Seismic inversion of the Marmousi synthetic section

To evaluate the performance of the fuzzy inversion methods presented in the manuscript, they were applied to the well-known synthetic Marmousi dataset. The Marmousi acoustic impedance model is shown in Figure 1a. A temporal sampling interval of 4 ms was used, and the seismic section was generated using a Ricker wavelet with a dominant frequency of 30 Hz. The Ricker wavelet is widely used in synthetic seismic processing because of its zero-phase characteristic, band-limited nature, mathematical simplicity, and ability to approximate a seismic source, making it suitable for modeling, inversion, and educational purposes (Gholamy and Kreinovich, 2014).

The seismic section shown in Figure 1b was obtained by convolving the Ricker wavelet with the reflection coefficients derived from the Marmousi model. Gaussian noise was then added to the seismic section, resulting in a signal-to-noise ratio (SNR) of 1 dB.

Model-based seismic inversion requires a low-frequency background model of the subsurface. In this synthetic example, the background model was generated from the Marmousi model using a Gaussian-weighted moving average filter (Figure 2a). A moving average is a smoothing technique that replaces each data point with the average of neighboring values within a specified window, thereby reducing local fluctuations and emphasizing broader trends.

The model-based inversion was then applied to the synthetic seismic section shown in Figure 1b using this initial model, and the resulting acoustic impedance model is presented in Figure 2b. Because the objective function of model-based inversion includes a smoothness constraint, thin layers are not well resolved in the inverted model. In addition, the range of acoustic impedance values in the inverted result is narrower than that of the true model (Figure 1a).

Numerous studies and methods have been proposed to improve and optimize model-based seismic inversion. However, since these approaches are beyond the scope of the present research, only the general formulation of the method is considered here.

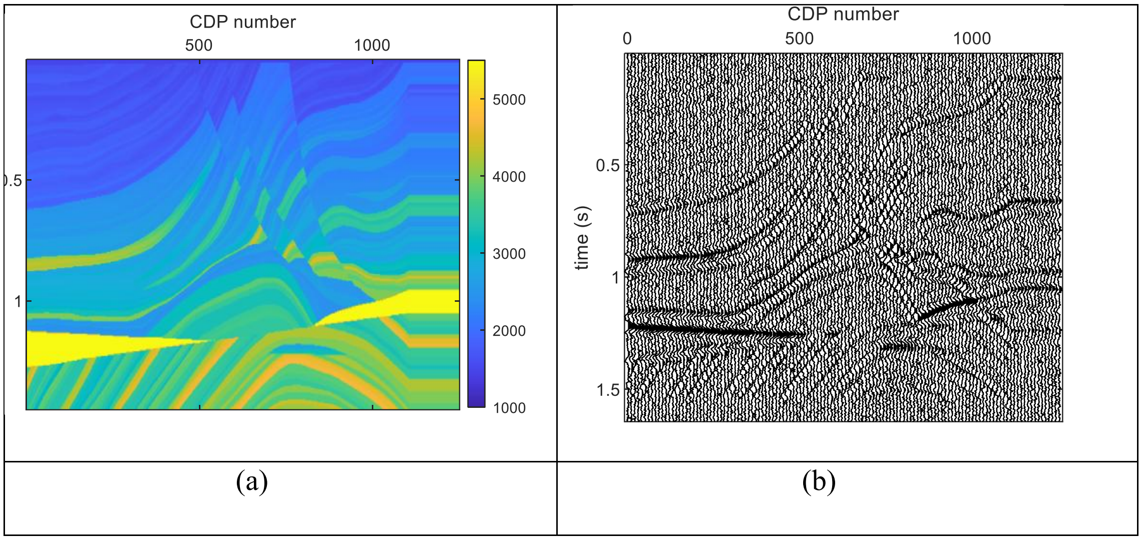


Figure 1: (a) The synthetic Marmousi acoustic impedance model, characterized by complex geological structures and a wide range of acoustic impedance values. (b) The seismic section generated from the model in (a) using a Ricker wavelet with a dominant frequency of 30 Hz. Gaussian noise was added to the seismic data, resulting in a signal-to-noise ratio (SNR) of 1 dB.

