about the statistical wavelet estimation, the initial model construction, and the inversion window size in the F3 case.
- Some claims in the abstract and conclusion should be moderated unless stronger evidence is added, particularly those related to denoising, deconvolution, wavelet insensitivity, and layer-content prediction.
- The figures should be improved where possible. Axis labels, color bars, and legends should be readable, and the membership-section figures should include axes or at least clear spatial/time references.
- Please check the reference format and journal names. For example, the citation information for Sun and Li should be formatted consistently with the target journal style.
- Please consider adding and discussing relevant references on sparse and broadband seismic inversion, including: Yang, S., G. Wu, J. Shan, and H. Liu, 2024, Prestack sparse envelope seismic inversion method adopting the L0-L2-norm regularization: Interpretation, 12(2), T149-T166; Yin, X., X. Liu, and Z. Zong, 2015, Pre-stack basis pursuit seismic inversion for brittleness of shale: Petroleum Science, 12, 618-627; Yang, S., G. Wu, J. Shan, and H. Liu, 2022, Multi-scale seismic envelope inversion method based on sparse representation theory: Journal of Applied Geophysics, 203, 104685; Zong, Z., Y. Wang, K. Li, and X. Yin, 2018, Broadband seismic inversion for low-frequency component of the model parameter: IEEE Transactions on Geoscience and Remote Sensing, 56, 5177-5184.