We appreciate the community review as it helped us make an important clarification for how the relevant quantities in the text are calculated.

The authors start by inventing a very loose-sounding concept of 'wealth'. Here, 'wealth' is determined to be the cumulative economic real output (eq. 6), which is very odd, as this includes everything that humanity has ever produced, and also consumed. Hence, it does not represent what one could generally think of wealth, as consumption in the past does not contribute to wealth now. Hence the definition used for 'wealth' is very absurd... The vagueness of the basic concept of 'wealth', its odd definition and faults in the model's elementary definitions give me the idea that the presented model is merely a mathematical contraption that does not represent the real world. Hence no real-world conclusions can be made from it.

Wealth is in fact very precisely defined mathematically as

$$W(t) = \int_0^t Y(u)\, du,$$

where Y is real, inflation-adjusted global output. Wealth, *as it is defined* is derived from standard economic quantities but we agree that despite its simplicity, it is not a standard economic quantity, nor is it the same as such traditional concepts of wealth (small "w") that are considered as components of economic capital. It is far more general, as we describe. What we show is that introduction of this mathematical identity confers a number of simplifications in the analytical treatment that lead to a compact definition of inflation that has not been previously described. We further show that this new economic definition is not merely an identity but that it can be directly linked to important physical quantities related to primary energy consumption E through the relationship $W = \bar{w}E$ where $\bar{w}$ is empirically argued to be a constant. The thermodynamics section of the article is essential to the interpretation of Wealth.

The mathematical treatment of what follows is absurd as well. As output is always positive, 'wealth' can only increase over time, according to its definition in eq. 6. However, the authors violate this definition right after that, by forcefully inventing a sink term in eq. 7. This seems completely inconsistent and makes everything that follows from eq. 7 incorrect. I assume this violation can lead to making any possible argument with the model, as the model is inconsistent with itself. (Additionally, no explanation is given to why the sink term should be of the form given in eq. 7. Yet, I think this is a minor violation, as there should be no sink term at all. Following this, the authors subsrtact real output from nominal in eq. 8. This again is inconsistent, as the variables are in different units (i.e. currencies of different years). This nonsensical definition of gamma then propagates further in the presented model.

To appreciate how Wealth can both rise and fall, and how, it is essential to read the component of the article related to the physical treatment of civilization Wealth, which contains the core identity that $W = \bar{w}E$. The economy may no longer be measurable monetarily during the phase of collapse, but it can be quantified physically. In the growth phase it is certainly possible for a sink term to be present in Eq. 7 while allowing for ongoing growth, provided it does not overcome nominal production.

Regarding the comment about currencies being used from different years, this is not in fact the case, but an important clarification was needed.

> Operationally, the value of γ can be calculated in a semi-continuous sense from observed time series for the global nominal and real GDP series that are tallied annually. From these statistics, the inflation rate can be calculated as a finite-difference approximation to $i = d\ln P/dt$ given by Eq. 12. For an accounting interval Δt – normally one year – the local gross inflation factor (not rate) is $\Pi = \exp(i^\Delta)$ where $i^\Delta = i\Delta t$. More generally $\Pi = P(t+\Delta t)/P(t)$. The unit-aligned nominal valuation Y_N used in Eq. 7 is then calculated for each year as $Y_N^\Delta = \Pi Y$, in which case the interval-averaged decay or devaluation coefficient γ becomes
>
> $$\gamma^\Delta = \frac{Y_N^\Delta - Y}{W} = (\Pi - 1)\,\frac{Y}{W}, \tag{1}$$
>
> where, for low inflation rates much less than 100% in a given year
>
> $$\Pi \simeq 1 + \Delta P/P = 1 + i^\Delta \tag{2}$$

For positive inflation, where over a chosen accounting interval $Y_N^{\Delta} > Y$, then the term $\gamma W > 0$ provides the desired mathematical sink term in the accumulation of W. Purely from an economic accounting perspective, γ represents the rate at which the unit-aligned nominal valuation of current production must be corrected within a given time interval so that the accumulation of Wealth dW/dt can remain evaluated in real terms. It is therefore not raw nominal GDP that is accumulated into W as a "savings" (namely, analogous to $s = 1$ in Eq. 2); rather, only the real component Y is accumulated, with γW representing the period-averaged correction required to keep W in real terms.

References