The fuzzy constraint used in the objective function of fuzzy seismic inversion requires prior clustering information about the model. This information can be extracted from available data. For the synthetic model, well-log and geological data are not separately available. Therefore, fuzzy clustering was performed on the acoustic impedance model at random locations corresponding to 250, 500, and 1000 seismic traces to estimate the number of clusters, the initial cluster centers, and their membership values. The fuzziness exponent, (q), was set to 2. Further details are provided in the following section.

Using the initial model shown in Figure 2a and eight cluster centers, fuzzy seismic inversion was applied to the synthetic seismic section of Figure 1b. The resulting acoustic impedance model is presented in Figure 2c. Despite the relatively high noise level in the seismic data, the inverted model exhibits better vertical and lateral continuity than the result obtained from conventional model-based inversion. It should be noted that a two-dimensional smoothness constraint, applied in both the lateral (offset) and vertical (time) directions, was used in this inversion.

Because the number of clusters is small relative to the number of geological layers, units with similar acoustic impedance values are grouped into the same cluster. This behavior is reasonable for real seismic data, where sequences of layers often share similar petrophysical properties. In effect, the clustering process groups geological units with comparable characteristics. Although the resulting model preserves the main geological trends, its stratigraphic resolution remains lower than that of the true model, and thin layers cannot be clearly distinguished.

The inversion result could be further improved by incorporating additional prior information, such as interpreted horizons, major layer boundaries, and a larger number of clusters in the model.

The final seismic inversion method investigated in this study is the multi-constraint fuzzy seismic inversion (MFSI) approach. Similar to the FSI method, the Marmousi seismic section was inverted using the same initial background model and prior fuzzy clustering information. The resulting acoustic impedance model is shown in Figure 2d.

Compared with the previous methods, the MFSI result exhibits reduced dependence on the initial model and improved continuity in both the vertical and lateral directions. Thin layers are more clearly resolved, indicating a higher level of stratigraphic detail. In addition, the range of acoustic impedance values in the inverted model is comparable to that of the true Marmousi model, demonstrating a more accurate reconstruction of subsurface properties.

