

Review Comments to the Authors

Reviewer 1:

The manuscript proposes a multi-objective fuzzy seismic inversion framework for acoustic impedance estimation from post-stack seismic data. The method is designed to integrate conventional inversion constraints with clustering information derived from well logs or geological prior knowledge. The study is relevant and potentially valuable. In particular, the generation of membership sections may provide an additional interpretation attribute if its geological meaning can be validated.

Nevertheless, the manuscript requires substantial improvement before publication. The main issues concern the clarity of novelty, mathematical consistency, parameter selection, and validation. In particular, the synthetic model used in the current manuscript is too simple to convincingly demonstrate the claimed robustness and interpretability of the method. A more complex benchmark test is needed.

Author:

We sincerely appreciate the reviewer's thorough evaluation and constructive suggestions.

Given that the manuscript is structured as a concise research paper of approximately 10 pages, we have focused on comprehensively addressing the essential evaluations, discussions, and key subjects raised, while appropriately referencing established literature for secondary procedural details. We have carefully revised and tailored the text to fully address your valuable feedback. These updates have been integrated smoothly to preserve the overall cohesion, integrity, and focus of the paper.

A detailed, point-by-point response to each comment along with the corresponding revisions is presented below.

Major comments

1. Novelty should be more clearly defined

Reviewer 1:

The idea of using fuzzy clustering or petrophysical clustering constraints in geophysical/seismic inversion has been reported in previous studies. The authors should clearly explain what is new in the present work compared with existing fuzzy clustering constrained inversion and guided fuzzy clustering inversion methods. The novelty should not be stated only as "using fuzzy clustering"; instead, the authors should specify whether the main contribution lies in the multi-constraint objective function, the windowed multi-trace implementation, the membership sections, or the proposed weighting strategy.

Author:

To address this valuable comment and explicitly highlight our specific contributions, namely the unique multi-constraint formulation and the coupled optimization framework, we have revised the abstract as follows:

“We propose a multi-objective clustering-based seismic inversion framework in which clustering information derived from prior geological knowledge, such as well logs and geological data, is explicitly embedded into the inversion objective function. Unlike conventional approaches, the proposed method formulates the problem as a coupled optimization system that balances data fidelity, resolution, model regularization, and clustering consistency.”

Furthermore, to reinforce how our approach differs from traditional strategies, we have explicitly emphasized the integration of these distinct constraints and our optimization strategy in the conclusion:

“The proposed objective function is a combination of five well-known objective functions, each of which ensures a desired property in the resulting model. A simple solution to the new objective function is presented.”

Reviewer 1:

The literature review would also benefit from a more balanced discussion of related sparse, envelope-based, basis-pursuit, and broadband seismic inversion methods. For example, the authors may consider discussing Yang et al. (2024), Yin et al. (2015), Yang et al. (2022), and Zong et al. (2018), which are relevant to sparse regularization, basis pursuit inversion, low-frequency recovery, and broadband seismic inversion. This would help position the proposed multi-constraint inversion framework more clearly within existing seismic inversion research.

Author:

We thank the reviewer for this constructive suggestion. We have integrated a new paragraph discussing alternative advanced methodologies, such as sparse regularization, basis-pursuit, envelope-based, and broadband inversion techniques, and have incorporated the recommended references to better highlight how our approach complements these existing paradigms.

The added text reads as follows: “Beyond smoothness, sparse regularizations like basis-pursuit are often used to enhance vertical resolution and produce compact reflectivity models. ...”

2. Some key equations and definitions appear inconsistent

Reviewer 1:

The derivation from acoustic impedance to reflectivity and log-impedance needs to be checked carefully. In Eq. (2), if x is defined as $\log(z)$, then the factor of $1/2$ must be retained in the reflectivity expression. If x is defined as $0.5 \log(z)$, this should be stated explicitly. Eq. (3a) also appears problematic because a local difference is equated to a cumulative summation of reflectivity. The authors should rewrite Eqs. (2)-(4) with consistent definitions of z , x , r , D , and H . The smoothness terms also require clarification. Some formulations combine

vertical and horizontal derivatives directly, but these derivatives have different sampling intervals and physical units. The role and scaling of the parameter alpha should therefore be explained.

