Supplement to Author Comment on RC2

This document provides supporting material for the author comment responding to Referee #2. It summarizes the planned global basin map, the planned symbol table, and the supplementary diagnostic for the Eq. (8) assumption.

Summary of supporting materials

Item	Purpose in RC2 response	Planned location
Figure A1	Shows the global spatial distribution and regional coverage of the 228 representative basins.	Main manuscript; replaces original Table 2.
Table A1	Provides a self-contained notation guide for the main symbols used in the manuscript.	Supporting Information.
Figure A2	Evaluates whether precipitation and runoff coefficient vary systematically across H-S orders, supporting the interpretation of Eq. (8) as a first-order approximation.	Supporting Information.

A1. Global Distribution of Representative Basins

This figure addresses the reviewer's request for a map showing the location and extent of the 228 selected basins. The planned map will replace the original Table 2 and will make the geographical coverage and regional distribution of the selected basins more transparent.

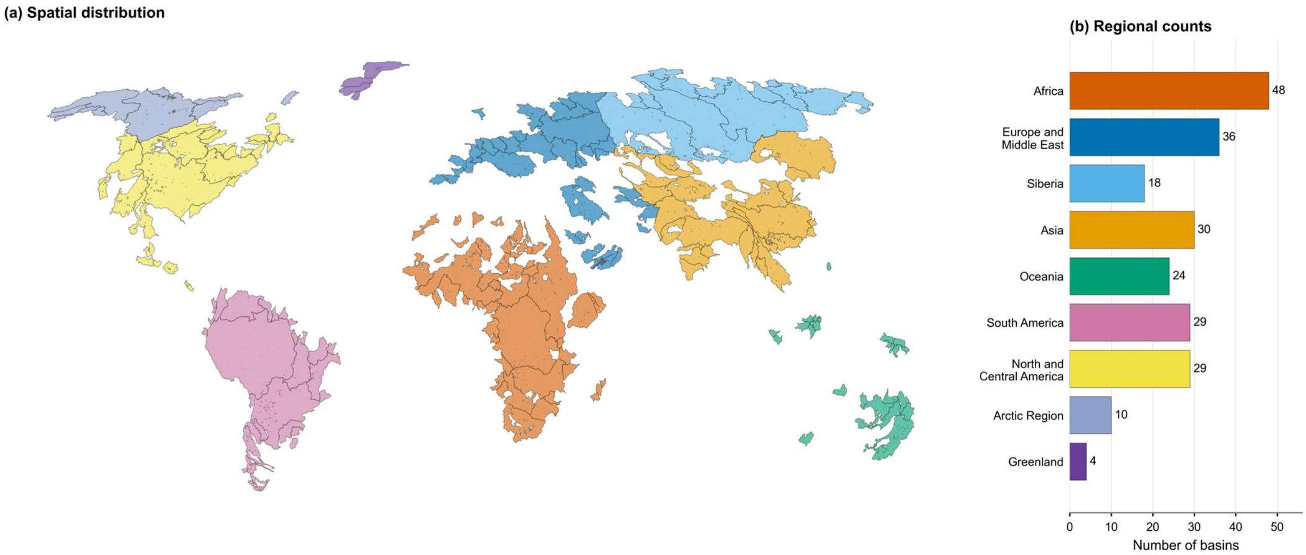

Figure A1. Global distribution of the 228 representative river basins. Basins are colored by HydroBASINS geographic region, and the accompanying bar plot summarizes regional basin counts.

A2. Planned Symbol Table

The notation system in the manuscript includes H-S order variables, hydrological ratios, fitted ratios, geometric ratios, and basic-unit parameters. To improve readability, the planned revision will add a symbol table to the Supporting Information and refer to it from the Methods section.

Table A1. Planned summary table of main symbols

The table below is planned for the Supporting Information.