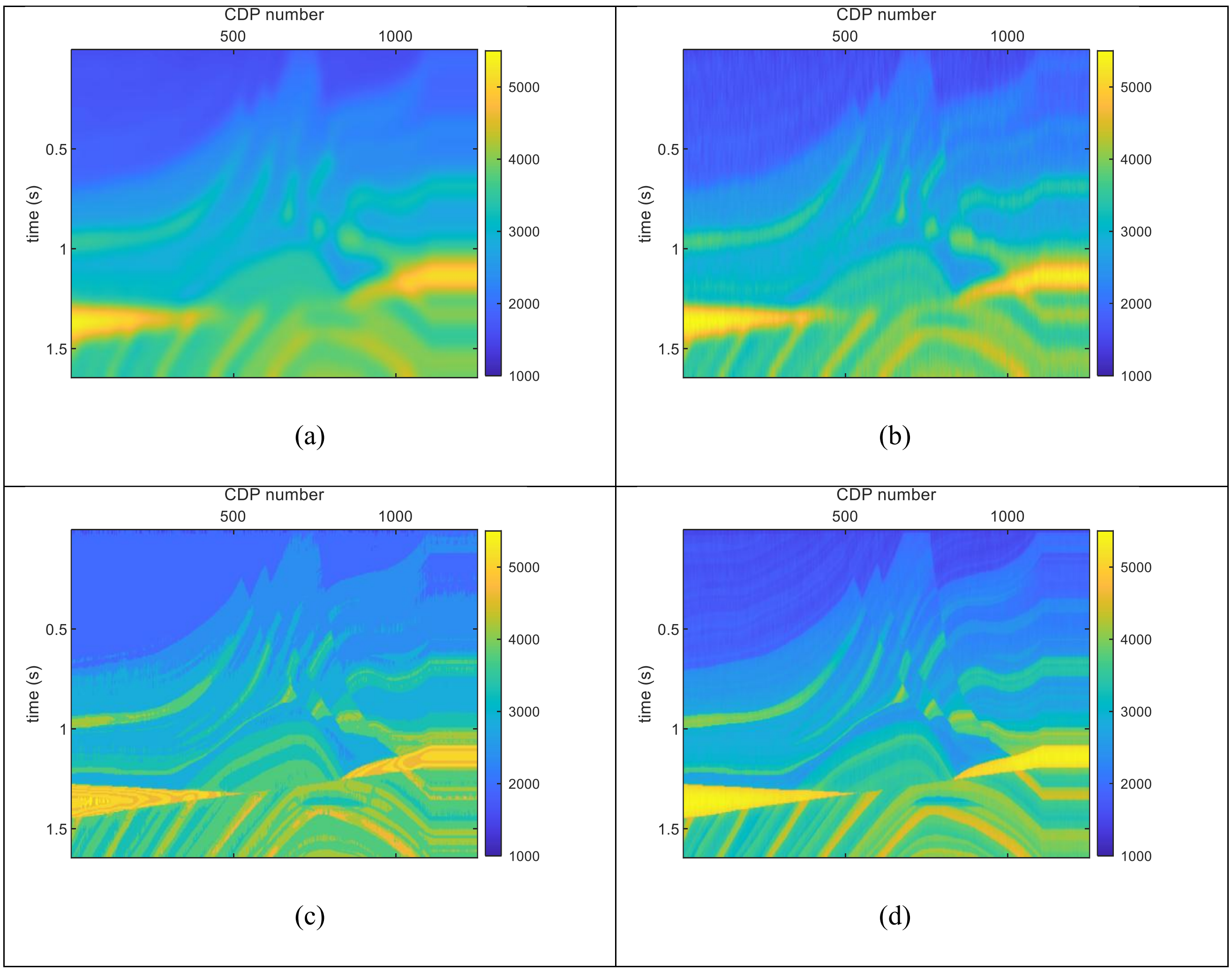


Figure 2: Seismic inversion results for the Marmousi seismic section. (a) Low-frequency initial model. (b) Model-based inversion result. (c) Fuzzy Seismic Inversion (FSI) result. (d) Multi-Constraint Fuzzy Seismic Inversion (MFSI) result.

Figure 3 compares the inversion results of the three methods with the initial model and the true model at common midpoint (CMP) 700. Model-based seismic inversion produces a smooth acoustic impedance profile that remains strongly influenced by the initial model. Although the major layers can be identified, both the impedance values and their variations differ from those of the true model.

The FSI results also show a significant dependence on the initial model, with the inverted impedance values distributed around the background model. The fuzzy constraint forces the impedance values within each cluster to remain nearly constant, resulting in little variation within a cluster. Consequently, the number of layers that can be resolved by this method depends on the number of predefined clusters. Improving layer resolution therefore requires increasing the number of clusters.

Among the three methods, MFSI produces acoustic impedance values that are closest to those of the true model. The influence of the initial model is less pronounced, indicating a reduced dependence on the starting solution. In contrast to FSI, the limited number of clusters has a smaller effect on the number of resolved layers, and most of the layers present in the true model are successfully reproduced in the MFSI result.

