

Physically Constrained Retrieval of Fog Visibility from Millimeter-Wavelength Cloud Radar Observations

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Abstract. Fog severely degrades visibility (Vis) and poses substantial risks to transportation and aviation, yet its quantitative monitoring remains challenging ~~, because conventional optical and passive remote sensing techniques lose sensitivity under dense fog conditions. Millimeter-wave~~ because conventional ground-based instruments provide spatially limited point observations and cannot resolve three-dimensional fog structure, while passive satellite retrievals have insufficient vertical resolution and difficulty separating fog from low cloud. Millimeter-wavelength cloud radar (MMCR) ~~provides a promising alternative due to its strong penetration capability and fine spatial resolution; however existing Vis retrieval algorithms largely can penetrate fog and provide fine spatiotemporal resolution, but many radar Vis retrievals rely on empirical parameterizations, resulting in considerable uncertainties. In this study, we develop~~ reflectivity-visibility relations. This study develops a physically constrained ~~fog Vis retrieval framework that explicitly links radar reflectivity(Z) to fog microphysical properties through radiative transfer theory. Visibility is inferred by estimating the extinction coefficient from independently derived fog,~~ observation-calibrated single-wavelength Ka-band MMCR retrieval that links reflectivity, liquid water content (LWC)~~and effective droplet radius(re), thereby avoiding purely empirical formulations. The method is applied to a Ka-band MMCR observations at the Datan Observation Base (DOB) Fujian province, China. LWC is retrieved using a,~~ geometric median radius, extinction, and Vis through lognormal droplet-size-distribution moments. A segment-wise mass-absorption ~~based algorithm developed in our previous study. A power-law relationship between the geometric mean radius (rm) and radar reflectivity (Z) is then established using 5 years of droplet size distribution data from an~~ LWC power law and an extinction-oriented radius power law are fitted to same-source FM-120 fog droplet spectrometer, enabling a reasonable estimation of re from observation. Based on the obtained LWC and droplet size parameters, the extinction coefficient (β_{ext}) and Vis are subsequently derived. Validation against ten sea fog events in spring 2025 demonstrates that the proposed algorithm achieves a fractional error of 36.61%, outperforming commonly used empirical approaches, which exhibit errors ranging from 38.61% to 77.08%. These results indicate that our proposed framework provides a more reliable and physically interpretable solution for high-precision fog visibility retrieval using single-frequency MMCR observations extinction-matching pseudo-targets, achieving a mean absolute fractional extinction error of 9.37% on the development dataset. In ten spring-2025 DNQ3-conditioned cases, the present

retrieval attains a mean absolute error of 0.092 km, while the local FM-120 and Liu et al. (2019) relations yield broadly comparable ranges and the Gultepe et al. (2009) relation departs more strongly. This method improves the physical self-consistency of single-band MMCR fog Vis retrieval without requiring auxiliary in-situ observations and provides technical support for three-dimensional coastal fog monitoring and traffic meteorological early warning.

1 Introduction

Fog ~~an atmospheric phenomenon~~, forms when moist air near the surface becomes saturated and ~~condenses to water droplets~~ ~~that can significantly reduce water vapor~~ condenses into droplets, often reducing horizontal visibility (~~Vis~~) ~~often Vis~~ to below 1 km. It is ~~highly prevalent in valley~~, ~~prevalent in valleys and in~~ mountainous, urban, and coastal regions, where radiative cooling, moisture advection, evaporation, or orographic lifting ~~lead the can cause~~ near-surface air to reach saturation. ~~It Fog~~ poses substantial risks to port operations, maritime traffic safety, and regional economic activities (Gultepe et al., 2009; Leung et al., 2020; Duran-Rosal et al., 2018). ~~Vis~~, ~~As~~ the core parameter for quantifying fog intensity and guiding operational decisions, ~~Vis~~ is defined as the maximum horizontal distance at which an observer with normal eyesight can distinguish a target against its background under current weather conditions (~~Long et al., 2021; Miclea et al., 2020~~) (~~Middleton, 1957; Long et al., 2021; Miclea et al., 2020~~). Despite its operational importance, ~~accurately measuring Vis and characterizing accurate measurement of Vis and characterization of~~ fog structure remain challenging ~~due to the because of~~ limitations in existing observational techniques.