Author:

a) Thanks for noticing these typographical inconsistencies. Eq. 2 and Eq. 3 are modified accordingly.

b). The smoothness and the alpha factor are already discussed in section 2.4

c) “The α is a small factor that controls the smoothness variation along these directions.”

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3. The objective-function weighting and L2-L1 formulation need a more rigorous explanation

Reviewer 1:

The normalization factors and weights in Eq. (14) are not fully justified. Setting the weights to sum to one does not automatically solve the regularization-parameter selection problem. The authors should provide sensitivity tests or objective criteria for choosing the weights. In addition, the sparsity term appears to have no independent weighting factor, which makes its relative contribution unclear.

Author:

As noted in the manuscript, “Weighting factors balance the misfit or data fidelity term and the other terms”. The normalization scheme introduced in Eq. (14) is specifically designed to remove the physical dimensions and disparate scales inherent to each term, as they represent different geological and mathematical constraints. By non-dimensionalizing these components, we ensure that the optimization process evaluates the relative contribution of each constraint rather than being biased by scaling discrepancies.

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Reviewer 1:

The transformation from Eq. (14) to Eq. (15) should be derived more explicitly. If weighted squared norms are stacked into a single least-squares system, the matrix coefficients should normally involve the square roots of the weights and normalization factors. The authors should verify whether the current formulation uses w or \sqrt{w} , and whether the dimensions of all matrices are consistent.

Author:

To prevent any ambiguity, we have double-checked all equations throughout the manuscript. Accordingly, the symbols and expressions in Eq. (14) and Eq. (15) have been modified to ensure complete mathematical clarity.

4. The inversion-clustering coupling is nonlinear and should be described as an alternating optimization

Reviewer 1:

The memberships and centroids depend on the current impedance model, while the impedance model is estimated using the clustering term. Therefore, the problem is not a simple fixed L2-L1 inversion unless the memberships and centroids are held constant within each iteration. The authors should clearly describe the alternating optimization process, convergence criteria, stopping conditions, and computational cost.

Author:

a) The proposed method is flexible for several scenarios of the initial data and approaches on the centroids (Section 3.2).

b) Please see section 2.6: “Solving Eq. **Error! Reference source not found.** for memberships and centroids and repeating the inversion converges the model, the centroids, and the memberships. In clustering methods, the centroids and memberships are updated repeatedly until they converge. However, here these updates happen just once per each iteration of the inversion using Eq. **Error! Reference source not found.** and Eq. **Error! Reference source not found.** You can opt to update the centroids and memberships based solely on the model, (\mathbf{x}) or include both the time (depth) of data points and the model, ($[\mathbf{t}, \mathbf{x}]$).”

6. Synthetic validation should be strengthened using a more complex benchmark model

Reviewer 1:

The synthetic model used in the manuscript is relatively simple and contains limited structural and stratigraphic complexity. Such a simple model is useful for illustrating the workflow, but it is not sufficient to support strong claims about robustness, high resolution, reduced dependence on the initial model or wavelet, and the interpretability of membership sections. I suggest that the authors add a more challenging synthetic test based on a well-known complex benchmark model, such as the Marmousi or Marmousi2 model. A Marmousi-type test would provide a more convincing evaluation because it contains stronger lateral heterogeneity, complex structures, and more realistic impedance contrasts. The authors could compare the proposed method with conventional model-based inversion or sparse inversion, and report impedance error maps, trace-by-trace comparisons, sensitivity to noise/wavelet errors, and the behavior of the membership sections under complex geological conditions.

Author:

The custom synthetic model initially presented in the manuscript was deliberately designed to clearly isolate and illustrate the fundamental mechanics of the clustering-based inversion. However, this framework has also been extensively evaluated on more complex structures as part of a broader doctoral research project.

