Reply to the reviewer2's comments

The paper proposes a physically-based retrieval of Dm from single-frequency nadir Doppler radars (PMM KuDPR, EarthCARE CPR, plus a hypothetical Ka), using specific attenuation k from PIA as a third constraint alongside Ze and Vp to remove air motion and constrain DSD shape, validated against a Monte Carlo coalescence/breakup model (MonDrop). The idea of using k as an independent constraint is worthwhile and the gap it addresses is real, since DFR is unavailable on these instruments. However, the validation does not stress the method's headline claims, several of the load-bearing assumptions are either unstated or not operationally realisable, and a few physical statements need correcting.

The motivation is timely and tied to real missions; the k-constraint is a genuine conceptual contribution distinct from the statistical EarthCARE approach; and using a Monte Carlo model is a defensible response to the scarcity of suitable observations. The error analysis, while incomplete (below), is a good-faith attempt to characterise sensitivity.

Thank you for your kind and detailed comments. Our replies are written in blue characters.

Major concerns:

1. **The validation rests on a single, highly idealized warm-rain simulation.** Despite "two settings" (W0 = 5 and 10 m/s), effectively one configuration drives the results. The simulation includes coalescence and breakup only and the cloud is initialised from a single gamma PSD at an unusually high liquid water content (2 g/m3). Because the entire retrieval error budget is dominated by the mismatch between the simulated DSD shapes and the gamma LUT, the realism of those shapes is doing most of the work, yet it is supported only by a qualitative resemblance of the Dm–R scatter to Bringi et al. (2003). A far stronger validation would (a) run the retrieval on disdrometer-measured PSDs (covering Rayleigh Ze > 10 dBZ) with added random vertical wind at, say, 0.5, 1, and 2 m/s standard deviation, and/or (b) compare against a vertically-pointing multi-frequency Doppler-spectra product that includes a W- or G-band channel for a reliable independent wind estimate. Either would test the method on realistic DSD variability and realistic air motion simultaneously.

Reply: Following the reviewer's comment, we applied this method to DSDs measured with a disdrometer in Maebashi, Japan during July and September 2022. As the reviewer noted, the primary selling point of this method is its correction for atmospheric air motion, therefore, we made estimations for data with added fixed atmospheric vertical velocity of ±1 m/s and random variations ranging from -0.5 to 0.5 m/s uniformly. The air velocities are -1.5 or +1.5m/s at the maximum for the estimation. In the Dm retrieval, the Doppler speeds are adjusted as Vp±△Vp so that the calculated specific attenuation (k) using the LUT matches the observed value. Since the wider range of △Vp added to the disdrometer data, the adjustment range for Vp was also expanded.

For such situation, the reflectivity factors (Z(Vp): Rayleigh Ze) are needed to calculate from the measured Ze and modified Vps instead of the measured Vp, and vary significantly which sometimes results in a double solution problem. There are two or more values of Vp which give the minimum difference between the estimated k and the observed value. For cases of no double solutions, i.e., the estimated ks increase/decrease monotonically with △Vp, the △Vp of the minimum difference between estimated and measured k (△k) is selected as mentioned in the manuscript. When the estimated values of k do not increase/decrease monotonically with △Vp, the double solution problem arises. To bypass the problem, we use k-Z relationship to calculate Z from k. First, select double solutions at △Vp1 and △Vp2, as △k (△Vp1), △k (△Vp2). Then two values of the Z at △vp1 and △vp2 using LUT are compared with the reflectivity calculated from a k-z relationship. The appropriate solution is selected so that the smaller difference between Z(△Vp1,2) and the reflectivity factor (Z') calculated from the k-Z relationship. The coefficients for the k-Z relationship were derived from the measured data in September. Note that the accurate k-Z relationship may not be necessary because the reflectivity factor △vp1 and △vp2 are generally relatively different associated with the large difference between △vp1 and △vp2.

Figure 1 shows the results of estimation of Dm for Ku-band. Atmospheric air velocities are sum of the fixed value (W_0) and the random variation ranging from -0.5 to 0.5 m/s as mentioned earlier. Results of the fixed air speed of W_0=0 (Fig.1a), -1m/s (Fig.1(b) and +1m/s (c) are plotted.
For W0=0 Est1 (black dots) underestimate whares Est2 (red dots) agree well with the measured Dm. For W0=-1 m/s, both Est1 and Est2 are in good agreement. These features are due to the difference of the measured shape of DSD and the LUT. For W0=+1 m/s, significant underestimated Dms of Est1 are observed associated with effects of updraft air motion, wheareas Est2 can correct the effect of the air velocities on the estimation of Dm.