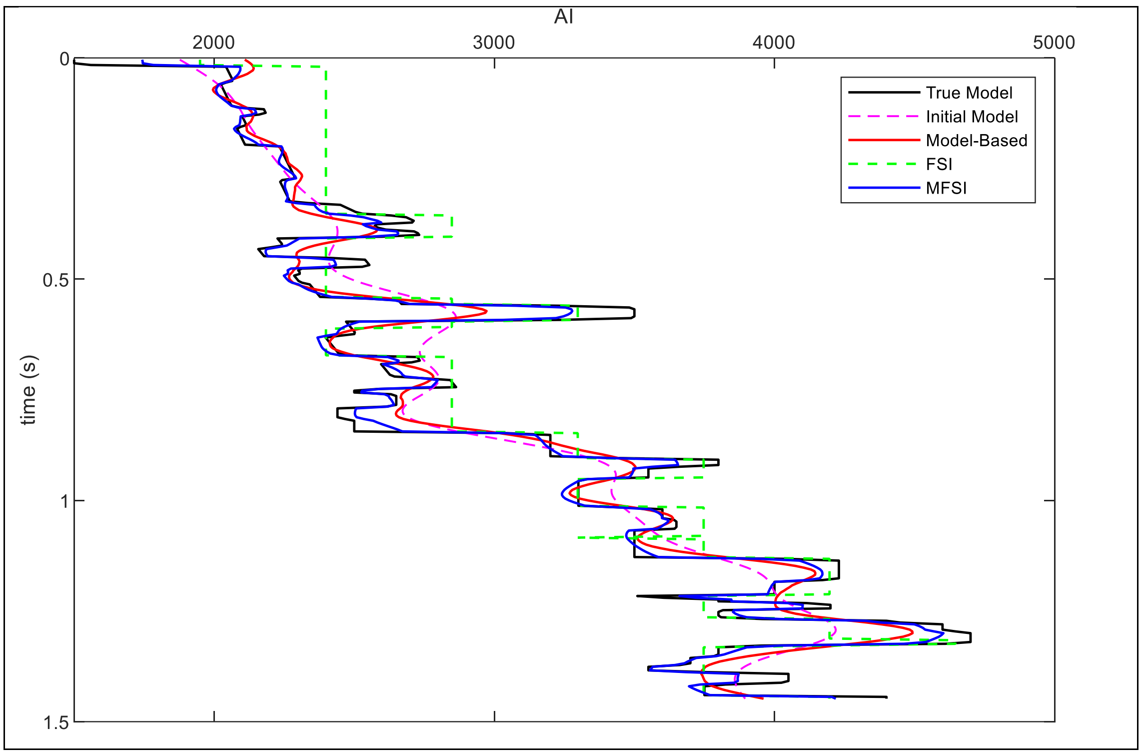


Figure 3: Comparison of the acoustic impedance profiles obtained from different seismic inversion methods at CMP 700 of the Marmousi dataset.

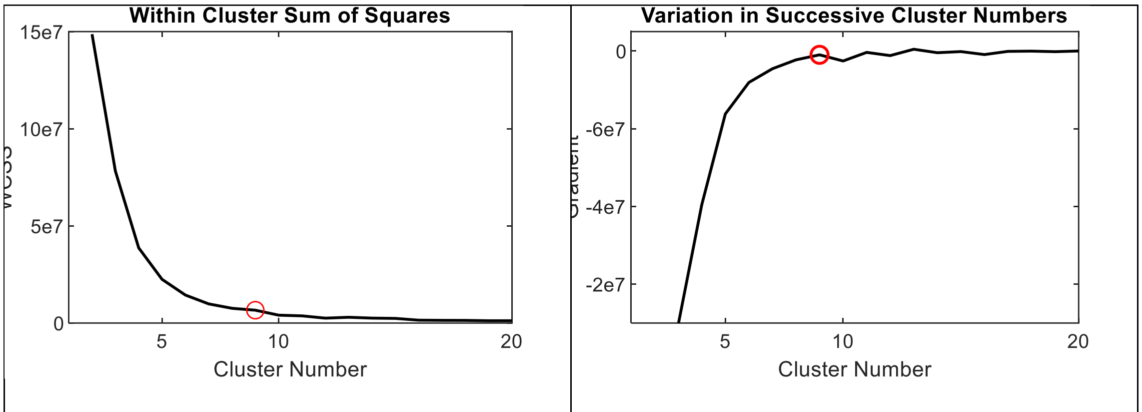
Selection of the Number of Clusters

To determine the optimal number of clusters, several validity measures such as the partition coefficient, classification entropy, the Dunn index, and other methods were considered. However, for brevity, only the results of three additional approaches are discussed here.

The first method, the within-cluster sum of squares (WCSS), evaluates the fuzzy clustering objective function (Equation 4.2) for different numbers of clusters. When the number of clusters exceeds the optimal value, further increases in cluster number lead to only minor changes in the objective function compared to previous cases. This approach generally performs better for hard clustering; however, in fuzzy clustering—where each data point belongs to multiple clusters with different membership degrees—it does not produce a clear “elbow” point.