To address your comment completely without exceeding the manuscript's length constraints, we have compiled these comprehensive benchmark tests on Marmousi model and appended them as a supplementary attached file to the comments (Additional File).

Minor comments

Reviewer 1:

Please correct typographical and grammatical errors, for example “Department op” should be “Department of”, “try and error” should be “trial and error”, and “Euclidian” should be “Euclidean”.

Author:

We thank the reviewer for pointing out these typographical and grammatical oversights. All of them, including the specific examples mentioned, have been thoroughly corrected throughout the revised manuscript.

Reviewer 1:

Please check all equation references. For example, the text appears to refer to Eq. (7b) as a smoothness term, but Eq. (7) is the K-means objective function.

Author:

We have audited the text and corrected this specific reference to Eq. (6b), and we have verified that all other equation citations across the manuscript are accurate and consistent.

Reviewer 1:

Please define all symbols before use and keep notation consistent, especially u , v , O , C , M , H , D_v , and D_h .

Author:

Those symbols are defined before or after their first appearance in the related equations. Such as in “ D_v and D_h are the first-order differential matrix along the temporal and spatial dimension, respectively.” Some others like “ u ” are defined as an equation.

Reviewer 1:

Please provide more details about the statistical wavelet estimation, the initial model construction, and the inversion window size in the F3 case.

Author:

The referred part is modified as: “A statistical wavelet was estimated from the seismic volume using an autocorrelation-based method in OpenTect. The extracted wavelet had a duration of 128 ms and was assumed to be stationary throughout the inversion process. The initial low-frequency model was constructed in Hampson-

Russell using the available acoustic impedance well logs, interpreted horizons, and kriging interpolation. The proposed inversion was performed using a moving window consisting of each three adjacent traces over the full-time length of each trace.”

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Reviewer 1:

Some claims in the abstract and conclusion should be moderated unless stronger evidence is added, particularly those related to denoising, deconvolution, wavelet insensitivity, and layer-content prediction.

Author:

Those claims are rewritten by considering the evaluations results. For example, in the abstract, now we have “the approach generated reflectivity sections with reduced noise and improved continuity of the seismic data, which are useful for future interpretation.” Or in the conclusion: “The resulting membership sections may assist interpreters in tracking horizons and distinguishing between sequences and layers, while providing supplementary information for geological interpretation.”

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Reviewer 1:

The figures should be improved where possible. Axis labels, color bars, and legends should be readable, and the membership-section figures should include axes or at least clear spatial/time references.

Author:

To ensure optimal resolution and print quality, we have uploaded the high-resolution figure files separately.

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Reviewer 1:

Please check the reference format and journal names. For example, the citation information for Sun and Li should be formatted consistently with the target journal style.

Author:

The references are already formatted in accordance with the official template provided by the journal.

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Reviewer 1:

Please consider adding and discussing relevant references on sparse and broadband seismic inversion, including: Yang, S., G. Wu, J. Shan, and H. Liu, 2024, Prestack sparse envelope seismic inversion method adopting the L0-L2-norm regularization: Interpretation, 12(2), T149-T166; Yin, X., X. Liu, and Z. Zong, 2015, Pre-stack basis pursuit seismic inversion for brittleness of shale: Petroleum Science, 12, 618-627; Yang, S., G. Wu, J. Shan, and H. Liu, 2022, Multi-scale seismic envelope inversion method based on sparse representation theory: Journal of Applied Geophysics, 203, 104685; Zong, Z., Y. Wang, K. Li, and X. Yin, 2018, Broadband seismic inversion for

low-frequency component of the model parameter: IEEE Transactions on Geoscience and Remote Sensing, 56, 5177-5184.

Author:

We appreciate this suggestion. Methodologies and references have been incorporated into the Introduction section as follows: “Beyond smoothness, sparse regularizations like basis-pursuit are often used to enhance vertical resolution and produce compact reflectivity models (Yin et al., 2015, Yang et al., 2024). Similarly, multi-scale envelope and broadband inversions help recover low-frequency data missing from bandlimited seismic tracks (Yang et al., 2022, Zong et al., 2018).”