Figure 2 shows result for W-band. For W_0=0, the estimated Dm are generally in good agreement both for Est1 and Est2. For W_0=-1 m/s, significant underestimation of Est1 appear, whereas Est2 are generally in good agreement. For W_0=1m/s, The results of Est1 show overestimation, whereas Dms of Est2 are in good agreement. There are some miss-selected estimated values of Est2 appear. Fig.3 is the frequency plot of the estimated errors of Est2. Most data are in the range of -0.5 to 0.5 m/s.

We added new section 6 to address these issues in the revised manuscript.

2. **The k constraint carries much less DSD-shape information than the paper claims.** You have three unknowns (Vair plus DSD concentration and shape) and three observables (Ze, Vp, k), but they do not span three independent dimensions. In the Rayleigh regime k and Ze are both essentially LWC-weighted and nearly collinear, leaving shape poorly determined. Crucially, even in the Mie regime the dependence of k on the shape parameter is weak: from Fig. 7 the spread between $\mu=1$ and $\mu=6$ is roughly 1 dB/km at a peak near 16 dB/km, under 10%. So the statement that "k depends on the DSD as well as LWC for the Mie regime" overstates the shape sensitivity. k is mostly a function of LWC and Dm, not μ. This directly undercuts the premise that k disambiguates DSD shape, and it is the real reason the Ku correction stays weak and the separate Ze/R technique (4.3) is needed. I'd recommend framing the whole method as an ill-conditioned inverse problem and quantifying the actual information content of k (e.g., via the k/Z ratio — see point 7).

Reply: For the Ku-band (Rayleigh regime), k is almost independent of the shape of the DSD, making it difficult to correct the effects of the DSD on the estimation (L320). For the W-band, whereas, k depends on the shape of DSD, particularly at large Dm values. While the correction of the effects of DSD may not be so effective, the effects of the DSD can be taken into account to some degree. Figure 4 shows the relationship of k/Ze (linear) and Dm for various values of μ. While no significant differences due to μ are observed at small Dm, larger differences appear at large Dm (20% at Dm = 2 mm), indicating the potential to reduce the effects of DSD on the retrieval to some degree. I would like to include this figure as Fig. 7b in the revised manuscript

3. **The Rayleigh-equivalent Z substitution at W/Ka (Section 4.2) is not operationally realizable and creates a logical circularity.** Z is not measured; obtaining it requires estimating the PSD parameters, but once those are known, the retrieval is unnecessary. As written, the method assumes the answer to produce the input. The LUT-based Z(Ze, Vp) conversion partially hides this but does not resolve it, and the additional error it injects is not cleanly separated in the budget. This needs either an operationally defensible route to Z or an explicit acknowledgment that the W/Ka results are a theoretical upper bound.

Reply: As the reviewer pointed out, exact conversions from Ze to Z are not possible. Even if the true Z is given, the estimated value of k derived from the observed Vp (assuming zero air motion) and Ze would differ from the true value because the DSD in the measurements is usually different from the DSD used in the LUT.

In the retrieval method, Z is derived from the observed Ze and modified Vp using the LUT, i.e., Z is a kind of 'normalized value' according to the specific LUT. The specific attenuation is also determined based on the LUT and is also 'normalized value'. Thus, Dm is estimated from both 'normalized values of Z and k. In other words, the results can be viewed as values converted according to the specific LUT employed; the LUT acts as an intermediary for estimating the true Dm. Similar techniques are frequently used in remote sensing, such as in space-borne radar (Iguchi et al, 2000) and space-borne visible radiometer (Kobayashi, GRL,2008 doi:10.1029/2008GL036140, 2007, doi:10.1029/2007GL029606).

while a Ze-to-Z conversion is not performed for Ku-band, the k value is—like that of the W-band—a value converted based on a LUT.

; consequently, if the observed DSD differs from the LUT's DSD, the estimated Z will differ from the true Z. The resulting Z is effectively a value converted based on the LUT's DSD. Similarly, the k value derived from the LUT using observed Vp (zero air motion) and Z is also a value converted based on the LUT. This method estimates Dm using Z and k, which are themselves converted from observed Ze and Vp (+delVp) via the LUT.

4. **The air-motion correction (the primary selling point) is barely exercised, and even the calm-air case already needs correction.** The clean validations (Figs 9, 11, 17, 18) are stated to be in still air for most data, so they test the secondary DSD effect, not the effect the authors themselves call dominant. Worse, the Ku-band discrepancy that motivates the μ-correction already appears with no air motion; adding realistic up/downdrafts and turbulence broadening should make errors larger, not smaller. The ±0.5 m/s random Vp sweep is too narrow given convective vertical velocities of several m/s unless the retrieval technique is dedicated to stratiform/sedimenting rain but that should be stated.

