

Reviews (2nd cycle) ShyBFM v1.0 (Alessandri et al., 2026)

Review #1

RC1.1: I would like to thank the authors for their hard work to answer all my critics and questions.

The revised version is much clearer, and i'm grateful for that. No doubt, SB shows improved seasonality and temporal variability, compared to NB. One question still remains though. Both SB and NB are quite different, and it is unclear how much of BFM's BGC's improvements in SB - seen in the skill score - is due to the coupling to SHYFEM-MPI, or the ER-optimized parameters, tides, ... I don't think we can say for sure, but the paper is now very clear about the models set-up for the reader to understand the outputs and comparisons.

I am now happy for this paper to be accepted as is.

Response: *We thank the reviewer for the positive assessment and for the constructive engagement throughout both rounds of review, which has substantially improved the manuscript.*

We fully agree with the remaining point. SB and NB differ simultaneously in several respects: the unstructured grid and its higher coastal resolution, the inclusion of tides and the BFM parameter optimization, so the improvement in the skill scores has to be regarded as their combined effect. A proper attribution would require a dedicated set of experiments, which we plan for a follow-up study.

Review #2

RC2.1: The revised manuscript is significantly improved in most respects. The addition of Figure 3 provides helpful physical context for interpretation of the biogeochemical evaluation, and this is reflected in the discussion. The addition of NB parameter values in the supporting information is also helpful in understanding the differences between SB and NB.

However, I believe the equations in section 2.1 require further attention. Based on the revised equations, I have further questions regarding the relevance of the idealized case considered in equation 5, the accuracy of the subsequently derived general boundary conditions, and the relationship between the equations presented in this section and the discretized model equations presented in Section 3.

Response: *We thank the reviewer for the careful reading of section 2.1 and for pointing out imprecisions in the derived equations. Prompted by these comments we re-derived the volume integral budget from scratch and we wrote the corrected equation (5) and boundary conditions. We wrote a further equation (new eq. 8; see response to RC2.3) that shows the link*

between the volume integral and the boundary conditions, explaining more clearly the role of the terms in the boundary condition definition, the assumptions required and the connection with the discretized equations.

RC2.2: In equation 5, it appears that something must have gone wrong in the last term on the left-hand-side, as the limits of integration refer to vertical positions and the integration is over dOmega (area).

Response: This was a typo; the limit of integration is over the horizontal area Omega. However, after re-deriving the volume integral we realized that also the bottom diffusive slope term was missing (the one proportional to $\nabla_h H$). We therefore rewrote Eq. 5 as:

$$\begin{aligned} \frac{d}{dt} \int_V C \, dV = \int_{\Omega} \left[-C_{\eta} (E - P - R) - w_s|_{z=\eta} C_{\eta} + w_s|_{z=-H} C|_{z=-H} + \right. \\ \left. + k_v \frac{\partial}{\partial z} C|_{z=\eta} - k_v \frac{\partial}{\partial z} C|_{z=-H} - k_h \nabla_h C|_{z=\eta} \cdot \nabla_h \eta - k_h \nabla_h C|_{z=-H} \cdot \nabla_h H \right] d\Omega + \int_V \frac{\partial C}{\partial t} \Big|_{bio} dV \end{aligned} \quad (5)$$

Note that the terms $k_h \nabla_h C|_{z=\eta} \cdot \nabla_h \eta$ and $k_h \nabla_h C|_{z=-H} \cdot \nabla_h H$ are written for completeness and go identically to zero because in the layer-integrated formulation the horizontal diffusive flux is tangent to the layers by construction. The new eq. (5) was written in the manuscript, and this text was added (lines 102-104):

“In the layer-integrated formulation of the model, horizontal diffusion occurs between different columns of the same layer, so its flux is tangent to the layers by construction, and the 6th and 7th terms on the RHS of (5) are identically zero.”

RC2.3: I am not sure about the $C_{\eta}(E-P-R)$ term on the RHS of (5). While $(P+R-E)$ would locally dilute tracer concentration at the surface, it would not contribute to the volume integral tracer amount if those terms consist of water with zero tracer concentration. Otherwise, shouldn't a time derivative tracer concentration term be present? Or is this a simplification of some kind? I am unsure of the relevance of equations 5-7 given that the actual model boundary conditions are more complex and imply a changing inventory of tracer over time within the model domain.