The WCSS curve was evaluated at CMP locations 500 and 1000 of the Marmousi model, and the results are shown in Figure 6a. As the number of clusters increases, WCSS decreases. To better understand its variation, the difference between successive WCSS values was also computed and is shown in Figure 6b. It can be observed that the slope of the changes varies in three stages: from 2 to 6 clusters, the decrease is rapid; from 6 to 8 clusters, the rate of change slows down; and beyond 8 clusters, it becomes nearly constant.

Given the large number of layers in the model and the presence of thin beds, 8 clusters are selected as the optimal number for fuzzy seismic inversion in this study.



(a) (b)

Figure 4: (a) Within-cluster sum of squares (WCSS) for different numbers of clusters. (b) Change in WCSS between consecutive cluster numbers.

The second method is the Silhouette index (SI), which is computed by comparing the average distance of a data point to other points within its assigned cluster with the average distance to points in the nearest neighboring cluster. The Silhouette values for different data points and for cluster numbers ranging from 2 to 8 are shown in Figure 5a. Values close to 1 indicate that the point is well separated from neighboring clusters, values near 0 indicate that the point lies on the boundary between two clusters, and values approaching −1 suggest that the point has been assigned to the wrong cluster.

Since this method is originally defined for hard clustering, the average Silhouette values for different numbers of clusters (dashed line in Figure 5b) indicate an optimal number of 10 clusters in this case. However, when membership degrees are incorporated into the computation, an adapted fuzzy version of the Silhouette index proposed by Campello and Hruschka (2006) yields different results (solid line), indicating an optimal number of 9 clusters.

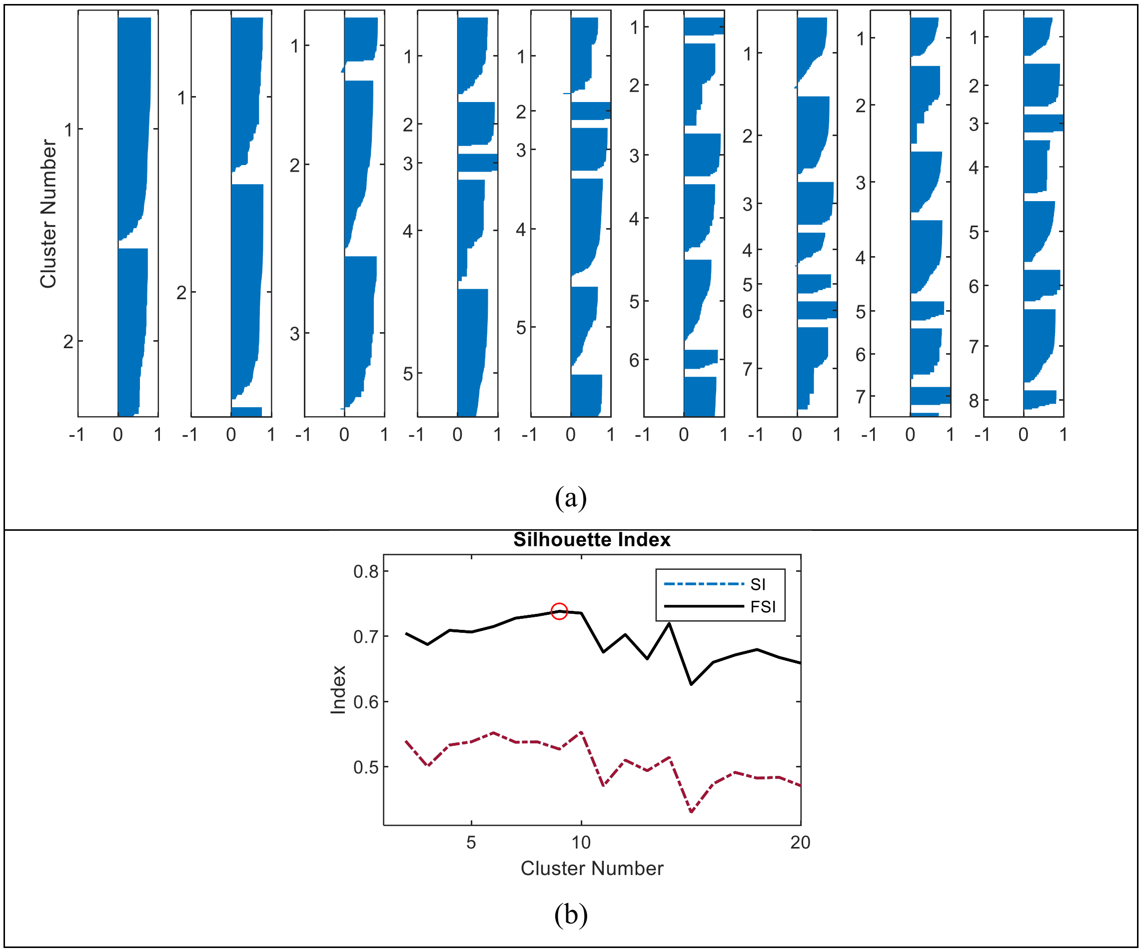
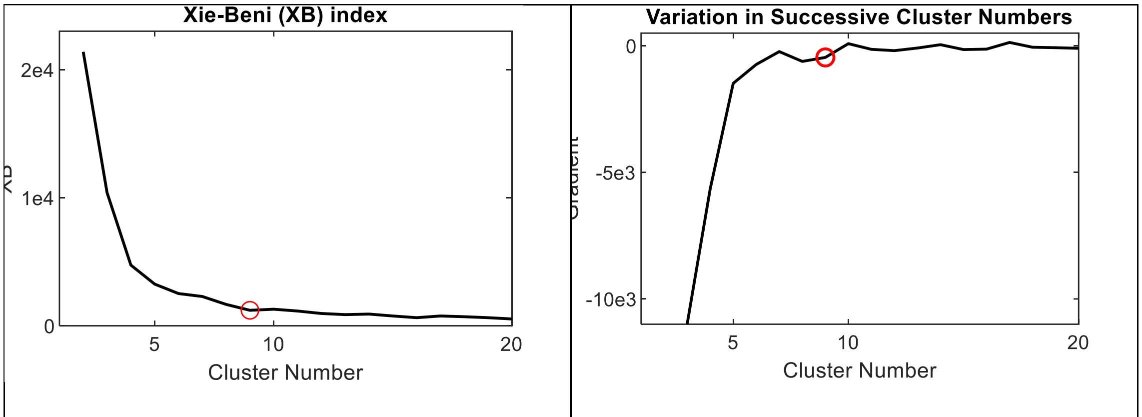