Various instruments have been used to conduct fog observations. Ground-based sensors such as optical visibility meters and fog droplet spectrometers provide high-precision point measurements but fail to capture the horizontal or vertical distribution of fog (Tjugum et al., 2005; Cheng et al., 2018; Zhou et al., 2024). Laser-based active remote-sensing instruments (e.g., lidar, ceilometers) offer vertical profiling capabilities; however, their signals suffer severe attenuation in dense fog, often leading to signal saturation or complete loss (Zang et al., 2019). Satellite passive imagers enable large-scale fog detection but struggle to distinguish fog from low-level clouds and lack vertical resolution to resolve internal stratification or microphysical structures (Ran et al., 2022; Guo et al., 2021). These limitations hinder reliable short-term fog nowcasting and proactive risk mitigation in transportation and maritime operations (Mahdavi et al., 2024; Antoine et al., 2023; Kamangir et al., 2022; Gultepe et al., 2007). In recent years, millimeter-wave cloud radar (MMCR) has gained prominence in fog monitoring due to its strong penetration, fine spatiotemporal resolution, and sensitivity to small hydrometeors. Field campaigns such as the 2021 Pingtan Sea Fog Experiment in China's Fujian Province have shown that Ka-band radar maintains robust signal integrity under dense fog conditions (Hu et al., 2022). ~~Bell (2021)~~ Bell (2021) utilized dual-frequency radar to observe multi-layer fog, and by assimilating these radar data into the Applications de la Recherche à l'Opérationnel à Méso-Echelle (AROME) model, they reduced the fog-top height error by 64 % and largely improved visibility forecasts. ~~In the absence of large hydrometeors, MMCR has gained prominence in fog monitoring for its strong penetration, fine resolution, and sensitivity to small hydrometeors~~ (Horvath, 1981; Long et al., 2021; Miclea et al., 2020). The development of millimeter-wave radar has thus provided a new means for fog detection.

Previous studies have made advancement in quantifying fog visibility. Many existing works aim to improve the conversion and simulation advanced the quantification of fog visibility by establishing the relationship between fog microphysical properties and Vis based on their physical definitions. Early work by Eldridge (1966) proposed an inverse relationship between Vis and fog liquid water content (LWC) Vis and LWC through bulk microphysical analysis, laying the foundation for empirical Vis modeling. Subsequent research demonstrated that droplet size distribution (DSD). Subsequent studies showed that DSD strongly modulates extinction and visibility, motivating the development of more sophisticated models. For example, Gultepe et al. (2006). Gultepe et al. (2006) incorporated droplet number concentration (N) into a multi-parameter into a multiparameter scheme, improving Vis estimates in Numerical Weather Prediction (NWP) Vis estimates in numerical weather prediction models by up to 50%. Song et al. (2019) further enhanced Vis retrieval performance by integrating %. Song et al. (2019) further incorporated effective droplet diameter. Collectively, these These studies indicate that incorporating fog microphysical properties can substantially improve the accuracy and improve the physical consistency of Vis visibility estimation (Ui et al., 2023; Jayakumar et al., 2025; Claxton, 2025). Despite these advances, most current Nevertheless, most radar-based Vis parameterizations remain largely empirical, typically derived Vis parameterizations remain empirical and are commonly obtained by directly fitting Vis to radar reflectivity (Z) Vis to Z (Gultepe et al., 2009). Although Yunlong Li et al. (2017) Li et al. (2017) introduced radar attenuation correction to obtain more accurate Z , the resulting Vis retrieval still relies primarily on empirical relationships. More recent radar-based approaches have begun to mainly on empirical relations. Approaches that account for microphysical influence; however, they generally rely on auxiliary instruments to provide key supplementary information such as attenuation measurement or detailed fog microphysical parameters. This reliance on additional observations significantly restricts operational flexibility and constrains the applicability in environments where supporting instrumentation is unavailable, thereby hindering robust fog detection and monitoring under practical condition effects often require auxiliary attenuation or microphysical measurements, restricting operational flexibility where supporting instruments are unavailable.

