

AUTHOR COMMENT AC8

Response to the comments of referee report RC3

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1 Introduction

This author comment responds to referee report RC3, which raises seven observations and concludes that the manuscript may constitute a contribution to hydroinformatics and to the hydrology of extreme events once it has been revised in detail. We thank the referee for a report that separates what remains to be exhibited from what remains to be corrected, and we state the scope with which we receive the referee's assessment of the theoretical originality: the revised manuscript presents the variational formulation as the specification of the storm model and not as a principle of inference distinct from the Gibbs and maximum entropy framework, in the terms already committed in response to report RC1 (AC6, C8). What the referee values as a new direction is, in those terms, the formulation by potential and the model that is built from it.

Each observation is addressed in the parts that apply: What we accept, What we maintain and Committed changes to the revised manuscript. The numbering continues that of author comments AC6 and AC7: the arguments run from 57 to 67 and the committed changes from C44 to C48.

Three of the seven observations coincide with points already addressed in response to reports RC1 and RC2 and are answered by reference to the item and the change where they are resolved, without repeating the argument. The remaining four receive their own development: the interpretation of the two central quantities, the derivation of Eqs. (10) and (11), the determination of the transition interval and the consistency of Eq. (27). The committed changes add to the body of the manuscript an interpretative paragraph in Section 2.1.8 and a step in Section 2.4; the developments are placed in the appendix and in the supplement already committed, so that Section 2 is shortened and not lengthened (item 64).

2 Comments of referee report RC3

2.1 Comment: the central concepts lack interpretation

Core concepts such as Knowledge potential $C(t)$ and Certainty $\aleph$ lack intuitive physical or mathematical interpretation.

2.1.1 What we accept

57. We accept the observation. The manuscript defines $C(t)$ and $\aleph$ operationally, through Eqs. (5) and (9) to (12), and proceeds at once to use them, without offering a reading that allows the reader to know what they represent and what changes when they change. The consequence is the one the referee points out: the two quantities remain definitions and not interpretable magnitudes.

2.1.2 What we maintain

58. The interpretation follows from four readings of the equations already written in the manuscript, three mathematical and one in the phenomenon.

First, as an identity of compensation: Eq. (9) establishes $\aleph_{\max} = C(t) - \ln f(t)$ for every t in the interval $[0, DT]$, so that where Knowledge is high the surprise is low, and the sum of the two does not depend on t . $\aleph_{\max}$ is that constant of compensation, the quantity the system keeps fixed while it distributes structure and surprise along the event.

Second, as the logarithm of the partition function and a decomposition between mean knowledge and entropy: Eq. (12) gives $\aleph_{\max} = \ln Z$, the same quantity that in statistical physics

is the logarithm of the partition function (the negative of the free energy in units of kT), and the correspondence with Jaynes (1957), established equation by equation in AC6, item 2, gives $\ln Z = E[C] + S$. Maximum Certainty is thus divided into two summands with a meaning of their own: the mean Knowledge that the modeller introduced into the potential and the entropy that remains after introducing it; a potential with a great deal of structure gives a high $E[C]$ and a low S , and one without structure the opposite. The limiting case fixes the scale: if C is constant, $Z = DT \cdot e^C$ and $\aleph_{\max} = C + \ln DT$, the entropy of the uniform distribution over the duration of the event plus the constant Knowledge; with $C = 0$, $\aleph_{\max} = \ln DT$.

Third, $C(t)$ through its two components: the manuscript defines it as a sum of monotonic functions, $C(t) = E(t) + Cr(t)$, with the Experience $E(t)$ decreasing and $Cr(t)$ increasing or constant, and in Fig. 1 with $E(t) = -\lambda t - \aleph_{\max}$ and $Cr(t) = 0$. Variations of E and of Cr generate the different families of distributions. $C(t)$ is an assumption about the phenomenon, with its parameter subject to calibration, and not a result of the principle, as was stated in AC6, item 12, and committed in AC6, C7.

Fourth, in the phenomenon. Under the interpretative hypothesis of Section 2.3.1 (AC6, item 13), $f(t)$ is the density of the time T during which the event remains in growth, and $-\ln f(t)$ is the surprise that growth ends at t . $C(t)$ is, apart from the constant $\aleph_{\max}$, the logarithm of that density: the Experience $E(t) = -\lambda t$ expresses that each additional instant of growth is exponentially less probable than the previous one, with constant rate $\lambda = \alpha_0/DT$, so that the ratio between the propensities of two instants separated by the same interval does not depend on the starting instant, and $Cr(t) = \ln(1 - e^{-\lambda t})$, in the quadratic form, expresses that the event does not begin at full intensity but passes through an initiation stage, the choice of which is empirical (AC6, items 17 to 20). With that reading, $\aleph_{\max} = \ln Z$ is the logarithm of the total mass of propensity to growth accumulated over the duration of the event, and its difference between two states, $\Delta\aleph_{\max}$, is what Section 2.3.1 converts into the development time t_d through the closure relationship already declared as such (AC6, C13).