Reply: Since DSD-related effects cannot be corrected in the Ku-band, errors appear even in calm wind conditions. As the primary selling point of this method is the correction of atmospheric flow, we conducted additional validation using disdrometer-measured PSDs by introducing larger variations of the atmospheric flow as ±1 m/s, along with random errors varying uniformly between -0.5 and +0.5 m/s (reply 1). Although the resulting atmospheric velocity reaches up to ±1.5 m/s, the method successfully corrects for these effects.

5. **"k and attenuation-corrected Ze are assumed given," and the error analysis is**

correspondingly optimistic. The PIA -> k inversion is itself hard and error-prone, especially at W-band, and its errors are correlated in range rather than random. Testing +-20% random k error therefore both understates the magnitude and misrepresents the structure of the real error. The error analysis does not span the expected range and should be redone with correlated errors and the wider wind range suggested in point 1; as it stands it shows too-optimistic performance.

Reply: We also calculated the error resulting from the specific combination of Ze, k, and Vp errors that maximizes the Dm estimation error,specifically, the case where the error signs are +, +, and −. While Fig. 20 does not include the Vp error, it assumes worst-case scenarios for Z and k; the Vp error is discussed in a newly added section. We present a validation using disdrometer-measured PSDs that incorporates an atmospheric flow of ±1 m/s and random errors varying uniformly between −0.5 and +0.5 m/s. The results demonstrate that the effect of atmospheric flow is effectively corrected. We will make error analysis with the air motion of wider range.

6. **Several claims are overstated.** The assertion that the vertical Ze–Dm correlation sign can identify the precipitation life stage from GPM DPR overstates DPR's capability: the dual-frequency Dm problem is under-constrained (PIA is not always available, the gamma PSD has three free parameters, and k is not measured but inferred from an adjusted k–Z relation), and single-profile Dm is too noisy at DPR's resolution and sensitivity to read stage from the correlation sign. Similarly, "good agreement" in the abstract should be tempered by the still-air and given-k caveats.

Reply: We have toned down the claim that the precipitation life stage can be identified.

L265 The behavior of the different correlations between Z_e and D_m in altitude, may be used to determine the life stage of rain from the GPM observations

→The behavior of the different correlations between Z_e and D_m in altitude, may be used to determine the life stage of rain from the GPM observation only given accurate measurements of k.

abstract

The validation results show generallygood agreement between the estimated D_m and the model values calculated from the simulated raindrop size distributions.

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The validation results show good agreement between the estimated D_m and the model values calculated from the simulated raindrop size distribution for weak velocities

Minor points:

7. **Ze variability with μ and the k/Z ratio are not discussed.** Most attenuation retrievals hinge on the k/Z (α) coefficient, which is itself the DSD indicator the method implicitly exploits.

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The paper neither shows how Ze varies with μ at fixed Dm nor frames the information content through k/Z. Adding this would clarify exactly what the constraint buys.

We added the relationship of k/Z (Fig.4)

8. **The Atlas et al. (1973) fall-speed formula is misapplied to the cloud regime.** V = 9.65 − 10.3 exp(−0.6D) crosses zero near D ≈ 0.11 mm and is negative (i.e. spurious upward motion) below it. Since the initial cloud is set at D0 = 0.1–0.2 mm, a meaningful fraction of the starting population lies where this raindrop formula is invalid, which contaminates the early coalescence dynamics and the cloud sedimentation. A small-drop (Stokes-regime) terminal velocity should be used below the formula's validity range or at least the issue should be noted.

We used Best fall v formula.

D>0.3

V=932(1-exp(-D/1.77)^1.147)

D0.05<0.3

V=932(1-exp(-D/1.77)^1.147)

D<0.05

V=3040+D^2

9. **The frequency of the simulated Ze is not consistently stated.** Line 170 notes C-band for one figure, but elsewhere (Figs 3, 5, 6) it is ambiguous. If these are Rayleigh values, say so explicitly and uniformly.

We remove C-band and rewrite Rayleigh.

10. **The W-band "measures only light rain below 20 dBZ" framing is physically misleading.** W-band detects the same rain rates as Ku; what differs is that Mie scattering lowers the apparent reflectivity and attenuation lowers it further, so the measured Ze caps

around 20 dBZ. The correct statement is that W-band attenuates quickly and drops below sensitivity at higher rain rates, so reliable retrievals are not expected above roughly 5–10 mm/h, not that the instrument is blind to heavier rain.

Done

11. **Doppler folding** (CPR Nyquist ~5–6 m/s, Fig. 16) is not a minor aside given the method's acute Vp sensitivity; its interaction with the Vp iteration deserves explicit treatment or mentioning that Vp needs to be unfolded before the retrieval is applied.