In this study, we develop a fog Vis physically based, observation-constrained visibility retrieval algorithm for single-wavelength MMCR. Building on radar Ka-band MMCR. Radar transmission theory and fog microphysics, we established quantitative physical linkages among Z , fog microphysical parameters—LWC provide moment-based links among reflectivity (Z), liquid water content (LWC), effective radius (r_e), and the extinction coefficient (β_{ext} , a 2nd-moment parameter). This new physically constrained framework does not rely on empirical coefficients, therefore largely reduces the errors introduced by traditional methods and substantially improves adaptability and stability across a wide range of fog conditions, enabling accurate Vis estimation. The observational instruments and theoretical basis of the proposed retrieval algorithm are described in Sections β_{ext} ; however, these physical relations alone cannot resolve the ill-posedness inherent in single-wavelength radar measurements. We therefore use five years of FM-120 droplet size distributions as a priori microphysical constraints to inform the retrieval. Specifically, the observed microphysical ranges are used to (i) constrain the initialization and bounds of the mass-absorption-based LWC inversion and (ii) establish an extinction-oriented power-law relationship between the geometric median radius (r_m) and linear reflectivity (Z_{lin}). Section 2 and describes the instrumentation and FM-120 processing. Section 3, respectively. Section presents the theoretical framework, the observed DSD regimes, and the two-step retrieval. Section 4

analyzes method sensitivity to key parameters using data from fog droplet spectrometer monitor (FM-120) manufactured by the Droplet Measurement Technology company. Section evaluates retrieval uncertainties associated with DSD shape, the constant- Q_{ext} approximation, and perturbations in LWC and radius. Section 5 evaluates the algorithm's performance using a typical demonstrates the algorithm through a 2025 sea-fog case and statistical validation across ten sea-fog cases. Section study and a ten-event consistency comparison. Section 6 finally summarizes the major findings and implications for operational fog monitoring summarizes the findings and limitations.

2 Instrumentation and data

The Datan Observation Base (DOB) is located on Dadeng Island, Xiamen (24.55°N, 118.32°E) within the Taiwan Strait and surrounded by ocean on three sides. The site is operated by the Xiang'an District Meteorological Bureau (XDMB) under the Fujian Meteorological Bureau (FMB) and is situated in one of China's most fog-prone coastal areas. Positioned within a transition zone between marine and continental climates, this area frequently experiences interactions of warm, moist marine air masses and cold air, creating highly favorable conditions for sea fog formation (Zhang et al., 2024, 2023; Niu et al., 2010). Dedicated sea-fog Dedicated monitoring campaigns have been conducted at DOB since 2019, yielding five years of fog observations that provide a valuable foundation for this study. Three specialized instruments are deployed at DOB for fog observation and analysis. 2019. The YLUKA2 solid-state Ka-band dual-polarization cloud radar provides radar reflectivity factor to drive the fog Vis retrieval algorithm reflectivity for the visibility retrieval. A DNQ3 Vis meter, integrated into an automatic observation platform, serves as the in-situ reference for validating the retrieved Vis. In addition, visibility meter provides the near-surface point record for the ten-event consistency comparison, and an FM-120 fog droplet spectrometer provides detailed measurements of droplet size distribution and microphysical properties, which supports spectrometer provides DSD and microphysical measurements for algorithm development and sensitivity analysis.