2.1.3 Committed changes to the revised manuscript

C44. Interpretative paragraph in Section 2.1.8, immediately after Eq. (12), with the four readings of item 58: the identity of compensation between Knowledge and surprise; the identification of $\aleph_{\max}$ with the logarithm of the partition function and its decomposition $\aleph_{\max} = E[C] + S$, with the limiting case $\aleph_{\max} = C + \ln DT$; the reading of $C(t)$ through its two monotonic components; and the reading in the phenomenon of $E(t)$, $Cr(t)$ and $\aleph_{\max}$. Fig. 1 is relabelled as an illustration of that compensation.

2.2 Comment: the full derivation of Eqs. (10) and (11) is missing

The full derivation of equations 10 and 11.

2.2.1 What we accept

59. We accept the observation. The manuscript states Eqs. (10) to (12) without the step that produces them, so that the exponential density and the partition function appear as adopted definitions and not as the result of maximising the functional. The derivation is brief, and its absence is what prevents the reader from reproducing the argument.

2.2.2 What we maintain

60. The derivation is written on the variable the manuscript employs, the cumulative $PT(t)$, with the endpoints the problem fixes, $(0, 0)$ and $(DT, 1)$:

$$\aleph(P_T(t)) = \int_0^{DT} P'_T(t) \{C(t) - \ln P'_T(t)\} dt, \quad P_T(0) = 0, P_T(DT) = 1, \quad P'_T(t) > 0$$

The integrand does not depend explicitly on $P_T(t)$, but only on its derivative, so that the Euler–Lagrange equation reduces to the requirement that the derivative of the integrand with respect to $P'_T(t)$ be constant along the interval: $C(t) - \ln P'_T(t) - 1 = k$. That first integral is Eq. (9) of the manuscript, with $\aleph_{max} = k + 1$. From it follows $P'_T(t) = e^{C(t)} \cdot e^{-(1+k)}$, and the boundary condition $P_T(DT) - P_T(0) = 1$ fixes the constant: $e^{1+k} = \int_0^{DT} e^{C(t)} dt$, which is Eq. (11); with it $P'_T(t) = f_T = e^{C(t)}/Z$, which is Eq. (10), and $\aleph_{max} = \ln Z$, which is Eq. (12). The integrand is strictly concave in $P'_T(t)$ for each t , with second derivative $-1/P'_T(t)$ negative for every admissible density, so that the extremal is the global maximum; it is the same conclusion given by the Gibbs identity of item 61. Normalisation does not enter as an external constraint: it is the boundary condition of the cumulative curve.

61. Two routes lead to the same result. Maximisation over the density, with normalisation imposed as a constraint, gives the same maximiser and the same maximum value, and the quantity that appears there as a multiplier is the same one that appears here as the constant of the first integral; both formulations describe the same variational problem (AC6, item 8). The manuscript writes on the cumulative because it is the curve the recording rain gauge registers and it is the object the model constructs, the cumulative depth of the event, and because the boundary conditions are imposed directly on it (AC6, item 11). Maximality is moreover global by the Gibbs identity, $\aleph[f] = \ln Z - \text{DKL}(f \parallel eC/Z)$, accepted in AC6, item 1, and committed in AC6, C1.

2.2.3 Committed changes to the revised manuscript

C45. Incorporation of the derivation of item 60 into the appendix that receives the general-scope material of Sections 2.1 and 2.2 (AC7, C26): functional on $PT(t)$ with its boundary conditions, first integral of the Euler–Lagrange equation giving Eq. (9), in which $\aleph_{max}$ is the conserved constant, determination of that constant by the boundary condition, and sign of the second derivative. It is stated there that maximisation over the density leads to the same maximiser and the same value, and that the formulation on the cumulative is retained because of the object the model constructs, consistently with AC6, C6. In Section 2.1.8 a sentence is kept stating the step, Eq. (10) follows from Eq. (9) by imposing the boundary condition $P_T(DT) = 1$, with a cross-reference to the appendix.

2.3 Comment: the relationship with the maximum entropy principle is not discussed

The differences, complementarity of PCM and Jaynes' Maximum Entropy Principle are not discussed.

