Review Report on "Maximum Certainty Principle applied to rainfall modelling and regionalisation in Ecuador" by Franklin Aparicio Beltrán Vega and Jhoan Alexander Beltrán Valarezo

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1 Summary of the Paper

The paper proposes a "Maximum Certainty Principle" (PCM), presented as a variational framework in which a functional combining a structural term, the "Knowledge Potential" C, and an informational term, the surprise $-\ln f$, is maximised to select a probability density. The maximiser is said to carry an invariant $\aleph_{\max}$. The principle is then applied to storm modelling in the Metropolitan District of Quito through a stochastic simulator (MIT-Q), calibrated against the EPMAPS IDF curves at the Quito-Observatory station and compared at four further stations. Finally the framework is used to reinterpret the exponent b of the Ecuadorian national (INAMHI) IDF formulae as a "number of rainfall bursts" α_v, and to derive "Potential IDF" (IDFP) curves by multiplying the official design intensities by a factor $\varphi(b)$ that reaches 1.51 at some stations.

The applied part of the paper contains material of genuine practical value. The theoretical part, which the paper advertises as its principal contribution and from which the operational product is derived, does not survive examination, and the applied part inherits several errors from it. We set out the reasons below.

2 Major Comments

2.1 PCM is the Gibbs construction, and the invariant is its defining identity

The certainty functional is, in effect, $\aleph[f] = \langle C \rangle_f + H[f] = \int fC\,dt - \int f \ln f\,dt$. Maximising this over normalised densities is the standard Gibbs variational problem. Introducing a multiplier for normalisation gives the stationarity condition $C(t) - \ln f(t) - 1 - \gamma = 0$ and hence $f = e^C/Z$ with $Z = \int_0^{DT} e^C dt$, which is Eq. (10). The cleanest way to see it is the identity $\aleph[f] = \ln Z - D_{\mathrm{KL}}(f \,\|\, e^C/Z)$: since the Kullback–Leibler divergence is non-negative, the maximum is attained exactly at $f = e^C/Z$ and its value is exactly $\ln Z$. This is not a stationarity result that might hide further structure; it is a global identity.

This is the standard statement that a Gibbs measure has constant free energy. It is not a conservation law in Noether's sense. A Noether construction requires an action, a continuous transformation leaving it invariant up to a divergence, and a derived conserved quantity. None is exhibited anywhere in the manuscript. The introduction's discussion of proper and improper divergence relations, citing Kosmann-Schwarzbach (2011), is particularly unfortunate, since an improper conservation law is precisely one following from a differential identity rather than from dynamics, which is exactly what $\aleph_{\max} = \ln Z$ is.

2.2 What is being modelled is never defined

The manuscript is not consistent about what f is. In Sects. 2.1.8 and 2.3.1, T is a random variable, "the residence time in the development state, or the ability to remain in optimal conditions until reaching an unsustainable point", and f_T is its density, so Eq. (13) is a probability statement about when a storm ceases developing. In Sect. 2.3, what is actually used is $\mathrm{Pre}(t)/\mathrm{PRE}$, the fraction of the event's total depth accumulated by time t, which for a given storm is a deterministic mass profile and not a probability at all. Both are non-negative and integrate to one on $[0, DT]$, but they are different objects. Sharing an interval, monotonicity and endpoint values does not make them equivalent.

The ambiguity matters because the two halves of the paper need different readings. The apparatus of Sect. 2.1, comprising surprise, entropy, the unexpectedness of an event and the elimination of trajectories, requires a probability, hence the first reading. The application requires the second.

2.3 The model actually used does not follow from the principle

Whatever one concludes about Sect. 2.1, the storm model does not inherit from it. The variational argument yields the truncated exponential cumulative form, Eq. (13). The model then discards it: Eq. (14) squares it, and the authors state plainly that "the squared form was adopted empirically because it provided the most consistent representation". Every subsequent equation, (15), (19), (20), (23), (25), (34) and (35), uses the square.

The squaring is not a refinement but a repair, and this deserves to be stated quantitatively because it shows what the principle actually predicts. The unsquared law gives rainfall intensity proportional to $\lambda e^{-\lambda t}$, which is monotonically decreasing: the storm begins at its peak intensity and decays from $t = 0$, for every value of α_0. This is physically impossible and is the opposite of the early-peaked but growing behaviour the paper attributes to storms. Squaring moves the peak to $t/DT = \ln 2/\alpha$. The initiation, development and dissipation structure that the paper presents as an outcome is supplied entirely by the empirical square.

Worse, the squared pattern lies outside the family PCM can generate. The implied density is proportional to PP', so the corresponding potential is $C(t) = \ln[2P(t)P'(t)] +$ const, which diverges to $-\infty$ at $t = 0$ and is non-monotone thereafter, and is therefore not of the postulated form $E + Cr$ with E decreasing and Cr increasing. The pattern used to fit the data is one the principle forbids.

The cleanest way to put the difficulty is as a question the authors should answer directly: which numerical result anywhere in the manuscript would change if PCM were deleted and Eq. (15) simply postulated as a two-parameter storm-accumulation model? As far as we can determine, none would. A reader may begin at Eq. (15) and reproduce everything that follows. The honest description of what was done is that a two-parameter shape was fitted to hyetographs and IDF curves.

2.4 The IDFP curves are a rescaling of the information used to construct them

Stripped of the terminology, the operational outcome of the paper is $\mathrm{IDFP}(t) = \varphi(b)\,\mathrm{IDF}(t)$: an upward multiplicative tuning of the existing official curves, raising short-duration design intensities by 8 to 51%. The difficulty is where φ comes from. The baseline IDF information constrains MIT-Q. The same IDF shape parameter b is mapped into α_v by the imposed tangency. That reinterpretation generates α_p and then φ. And φ is applied back to the original curves. The chain runs from IDF to b to α_v to α_p to φ and back to IDF. The product being corrected is the principal source of the information used to generate the correction.

The "strong correlation" between φ and b in Fig. 8a is therefore not an empirical finding but a formula plotted against its own argument, and fitting $\varphi(b) = b^{-0.237}$ to it and reporting the fit quality is fitting a curve to an identity. The physical motivation, that a sparse network systematically misses the storm activity centre, is reasonable. But a motivation determines the sign of a correction, not its magnitude, and cannot validate it.

3 Recommendation

We recommend rejecting the paper in its current form. These are not matters of exposition, and we do not believe they can be settled by a normal revision. Either the theoretical novelty must be reformulated from the ground up and distinguished rigorously from the existing Gibbs and maximum-entropy framework, or the manuscript must become a considerably narrower one presenting MIT-Q as an empirical stochastic rainfall model without claim to a new principle of inference.The main unresolved issues include:

- PCM is the Gibbs and maximum-entropy variational construction with the Lagrange multiplier renamed "Knowledge Potential", and, unlike maximum entropy, it fixes that potential only by assuming the answer.
- The object being modelled is never defined: a residence-time density and a normalised rainfall accumulation are used interchangeably, and the trajectory apparatus of Sects. 2.1.2–2.1.7 is never used again.
- The temporal pattern actually fitted, Eq. (14), is adopted empirically, lies outside the family the principle can generate, and supplies the storm shape that the principle itself cannot produce.
- The IDFP curves are a multiplicative rescaling of the same IDF information used to derive the rescaling, and are supported by no independent observational test and no uncertainty estimate.