The YLUKA2 MMCR, manufactured by Aerospace New Sky Technology, is installed at the highest point of DOB, with an antenna height of approximately 25 m above the ground. Operating in the Ka-band (34.5 GHz–35.5 GHz), It operates at 34.50 GHz–35.50 GHz with an adjustable center frequency), the radar adopts an advanced technical architecture that and integrates solid-state transmission, dual-transmitter/dual-receiver linear polarization, and fully coherent pulse-Doppler technology. Its key performance specifications include a spatial resolution of 30 m, a Manufacturer specifications list 30 m spatial resolution, a 30 km maximum detection range of 30 km, a ground clutter suppression ratio of ≥ 42 dB, a 42 dB ground-clutter suppression ratio, and a minimum detectable reflectivity factor of ≤ -30 dBZ ~ 30 dBZ at 10 km. These capabilities ensure reliable detection of weak fog echoes. The radar's minimum detectable fog vertical height is determined by radar's minimum detection range (150 m), the antenna beamwidth of 0.39° , the radar scanning elevation angle (2°) and the antenna height above the ground (25 m). This value is estimated as $150 \times \sin(2^\circ - 0.39^\circ/2) + 25 = 30.2$ m, enabling effective observation of near-surface fog structures while reducing ground-clutter interference. Doppler and ground-clutter suppression techniques are further employed to eliminate ground-clutter effects due to antenna side lobes. The YLUKA2 MMCR performs sector Plan Position Indicator (PPI) scans at a fixed elevation angle of 2° , conducting continuous azimuthal surveillance between 60° and 300° . The YLUKA2

125 MMCR data used in this study are primarily affected by clear-sky ground clutter. As this method places high demands on data continuity, we recorded the ground clutter under clear-sky conditions and subsequently subtracted this clutter contribution to complete the ground clutter removal (Li et al., 2013; Kanemaru et al., 2021).

The FM-120 ~~has been continuously running-operated continuously~~ from 2019 through 2023. ~~The FM-120 measures droplets with a size range of $2\mu\text{m}$ – $50\mu\text{m}$ at a sampling frequency of 1 Hz, dividing measurements into~~ It measures droplets from
130 ~~2– $50\mu\text{m}$ at 1 Hz in~~ 30 non-equidistant size bins. The DSD ~~data acquired by the FM-120 observations~~ are used to ~~retrieve key derive~~ fog microphysical parameters ~~such as β_{ext} , LWC, and r_m , providing critical foundational data, including β_{ext} , LWC, and r_m ,~~ for algorithm development and ~~systematic~~ sensitivity analysis.

The DNQ3 ~~Vis-meter, deployed on an automatic observation platform at DOB, operates based on the forward-scattering visibility meter operates on the forward-scattering~~ principle. It emits infrared ~~light~~ pulses with a central wavelength of $0.87\mu\text{m}$ ~~via~~
135 ~~an infrared light-emitting diode. Scattered light at a 35° forward angle is received by the sensor and converted into electrical signals, which are subsequently processed to derive Vis. The DNQ3 Vis-meter is located, and the sensor receives light scattered at a forward angle. The instrument is~~ approximately 153 m from the radar along ~~the 150° azimuth direction. To ensure consistency between MMCR and visibility measurement, collocation is performed by selecting the second valid radar range gate along the 150° azimuth, accounting for radar signal detection. Corresponding azimuth 150° . Because this position is near the~~
140 ~~radar's minimum range, a nearby radar point whose temporal variation is most consistent with the DNQ3 measurements from the same observation period are then used for matched analysis and algorithm validation. trend is selected for matching. The ten spring-2025 cases were included when DNQ3 visibility remained continuously below 1000 m for at least 30 min and no other precipitation weather occurred during the interval.~~

The DOB location and MMCR installation are shown in Fig. 1.

145 3 **Retrieval method**

~~Vis (unit: m) is defined as the distance at which the contrast between an object and its background is reduced to 2% of its original value due to fog droplets extinction. This definition is grounded in the classical Koschmieder Law, developed in~~

2.1 FM-120 processing and observational constraints

The FM-120 fog droplet spectrometer operated at the Datan Observation Base from 2019 through 2023. It measured droplets
150 over a nominal diameter range of $2\mu\text{m}$ – $50\mu\text{m}$ in 30 non-equidistant channels at 1 Hz.