Reply: Some EarthCARE CPR data reveal occurrences of Doppler folding errors. We added that Vp needs to be unfolded before the retrieval is applied.

12. **Figure 16 should be a 2-D histogram / density plot.** The black crosses obscure the point concentration and fully cover parts of the panel.

We will modify Fig.16.

13. **Citations and language need a careful pass.** "Moroz" should be "Mroz" (l. 309); recurring typos include "evolves DSD," "understating," "Martial-Palmer" (Marshall–Palmer), "Gillespi"/"Gillespie," "relializations," "siginicant," "diffes," "DmEs1t."

Thank you for your comments, We corrected.

14. Conclusions should mention that the algorithm is effectively a no-prior limit of the variational/optimal-estimation retrievals it is contrasted against, and the operational error chain is not accounted for. The scheme is a deterministic fixed-point iteration that adjusts Vp until the LUT-predicted k matches the observed k given Ze (and the Rayleigh-equivalent Z at W/Ka).

We will add the sentence

'The algorithm is effectively a no-prior limit of the variational/optimal-estimation retrievals it is contrasted against, and the operational error chain is not accounted for.'

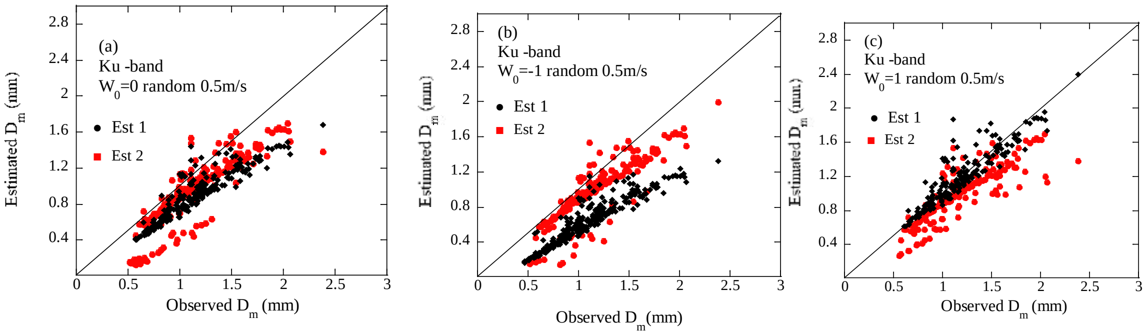

Fig.1 Estimated Dm for Ku-band

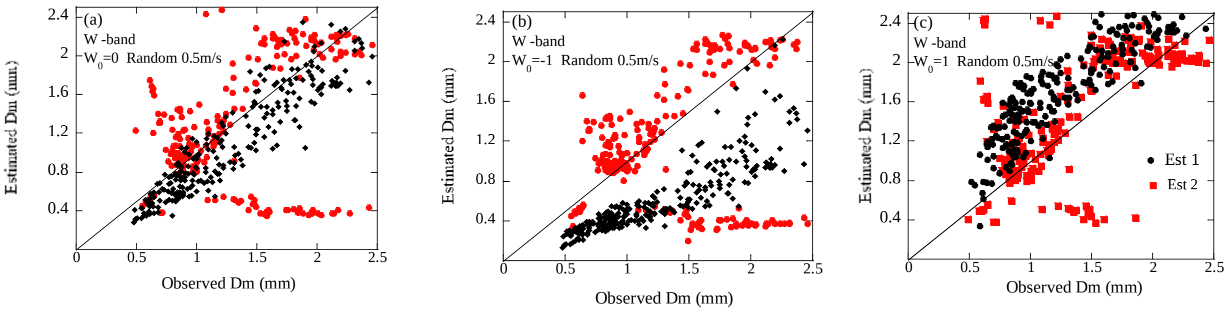

Fig.2 Estimated Dm for W-band

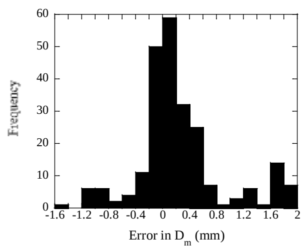

Fig.3 Frequency plot of estimated error

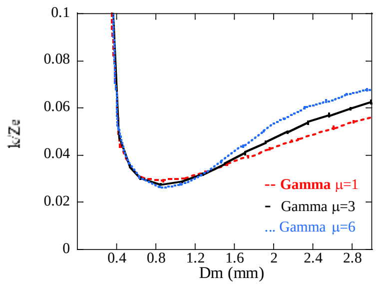

Fig.4 k/z vs Dm for various μ