Response: The $C_{\eta}(E-P-R)$ term in eq. 5 is due to the kinematic condition (eq. 3) and represents the vertical advective tracer flux at the surface ($z=\eta$) due to the water flux ($E-P-R$). The same boundary condition is used for salinity in general circulation models with free surface and water fluxes and even there we gain/lose only water as a physical process. However, if we want to get conservation of the salinity or tracer with water fluxes, we need to impose that the vertical advective flux term is perfectly balanced by the turbulent diffusion term (eq. 8 of the submitted version, now eq. 6) representing only a dilution/concentration effect. We have also added a reference for salinity (Verri et al., 2020; [10.1016/j.ocemod.2020.101587](https://doi.org/10.1016/j.ocemod.2020.101587)), which validity extends also to any generic tracer.

If there are sources/sinks of tracers at the air-sea interface (non-conserving tracer boundary condition), we can impose the tracers' fluxes associated with water entering or exiting the sea by advection and by dilution/concentration effects: these are the additional terms added in eq.

8 (now eq. 6). At the bottom boundary, we use a benthic return parameterization, so the particulate matter that we lose by deposition, returns as dissolved nutrients. This is highlighted with the term $F_{rem}(C_{bent})$ in eq. 9 (now eq. 7).

We understand that the old set of equations 6 to 9 could be confusing because eq. 8 and 9 are simply eq. 6 and 7 with the addition of the external tracer input. We decided to keep just eq. 8 and 9 (that will be the new eq. 6 and 7 in the revised manuscript) that are the actual boundary conditions and already contain the external forcing terms. Moreover, we added a final equation (eq. 8 in the revised manuscript) where boundary conditions are inserted into Eq. 5 and it shows that the tracer content in a laterally closed domain changes only through riverine (RC_{rw}), atmospheric wet (PC_{pw}) and dry ($F_g(C_{air})$) inputs at the surface, net pelagic-benthic exchange ($w_s|_{z=-H}C|_{z=-H} + F_{rem}(C_{bent})$) and internal biogeochemical transformation.

$$\frac{d}{dt} \int_V C \, dV = \underbrace{\int_{\Omega} (C_{rw}R + C_{pw}P + F_g(C_{air})) \, d\Omega}_{\text{external loading}} + \underbrace{\int_{\Omega} w_s|_{z=-H}C|_{z=-H} + F_{rem}(C_{bent}) \, d\Omega}_{\text{net pelagic-benthic exchange}} + \underbrace{\int_V \frac{\partial C}{\partial t}|_{bio} \, dV}_{\text{net reaction}} \quad (8)$$

In this case the tracer volume integral is not conserved but the equation is physically consistent with what happens in the model framework. After the new equation we added the following text (lines 122-123):

“The rate of change of tracer inventory evolves in time due to surface fluxes, equal to the sum of river contribution and wet and dry deposition, the net pelagic-benthic exchange and the net biogeochemical reaction rates.”

RC2.4: Equation 8: what is the meaning of $C_{\eta}(E-P-R)$ here?

Response: In Eq. 8 (new eq. 6) the term $C_{\eta}(E-P-R)$ is the part of the prescribed surface diffusive flux that compensates the vertical advective flux generated by the kinematic condition (3). Once Eqs. 8 and 9 (new Eq. 6 and 7) are substituted into Eq. 5 the role of this term becomes clear: it cancels the advective contribution exactly, so that the net effect on the inventory reduces to the external riverine, wet and dry inputs.

RC2.5: line 107: Would it be correct to simply state the surface boundary condition as one of specified surface fluxes across the air-sea interface equal to the sum of wet deposition, dry deposition, and river contributions?