A new event was defined when the gap between consecutive valid minutes exceeded 30 min. After quality control, the spectral-regime dataset aggregates 236037 contributing 1-Hz records into 4234 valid 1-min spectra from 28 events by summing particle counts and dividing by the summed sampling volume. For channel i , the 1-min number concentration was $N_i = C_i/V_s$, where C_i is the ~~1920s (Horvath, 1971), which expresses Vis as: particle count and V_s is sampled volume. With D_i in metres,~~

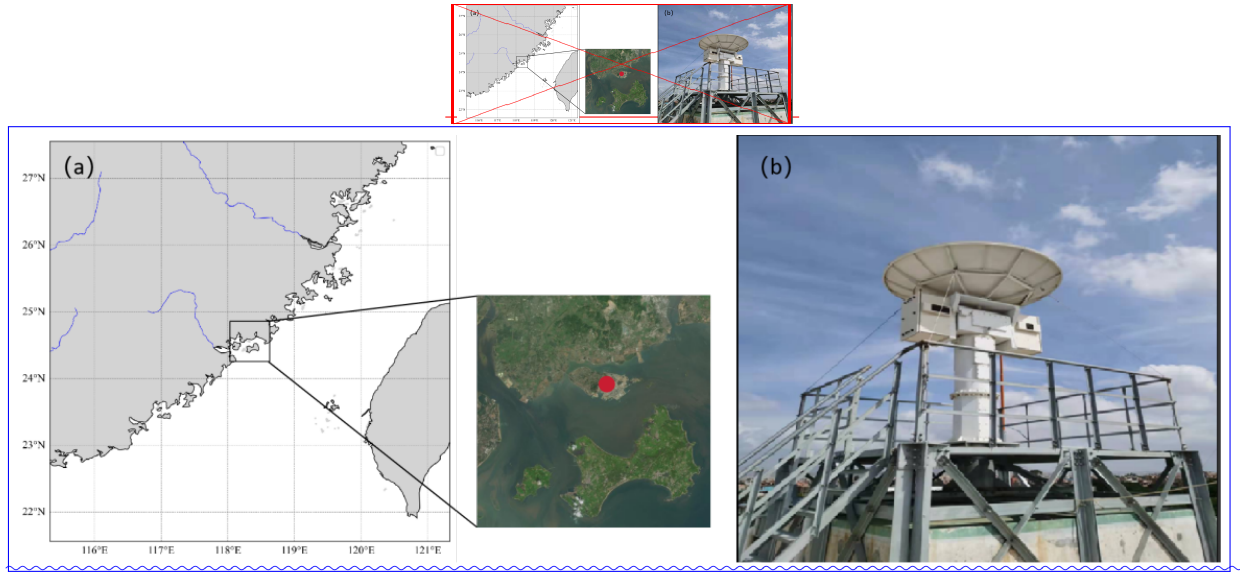


Figure 1. (a) Geographical location of DOB in Xiang'an District, Xiamen. ~~The red~~ mark denotes the ~~position of the~~ observation station. ~~Map source: Baidu Maps, copyright © Baidu Inc~~ (b) The YLUKA2 MMCR installation at the observation site., ~~used for academic research only.~~

155 D_i in millimetres, $N(D)$ in $\text{m}^{-3} \text{mm}^{-1}$, and N_T in m^{-3} , the discrete calculations were

$$\begin{aligned}
 \text{LWC} &= \frac{10^3}{\rho_w} \sum_i N_i \frac{4\pi}{3} r_i^3 \quad [\text{g/m}^3], \\
 Z_{\text{lin}} &= \sum_i N_i D_i^6 \quad [\text{mm}^6/\text{m}^3], \\
 \beta_{\text{ext}} &= \sum_i N_i Q_{\text{ext},i} \pi r_i^2 \quad [/\text{m}], \\
 r_e &= \frac{\sum_i N_i r_i^3}{\sum_i N_i r_i^2},
 \end{aligned} \tag{1}$$

160 where $Q_{\text{ext},i}$ is the size-dependent Mie extinction efficiency evaluated at 550 nm. Total number concentration and the number-weighted radius summary were obtained from the same discrete spectrum. All of these quantities share the same DSD source and therefore provide sample-internal conditioning variables rather than independent visibility observations; no separate historical interpolation or concentration-residual processing chain is implied by Eq. (1).