2.3.1 What we accept

62. We accept the observation, and the discussion has already been made in response to report RC1. The correspondence is established equation by equation against Jaynes (1957): his Eq. (2-12) gives $\lambda_0 = \ln Z$ and his Eq. (2-13) gives $S_{max} = \lambda_0 + \sum \lambda_r \langle fr \rangle$, which with $C(x) = -\sum \lambda_r fr(x)$ reads $\ln Z = S + E[C] = \aleph$; Eqs. (6), (8) and (9) of the manuscript are his Eqs. (2-10) to (2-13) and $\aleph_{max}$ is his λ_0 (AC6, item 2). The difference is not one of result but of point of entry: inference by maximum entropy starts from given constraints and from them obtains the multipliers, whereas here one starts from a potential postulated from the phenomenon, with the parameter subject

to calibration; in both cases the information enters from outside, and that difference does not constitute a distinct principle of inference (AC6, item 12).

The complementarity the referee mentions can be stated precisely: every potential C corresponds to a set of Jaynes constraints, and conversely. The Experience $E(t) = -\lambda t$ is exactly the Jaynes mean constraint on T , truncated at DT ; the term $Cr(t) = \ln(1 - e^{-\lambda t})$ of the quadratic form corresponds to a constraint on $E[\ln(1 - e^{-\lambda t})]$, which is not a usual moment and which the formulation by potential allows one to write directly from the phenomenon. It is the same variational problem read from opposite ends: maximum entropy goes from the constraints to the potential; the formulation of the manuscript goes from the potential to the constraints it implies. The revised manuscript incorporates this discussion into the changes already committed in AC6, C1, C7 and C8, with the correspondence paragraph added in C44.

2.4 Comment: the potential curves have no independent validation

the potential IDF curves are adjustments of existing INAMHI curves – no independent stations or events are used for validation.

2.4.1 What we accept

63. We accept the observation, which coincides with the fourth main comment of report RC1 and was already accepted there. The potential intensities are a transformation of the official curves, not an independent estimate, and their magnitude is not contrasted against records that did not take part in their construction. They are accordingly presented as intensification scenarios conditional on the model, and not as a different observed exceedance frequency, with the express statement that their magnitude requires independent contrast (AC6, item 26, and C19). We state which contrast applies. What the IDFP curves assert is that a sparse network samples away from the storm centre; the contrast that proves or refutes this requires observations that resolve the centre independently of the fixed point: a dense network of recording rain gauges, weather radar or high-resolution satellite estimates, comparing event by event the maximum intensity over the domain with the one recorded at the reference station. That ratio is an empirical scale factor that can be confronted with the $\varphi(b_1, b_2)$ of the model. The revised manuscript states that test as the one that applies and declares that it has not been carried out (C48).

2.5 Comment: Section 2 is long and displaces the technical content

The theory/methodology section 2 is overly long, with extensive philosophical discussion that crowds out technical details.

2.5.1 What we accept

64. We accept the observation, which coincides with the comment of report RC2 on the interjection of general theoretical background into technical sections, and the corrections are already committed: removal of the passage on times between earthquakes, radioactive decay and component lifetimes (AC7, C24); transfer to an appendix of the general-scope material of Sections 2.1 and 2.2 that does not take part in the construction of the model (AC7, C26); a new subsection setting out in five steps the chain that runs from the functional to the storm model (AC7, C25); and separation of the presentation of the model from the discussion of results (AC7, C29). The developments that this author comment adds are placed in that same appendix and in the supplement, and not in the body (C45 and C46).

2.6 Comment: the criterion for selecting t_1 and t_2 is not justified

The transition interval is defined by two tangency points Eqs.2, but criteria for selecting t_1, t_2 are not justified.

2.6.1 What we accept

65. We accept that the manuscript does not show the step. The text announces that, with Eqs. (23) and (30) substituted into Eq. (31), t_2 is solved for, but the expression printed as Eq. (32) is a rewriting of Eq. (23) and does not contain that solution. Without it, the reader cannot verify that the transition interval is determined, and the natural reading is that t_1 and t_2 are chosen.

2.6.2 What we maintain

66. There is no criterion of choice because there is no choice: t_1 and t_2 are obtained by solving the tangency system. The transition storm is a single storm tangent to the two branches of the official curve, which imposes $DT_1 = DT_2$ and $\alpha_{v_1} = \alpha_{v_2}$, from which Eq. (30) gives $t_1 = t_2 \cdot (\tau_1/\tau_2)$, and the equality of depths of Eq. (31), with Eq. (23) on each branch, determines t_2 . The solution is explicit:

$$t_2 = \left\{ \frac{a_2}{a_1} \cdot \left(\frac{\tau_1}{\tau_2} \right)^{b_1-1} \cdot 60^{b_1-b_2} \cdot \left[\frac{1 - e^{-\tau_1}}{1 - e^{-\tau_2}} \right]^2 \right\}^{\frac{1}{b_2-b_1}}$$

