Authors’ Responses to Reviewer 2

General Comment:

The authors proposed a potentially novel and useful framework for diagnosing processes related to MJO simulation errors. I see the proposed framework could be very helpful to the community, but I believe the authors should further clarify the techniques so the community can actually apply them. Further discussion of the advantages and disadvantages of the new framework relative to prior work would make the paper more valuable. In addition, having a strong visualization of the diagnostic makes it more attractive, which is another area I suggest the authors improve.

Comments:

1. It would be helpful to clarify the use of LIM in this work. I am confused about whether the authors are only using the “concept” of LIM to develop the framework or are actually using LIM as part of the framework. If it is the latter, I did not understand exactly how LIM is used in constructing the model diagnostics.

   We appreciate the valuable feedback from the reviewer. We do not employ LIM strictly as a forward-integrating model; rather, we apply its theoretical concept to our analysis. By projecting the moisture budget equation onto the MJO phase space, we construct a formulation mathematically equivalent to LIM. Consequently, standard LIM-derived metrics—including growth rate and frequency—can be readily evaluated. This type of projection-based approach has been used to study the annular mode (via projecting the zonal momentum equation onto the EOF of zonal-mean zonal wind) and to develop the Bjerknes-Jin index for ENSO. To avoid confusion, we briefly summarize the changes made in this revision.

   a. We explained the purpose of using LIM (L116 – L118).
   b. We further clarified that we only use the concept of LIM (L128 – L130).

2. I would like more explanation of how to calculate the terms on the right-hand side of Equation 7. Are you projecting the calculated moisture budget term onto the EOF of ERA5 moisture to find “M” for each term, then finding its time tendency?

   Equation 7 (Equation 8 in the revised manuscript) is derived by projecting the moisture budget term onto the EOF space. We do not first find 'M' (reduced-dimension field) and then take its time tendency; rather, because the moisture budget terms already have a unit of tendencies (kg/kg/s), projecting them onto the EOF modes directly yields the tendency of the principal component (∂M/∂t) with a dimension of 1/s. To avoid confusion, we include more discussion in L159 – L165.

3. Readability of Figs. 6-8 should be improved. It was extremely difficult to identify which term is plotted where and to find the supporting evidence for the arguments made in the text. Below are suggested changes that could help the readability of these figures:
    a. Plot leading order terms only (instead of all 28 terms)
    b. Avoid using light colors for the horizontal axis labels (term colors), as they were very difficult to see.
    c. Darken the box-and-whisker plots to make them more visible. I did not even notice at first that they were on the figures.
    d. Make the text of all labels larger. I had to zoom in a lot to read them.

We appreciate the valuable feedback from the reviewer. We have made some adjustments to Figs. 6 to 8 to improve the readability, including

- We removed the terms with secondary contribution (originally plotted in black and gray) from the main figures.
- We have increased the opacity of those statistically insignificant terms.
- The boxes and whiskers are plotted with darker colors.
- The sizes of tick labels have been increased.

To present the complete results, the original figures containing all terms are placed in the appendices.

4. The authors should further clarify the novelty of their diagnostic framework relative to prior work, including Anderson and Kuang (2012) and Adames and Ming (2018). Those studies also show the process diagnostics for the maintenance and propagation of a mode. I think those diagnostics are also phenomenological and process-oriented. Based on my understanding, the authors' new diagnostic is similar to the one used in the aforementioned prior work but requires an additional step involving EOF projections. A potential advantage of using EOF projections is to simplify the diagnostic and remove potential noise, but at the same time, the resultant diagnostic could lose information. For example, when the MJO structure deviates from the structure captured by the first EOFs (e.g., during El Nino, Kessler 2001), the EOF-based diagnostics could become less accurate. Additional discussion to clarify the advantages and disadvantages of the authors' new diagnostic framework relative to prior work would be helpful.

We thank the reviewer for this insightful comment and the opportunity to clarify the advantages, disadvantages, and overarching novelty of our new diagnostic framework relative to the pioneering works of Andersen and Kuang (2012) and Adames and Ming (2018).

While we agree that those prior diagnostics are exceptionally valuable, phenomenological, and process-oriented, our framework introduces distinct methodological choices designed to address different research objectives. Below, we discuss the specific advantages and disadvantages of our EOF-based, LIM-coupled framework, as well as the points raised by the reviewer regarding information loss.