3 Observation-constrained retrieval development

165 The classical Koschmieder law (Koschmieder, 1924; Horvath, 1971) expresses Vis as

$$\text{Vis} = -\frac{\ln \varepsilon}{\beta_{\text{ext}}} = -\frac{\ln 0.02}{\beta_{\text{ext}}} \quad (2)$$

~~where ε is the brightness contrast threshold. It represents the minimum contrast detectable by a standard observer with normal vision and is where ε is the brightness-contrast threshold,~~ typically set to 0.02, ~~and β_{ext} is the extinction coefficient (unit: m^{-1}) at the visible light wavelength of 550 nm. According to Mie scattering theory, the β_{ext} is given by:-~~

$$170 \quad \beta_{\text{ext}} = \int_0^{\infty} Q_{\text{ext}}(r, \lambda) n(r) \pi r^2 dr$$

$$\beta_{\text{ext}} = \int_0^{\infty} Q_{\text{ext}}(r, \lambda) n(r) \pi r^2 dr \quad (3)$$

~~where Q_{ext} where Q_{ext} is the extinction efficiency of droplets with radius r , quantified as the ratio of actual extinction cross-section to the geometric cross-section. Q_{ext} depends on both droplet radius r and incident wavelength. The function $n(r)$ denotes the fog droplet size distribution function for a droplet of radius r , and $n(r)$ is the fog DSD (unit: m^4), typically assumed to follow which is commonly represented by~~ a lognormal distribution (Bernardin et al., 2010; Podzimek, 1997):

$$n(r) = \frac{N_T}{\sqrt{2\pi} r \sigma} \exp \left[-\frac{\ln^2(r/r_m)}{2\sigma^2} \right] \quad (4)$$

~~where N_T is the total fog Here, N_T is total droplet number concentration, r_m is the r_m is geometric median radius, $r_m = \exp(\overline{\ln r})$, and σ is the and σ is logarithmic spectral width. For a lognormal distribution, $\sigma = \sqrt{(\overline{\ln r} - \ln r_m)^2}$.~~

180 ~~Based on lognormal distribution statistics, any k -th moment of DSD ($M_k = \int_0^{\infty} r^k n(r) dr$) can be simplified to an analytical form:-~~

$$M_k = N_T \exp \left(k\mu + \frac{k^2\sigma^2}{2} \right) = N_T r_m^k \exp \left(\frac{k^2\sigma^2}{2} \right)$$

the k th DSD moment can be written as

$$M_k = N_T r_m^k \exp \left(\frac{k^2\sigma^2}{2} \right) \quad (5)$$

185 ~~Notably, MMCR operates in the millimeter wave-millimetre-wave band, where most fog droplets ($2\text{ }\mu\text{m}$ – $50\text{ }\mu\text{m}$ in diameter) fall within the Rayleigh scattering regime. Under this approximation, the radar scattering cross-section of an individual droplet scales with the 6th power of its diameter. As a volumetrically integrated parameter, the are in the Rayleigh regime. The attenuation-corrected unattenuated radar reflectivity factor (Z , unit: mm^6/m^3 ; commonly representing the total backscattering~~

from all droplets within the sampling volume) is mathematically expressed as the sixth-moment integral of DSD. Z is proportional