Symbol	Definition	Notes / calculation
ω	H-S stream order.	Horton-Strahler order index.
Ω	Maximum H-S order of a river network.	Highest order after applying H-S ordering rules.
N_ω	Number of river segments of order ω.	Used for branch-ratio calculations.
$R_B(\omega)$	Branch ratio between consecutive H-S orders.	$R_B(\omega) = N_{\omega-1} / N_\omega$.
r_ω	Cumulative runoff volume for order ω.	Aggregated from runoff depth and catchment area.
d_ω	Mean discharge for order ω.	Mean annual discharge summarized by H-S order.
$R_r(\omega)$	Adjacent-order runoff ratio.	$R_r(\omega) = r_\omega / r_{\omega-1}$.
$R_d(\omega)$	Adjacent-order discharge ratio.	$R_d(\omega) = d_\omega / d_{\omega-1}$.
$\tilde{R}_r$	Fitted runoff ratio.	Estimated from log-linear fitting across orders.
$\tilde{R}_d$	Fitted discharge ratio.	Estimated from log-linear fitting across orders.
C_ω	Mean confluence catchment area for order ω.	Used in the Eq. (8) approximation.
A_ω	Mean upstream watershed area for order ω.	Used in the Eq. (9) approximation.
$R_C(\omega)$	Confluence-area ratio between consecutive H-S orders.	$R_C(\omega) = C_\omega / C_{\omega-1}$.
$R_A(\omega)$	Watershed-area ratio between consecutive H-S orders.	$R_A(\omega) = A_\omega / A_{\omega-1}$.
P_ω	Mean precipitation for order ω.	HydroATLAS precipitation attribute, summarized by order.
α_ω	Runoff coefficient for order ω.	Estimated as runoff depth divided by precipitation, $\alpha_\omega = \text{runoff}_\omega / P_\omega$.
λ	Integer topological characteristic length of a basic unit.	Defined by the number of inner links composing a basic unit.
$R(\lambda)$	Normalized runoff as a function of characteristic length.	Used in the basic-unit length analysis.
$D(\lambda)$	Normalized discharge as a function of characteristic length.	Used in the basic-unit length analysis.
R_r^{unit}	Runoff ratio of a basic unit.	Basic-unit parameter in the pyramid decomposition framework.
R_d^{unit}	Discharge ratio of a basic unit.	Basic-unit parameter in the pyramid decomposition framework.
$R_r^{unit\_in}$, $R_r^{unit\_out}$	Runoff ratios of inner and outer basic units.	Used to distinguish inner and outer unit behaviours.
$R_d^{unit\_in}$, $R_d^{unit\_out}$	Discharge ratios of inner and outer basic units.	Used to distinguish inner and outer unit behaviours.

A3. Diagnostic for the Eq. (8) Assumption

The reviewer correctly noted that Eq. (8) involves a simplifying assumption that mean precipitation and runoff coefficients do not dominate the adjacent-order runoff scaling. To evaluate this assumption, we computed a supplementary diagnostic using RiverATLAS hydro-climatic attributes.

For each representative basin and H-S order, all RiverATLAS reaches with the same MAIN_RIV were aggregated using catchment-area weighting. Mean precipitation was obtained from the HydroATLAS precipitation attribute, and runoff coefficient was estimated as runoff depth divided by precipitation. The diagnostic included 5,054,231 RiverATLAS reaches from the 228 representative basins.

For orders 1-7, which contain nearly complete basin-order samples, the basin-normalized median precipitation ranged from 0.94 to 1.03, while the basin-normalized median runoff coefficient ranged from 0.98 to 1.01. Higher orders showed larger uncertainty because they were represented by fewer basins. These results support the interpretation of Eq. (8) as a first-order topological approximation, while also confirming that precipitation and runoff-coefficient heterogeneity should be acknowledged as a source of scatter rather than ignored.

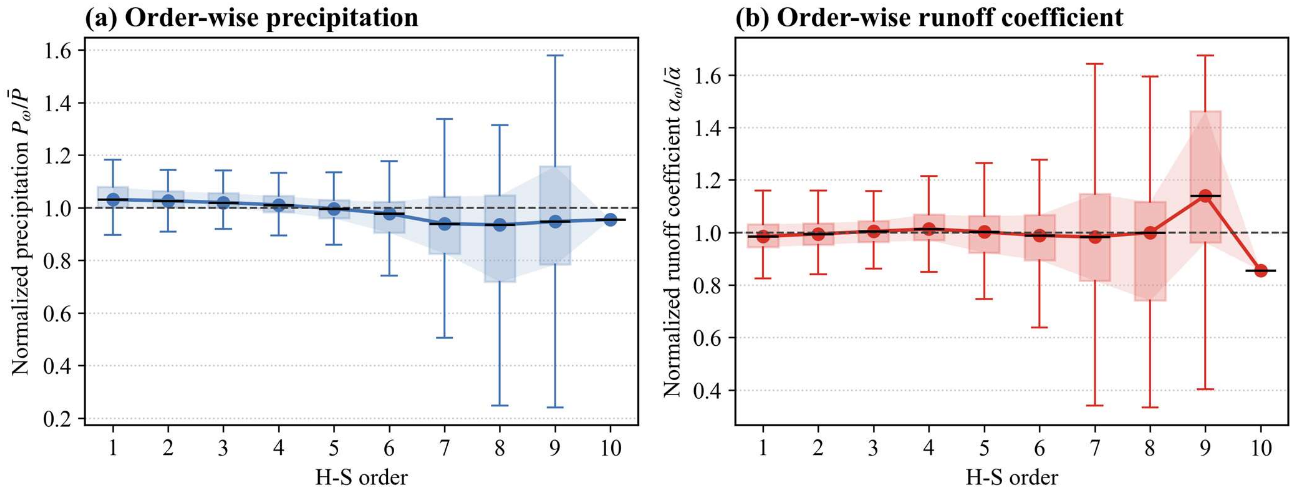

Figure A2. Order-wise precipitation and runoff-coefficient diagnostic for Eq. (8). Panel (a) shows basin-normalized precipitation by H-S order. Panel (b) shows basin-normalized runoff coefficient by H-S order. Boxes show interquartile ranges, points show medians, and the dashed line indicates the basin-normalized reference value of 1.