1. Structural Deviations and Information Loss (Disadvantages & Limitations)

The reviewer raises a valid point regarding the potential drawbacks of EOF projection:

- **Structural representation during mean-state drift:** We agree that when the MJO structure deviates significantly from the first two leading EOF modes —such as during strong El Niño events (Kessler, 2001), or due to the inherent diversity of MJO events (Wang et al. 2019)—projecting data onto the fixed EOFs of the ERA5 reference state can introduce errors or underestimate the anomaly's amplitude. We have explicitly acknowledged this limitation in the revised manuscript (L301 – L303).

- **Information loss vs. Noise filtering:** This presents a classic methodological trade-off. While space-time filtering retains the full spatial field of a specific frequency band, it also preserves localized noise. In contrast, our EOF projection acts as a physical filter, extracting only the large-scale spatial patterns responsible for the bulk of the variance. Although this means some fine-scale or state-dependent information is inevitably lost (L303 – L305), this abstraction allows us to map high-dimensional grid-point budget fields directly onto a highly simplified and intuitive 2D phase space (L306 – L308).

2. Key Advantages and Novelty Relative to Prior Work

Despite these tradeoffs, our framework offers several novel advantages over the traditional grid-point or wavenumber-frequency budget diagnostics utilized by Andersen and Kuang (2012) and Adames and Ming (2018):

- **Direct Bridge to Real-Time Performance Metrics:** Traditional budget diagnostics typically operate in physical or wavenumber-frequency space. By projecting the moisture budget into an EOF-defined phase space, our framework establishes a direct, mathematically closed link between process-level biases (the right-hand side of Eq. 7) and widely recognized phenomenological performance metrics (the MJO indices on the left-hand side) (L306 – L308). Because these indices are standard and routinely monitored for operational purposes, this approach allows model developers to efficiently pinpoint which specific physical processes (e.g., horizontal vs. vertical advection) are driving errors in simulated MJO propagation and amplitude (L329 – L333).

- **Intuitive Unit Circle Decomposition:** While conventional approaches typically define maintenance or propagation by projecting individual budget terms onto the total field or raw tendency, our framework translates these terms into a quantitative vector field ($\partial M/\partial t$) within the phase space. By decomposing these vectors into tangential and radial components via "Unit Circle Analysis," we can intuitively isolate the specific physical processes driving errors in propagation speed (tangential component) versus maintenance or decay rates (radial component) (L309 – L312).
- **A Prognostic Framework for Direct Theoretical Validation:** The core distinction lies in how theory is validated. Excellent frameworks like those by Andersen and Kuang (2012) and Adames and Ming (2018) operate primarily as *diagnostic tools*. They look at a pre-existing simulation and evaluate the balances between various physical terms to see if they align with theoretical expectations. However, they do not construct an independent, predictive theory from those fragments.

  By definition, a **predictive theory** requires a minimalist model capable of actively simulating and predicting all the essential ingredients of a phenomenon from a set of closed equations. A clear historical parallel can be drawn from El Niño-Southern Oscillation (ENSO) research : while individual conceptual fragments of ENSO theory were heavily documented throughout the 1970s, the theory was not considered complete or fully validated until the development of the minimalist, closed, and prognostic Zebiak-Cane model (1987), which actually simulated the coupled loop.

  By incorporating Linear Inverse Modeling (LIM), our framework functions as this missing prognostic engine. Instead of just calculating passive diagnostics from a GCM, we project the physical governing equations (moisture-mode theory) into a reduced phase space while preserving their dynamical invariance. This turns our framework into a fully realized, active testbed where theoretical instabilities, growth rates, and frequencies emerge organically from the minimalist operator and can be directly simulated and compared against observations.

We have expanded the discussion in L313 – L325.

- **Scalability to Multi-Variate Systems:** While this study uses moisture-mode theory as a testbed, the underlying methodology—combining statistical mode extraction with linear inverse modeling (LIM)—is fundamentally scalable and not limited to a single prognostic variable. In more complex frameworks, such as the trio-interaction theory (Wang et al., 2016) or moisture vortex instability (Adames and Ming, 2018), the system is governed by coupled equations involving multiple interacting variables (e.g., moisture, vorticity, and geopotential height). Traditional budget analyses often struggle to link these distinct fields without relying on heavy *a priori* assumptions. In contrast, our framework can naturally incorporate multiple variables into a single state vector. This provides an objective method to diagnose cross-variable interactions and resulting model biases without being constrained by a single, pre-selected thermodynamic framework (L326 – L329).

References:

- Zebiak, S. E., & Cane, M. A. (1987). A Model El Niño–Southern Oscillation. Monthly Weather Review, 115(10), 2262-2278.
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