190 to the sixth DSD moment:

$$Z = 64M_6 = 64N_T r_m^6 \exp(18\sigma^2) \quad (6)$$

Both Z and β_{ext} are governed by different moments of the fog DSD, which precludes the establishment of a direct quantitative linkage between them. Therefore, we introduce two key fog microphysical parameters—LWC and r_e —both of which are also defined via DSD moments, to serve as intermediate variables for bridging Z and β_{ext} . Because Z and β_{ext} depend on different DSD moments, they cannot be linked uniquely without intermediate microphysical information. LWC and effective radius are defined by

$$\text{LWC} = \frac{4}{3}\pi\rho_w M_3 = \frac{4}{3}\pi\rho_w N_T r_m^3 \exp\left(\frac{9}{2}\sigma^2\right) \quad (7)$$

$$r_e = \frac{M_3}{M_2} = r_m \exp\left(\frac{5}{2}\sigma^2\right) \quad (8)$$

200 where ρ_w is the density of water, LWC is the mass of liquid water per unit volume of fog (unit: g/m^3 , a direct function of the 3rd DSD moment (M_3)), and r_e is the volume-weighted average fog droplet size (unit: μm , defined by the ratio of the 3rd to 2nd DSD moments (M_3/M_2)). It is clear that both LWC and r_e are fundamentally linked to the DSD and are derived from its statistical moments. Consequently, they implicitly incorporate the unmeasurable DSD parameters N_T and σ . By eliminating these two unmeasurable variables from Eqs. (5)–(7), r_e can be explicitly expressed as a function of LWC and Z (Stephens, 1978): where ρ_w is water density. Eliminating N_T and σ from the moment relations gives (Stephens, 1978)

$$r_e = r_m^{\frac{4}{9}} \left(\frac{\pi\rho_w Z}{48\text{LWC}} \right)^{\frac{5}{27}} \quad (9)$$

In the visible band (e.g., green light with a wavelength of $\sim 0.55\mu\text{m}$), fog droplet sizes are much larger than the incident wavelength, placing them within the scope of Mie scattering theory. For fog conditions, the extinction efficiency factor Q_{ext} can be approximated as In the visible band, the extinction efficiency of sufficiently large fog droplets approaches 2 (Mazoyer et al., 2022). Substituting Eqs. (3), (7), (8) and (9) into the form of β_{ext} yields a closed-form relationship between β_{ext} , LWC and r_e :

$$\beta_{\text{ext}} = \frac{3\text{LWC}}{2r_e} = \frac{3}{2} r_m^{-\frac{4}{9}} \left(\frac{\pi\rho_w Z}{48} \right)^{-\frac{5}{27}} \text{LWC}^{\frac{22}{27}}$$

(Mazoyer et al., 2022). Figure 2 shows its radius dependence at 550 nm.

For dimensional consistency in Eq. (

215 For $Q_{\text{ext}} \approx 2$, extinction is $3\text{LWC}/(2\rho_w r_e)$. Substitution of Eq. (9) gives

$$\beta_{\text{ext}} = \frac{3\text{LWC}}{2\rho_w r_e} = \frac{3}{2\rho_w} r_m^{-\frac{4}{9}} \left(\frac{\pi\rho_w Z}{48} \right)^{-\frac{5}{27}} \text{LWC}^{\frac{32}{27}}. \quad (10)$$

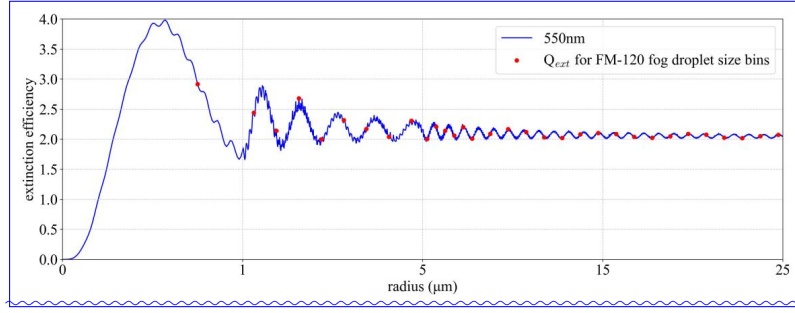


Figure 2. Size dependence of Q_{ext} at 550 nm. Red points mark radii associated with the FM-120 size bins. The constant value $Q_{\text{ext}} = 2$ is an asymptotic approximation rather than an exact representation over the sampled small-droplet range.