$$t_1 = t_2 \cdot \frac{\tau_1}{\tau_2}$$

with t_2 in hours when a and b are the coefficients of the official formulation. The daily intensity ID_{tr} and the factor in α_v cancel when the two branches are equated, so that the transition interval is fixed solely by the four published parameters of the station, a_1 , b_1 , a_2 and b_2 , through τ_1 and τ_2 , which are obtained from b by solving Eq. (25) with the corrected denominator (AC6, C14), a transcendental equation whose solution is numerical. It is the same argument with which AC6, item 24, replaced the fitting of φ by its closed form, applied now to the transition interval: of the deterministic chain that runs from (a_1, b_1, a_2, b_2) to the transition storm no degree of freedom remains. The solution is exact and not approximate: checked against the calculation workbook of the 66 INAMHI stations it reproduces t_1 and t_2 to the precision with which τ_1 and τ_2 are solved by scalar iteration; at M003-Izobamba it gives $t_1 = 0.5373$ h and $t_2 = 1.0145$ h, which are the values of Fig. 7b.

2.6.3 Committed changes to the revised manuscript

C46. Replacement of Eq. (32), which at present rewrites Eq. (23) instead of solving for t_2 , by the explicit expression of t_2 as a function of a_1 , b_1 , a_2 , b_2 , τ_1 and τ_2 , with its unit, accompanied by the statement that t_1 and t_2 are determined by the published parameters of each station and are not selected. The development that leads from Eqs. (23), (30) and (31) to that expression, and the check over the 66 stations, are incorporated into the supplement already committed in AC6, C17, together with the check of φ .

2.7 Comment: consistency of ΔX_{max} in Eq. (27)

Eq27 ΔX_{max} consistent with earlier definition? Check notational consistency.

2.7.1 What we maintain

67. The relationship is consistent and follows from the preceding equations. From Eqs. (16) and (17), multiplying by α_0/DT , one obtains $\alpha_0 \cdot t_d/DT = \ln(\alpha_0/(1 - e^{-\alpha_0}))$, whose left-hand side is the definition of τ of Eq. (24) evaluated at $t = t_d$, and whose right-hand side is ΔX_{max} . Eq.

(27) is that same identity written with α_v , the parameter inferred from the advected record, instead of α_0 . What the manuscript lacks is the intermediate step and a notational precision: in Section 2.3.1 $\Delta\aleph_{\max}$ corresponds to the generated event, with parameter α_0 , and in Eq. (27) to the point record, with parameter α_v , so that it is preferable to write $\Delta\aleph_{\max}(\alpha_0)$ and $\Delta\aleph_{\max}(\alpha_v)$ and not the same symbol without an argument. The two errata in the neighbouring equations are already corrected in earlier commitments: the denominator of Eq. (16), which is $+\lambda$ (AC6, C13), and that of Eq. (25), which is $1 - e^{-\tau}$ (AC6, C14).

2.7.2 Committed changes to the revised manuscript

C47. Incorporation of the step that leads from Eqs. (16) and (17) to Eq. (27), and distinction of the argument in the notation, $\Delta\aleph_{\max}(\alpha_0)$ for the generated event and $\Delta\aleph_{\max}(\alpha_v)$ for the point record, in Section 2.4 and in the list of symbols.

C48. Statement, in Section 3.1 and in the conclusions, of the contrast that applies to the IDFP curves: an event-by-event comparison between the maximum intensity over a dense network or available radar or satellite product and the one recorded at the reference station, as an empirical scale factor against the $\varphi(b_1, b_2)$ of the model; with the declaration that this contrast has not been carried out and that carrying it out over the 5 min network of the DMQ is the next step of the work

3 Conclusions

Report RC3 agrees with RC1 and RC2 on what the manuscript defines and uses without interpreting, and on the need to shorten Section 2; both are resolved by earlier commitments and by those added here, which go to the appendix and to the supplement. The three observations that receive their own development are closed with exact results: the derivation of Eqs. (10) to (12) as a first integral of the Euler–Lagrange equation on the cumulative, with normalisation as the boundary condition (item 60); the explicit expression for the transition interval, which shows that t_1 and t_2 are not selected but determined by the four official parameters of each station (item 66); and the identity that links $\Delta\aleph_{\max}$ with τ in td , with the notation that distinguishes the generated event from the point record (item 67). On the relationship with maximum entropy and on the IDFP curves, the position is that of AC6: the same variational problem read from the potential instead of from the constraints, without claiming a distinct principle of inference; and scenarios conditional on the model whose test is defined and pending. The five committed changes, C44 to C48, are added to the forty-three earlier ones with the same running numbering.

4 References

Jaynes, E. T.: Information theory and statistical mechanics, *Physical Review*, 106, 620–630, <https://doi.org/10.1103/PhysRev.106.620>, 1957.

Sincerely,
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