Figure 5: Results of the Silhouette index analysis. (a) Silhouette values of the data points for cluster numbers ranging from 2 to 10. (b) Average Silhouette index for different numbers of clusters (dashed line) and the fuzzy-modified average Silhouette index for different numbers of clusters (solid line).

The third method is the Xie–Beni (XB) index, which is a widely used criterion for evaluating clustering quality by measuring the balance between intra-cluster compactness and inter-cluster separation. A lower XB value indicates well-separated clusters with minimal internal variation, suggesting a better clustering result. This index has also been adapted for fuzzy clustering (Campello and Hruschka, 2006). However, relying solely on the XB index to determine the optimal number of clusters can be misleading, as a valid clustering solution may correspond to multiple XB values. Therefore, for a more reliable assessment, XB results are typically compared with other cluster validity measures such as the Silhouette index.

Similar to the elbow method, the XB index was computed for different numbers of clusters using acoustic impedance values at CMP locations, and the results are shown in Figure 6a. In this case as well, the variation shows three distinct regimes (Figure 6b). Based on the same reasoning as before, and considering the large number of layers and the presence of thin beds, the optimal number of clusters is selected as 9 at the end of the second slope region.

The results of different clustering validity methods for selecting the optimal number of clusters may vary. Although using fewer clusters can simplify interpretation, it may fail to capture important details and complexities in the data. This issue becomes more pronounced in subsurface data with strong heterogeneity and fine-scale features. Therefore, in this study, when discrepancies arise among validity measures and geological considerations, the larger number of clusters is preferred.



(a) (b)

Figure 6: (a) Xie–Beni (XB) index for different numbers of clusters. (b) Variation of the XB index between consecutive cluster numbers.