Response: We added a new equation, eq. 8, that clarifies the role of the surface boundary condition when there is an external forcing (wet and dry deposition and river contribution). The surface boundary condition contains also the dilution/concentration term $C_{\eta}(E-P-R)$ that compensates the advective term in (3), so that the net effect is composed of river contribution and wet and dry deposition. We revised the sentence as follows in lines 105-109:

“In the case of surface inputs of tracer, the surface boundary condition is equal to the sum of wet deposition by precipitation, PC_{pw} , dry deposition, $F_g(C_{air})$, and river contributions, RC_{rw} , plus the term $C_{\eta}(E - P - R)$ which ensures that the freshwater volume flux does not alter the tracer inventory and accounts for the dilution/concentration effects (Verri et al., 2020). Because

particulate matter does not settle out through the air-sea interface, $w_s|_{z=\eta}C_\eta = 0$. The most general surface vertical flux boundary condition is therefore:"

$$k_v \frac{\partial}{\partial z} C|_{z=\eta} = C_\eta (E - P - R) + RC_{rw} + PC_{pw} + F_g(C_{air}) \quad (6)$$

Please note that we removed the sinking velocity from the surface boundary condition, because $w_s|_{z=\eta}C_\eta = 0$ and does not enter the boundary condition.

RC2.6: In equation 9, is the C in $F_b(C)$ the concentration within the benthic compartment?

Response: Yes, and this represent the remineralization flux, however the current notation was not correct, because C was defined as the tracer concentration in the water column. We defined a new variable C_{bent} for the concentration within the benthic compartment. Moreover, we needed to make a clear distinction in this section between the behavior of particulate matter and dissolved tracers. We modified section 2.1 as follows (lines 113-120):

"Particulate organic matter (POM) is deposited into the benthic compartment, where it is remineralized using constant rates and returned to the water column as dissolved inorganic nutrients:

$$k_v \frac{\partial}{\partial z} C|_{z=-H} = -F_{rem}(C_{bent}) \quad (7)$$

$F_{rem}(C_{bent})$ is the remineralization flux returned from the sedimented pools. Here C_{bent} denotes the areal content of the tracer in the benthic compartment, i.e. the sedimented POM and benthic nutrient pools. For particulate, $F_{rem} = 0$, since organic matter is returned to the water column as dissolved inorganic nutrients and (7) reduces to a vanishing diffusive flux at the bottom, the whole exchange is the settling flux. Dissolved nutrients have no settling velocity ($w_s = 0$) and the benthic return imposes a remineralization flux from the bottom toward the water column, $F_{rem}(C_{bent})$."

Please note that we removed the settling flux ($w_s|_{z=-H}C|_{z=-H}$) in the bottom boundary condition (9) (new eq. 7) because this would prevent particulate matter to exit the water column.

RC2.7: In general, perhaps it would be helpful to explain how the boundary conditions enter in the discretized equations?

Response: We expanded section 3 to show where in the discrete model the boundary conditions enter in the system. The surface and bottom boundary conditions are added to the explicit term in eq. (15) (new eq. 14) which was modified as follows (lines 183-186 of the revised manuscript):

$$h_l^n \tilde{C}_l^n = h_l^n C_l^n + S_{phys}^{expl}(C_l^n)\Delta t + h_l^n S_{bgc}(C_l^n)\Delta t + \Delta t [\delta_{t1}F_s + \delta_{tL}F_{rem}(C_{bent})]_l^n \quad (14)$$

“Where Δt is the timestep and δ is the Kronecker delta with L the index of the deepest layer of the water column and “1” indicating the first layer. The last term on the RHS of (14) accounts for the surface and bottom boundary conditions and is active only at the surface ($l=1$) and bottom layers ($l=L$). $F_s = C_\eta(E - P - R) + RC_{rw} + PC_{pw} + F_g(C_{air})$ is the prescribed surface flux while $F_{rem}(C_{bent})$ is the remineralization flux, as discussed in section 2.1.”

We added a further clarification for the definition of eq. (15) at the surface and bottom layers (lines 190-192):

“At the surface and bottom layers, the vertical diffusive flux is not evaluated but prescribed by (6) and (7). Accordingly, $K_{V(0)}^{n+1} = K_{V(L)}^{n+1} = 0$ in (15), the first and last rows of the system reduce to two terms, and the prescribed fluxes enter the solution through the last term of (14).”

RC2.8: line 526: I think it might be clearer to specifically say "core-hours" rather than "computational resources" here.

Response: *We agree with the reviewer and changed the sentence as follows (line 536-537 of the revised manuscript):*

“NB was almost 3 times faster but used around 2.5 times more core-hours compared to SB.”