All quantities are evaluated in consistent SI units, including ρ_w , LWC, radius, and the conventional mm^6/m^3 reflectivity factor. Combining Eq. (10), LWC is converted to kg/m^3 and r_e to m during calculation, with the conversion relations being $1 \text{ g}/\text{m}^3 = 10^{-3} \text{ kg}/\text{m}^3$ and $1 \mu\text{m} = 10^{-6} \text{ m}$. From the above derivations, the parameterization scheme for Vis can be expressed as :-

$$220 \quad \text{Vis} = 3.912 \cdot \frac{2}{3} r_m^{\frac{4}{9}} \left(\frac{\pi \rho_w Z}{48} \right)^{\frac{5}{27}} \text{LWC}^{-\frac{22}{27}}$$

with Koschmieder's relation yields

$$\text{Vis} = 3.91 \frac{2\rho_w}{3} r_m^{\frac{4}{9}} \left(\frac{\pi \rho_w Z}{48} \right)^{\frac{5}{27}} \text{LWC}^{-\frac{32}{27}}. \quad (11)$$

Equation (10) requires both LWC and r_m , neither of which is uniquely determined by one MMCR reflectivity measurement. We therefore use two observation-constrained steps. The mass-absorption/TRR routine retrieves LWC and applies liquid-water attenuation correction, and the FM-120 development data provide the conditional radius–reflectivity relations.

In Eq. (10), both LWC–

3.1 FM-120-observed spectral regimes and retrieval context

This section characterizes the morphology of fog droplet size distributions (DSDs) and the variation of microphysical parameters across fog intensity grades, drawing on long-term FM-120 observations. The results provide observational constraints for the visibility-retrieval parameterization developed in subsequent sections and delineate the applicability boundary of the single-lognormal closure assumption.

3.1.1 Data processing and spectral shape classification

Following standard classification criteria for fog DSD morphology, each one-minute spectrum is assigned to one of four definite shape categories: narrow unimodal, broad unimodal, shoulder, and bimodal. An additional uncertain status is reserved

235 for samples with indistinct morphological features: spectra with a stability score below 0.80 are labeled as uncertain. Among them, 584 samples exhibit a weak two-peak pattern but are not classified as bimodal due to insufficient peak separation.

Across the full dataset, shoulder and broad-unimodal spectra account for the largest proportions, at 41.83 % and 33.54 %, respectively. Bimodal spectra make up 6.38 %, and r_m are required for the calculation of Vis; however, neither parameter can be directly obtained from cloud radar measurement. Therefore, appropriate strategies are needed to express these variables in terms of radar-measurable quantities. Herein, LWC is first retrieved by applying an adaptive physical constraint derived from liquid water absorption attenuation, as developed in our previous work (Ge et al., 2023). The retrieved LWC is then used to correct the attenuated radar reflectivity (Z_m). narrow-unimodal spectra account for 4.32 %. The remaining 13.93 % fall into the uncertain status. Median normalized spectra and interquartile ranges for each shape category are shown in Fig. 3a. Narrow-unimodal spectra have the smallest peak diameter and narrowest spectral width, while bimodal spectra exhibit a primary mode near 5 μm and a secondary mode near 25 μm , with an overall spectral width substantially larger than that of unimodal spectra.

3.1.2 Evolution of spectral shapes across visibility grades

To examine how DSD shapes vary with fog intensity, samples are grouped into four visibility grades according to meteorological fog classification standards, based on DSD-derived visibility: heavy fog ($500\text{ m} \leq V < 1000\text{ m}$), dense fog ($200\text{ m} \leq V < 500\text{ m}$), very dense fog ($50\text{ m} \leq V < 200\text{ m}$), and extremely dense fog ($V < 50\text{ m}$). An event-balanced statistical approach is applied: the fraction of each shape category within each visibility bin is first calculated per event, then averaged across all events with equal event weights. The spectral shape composition across visibility grades is presented in Fig. 3b.

Statistical results show that the proportion of bimodal spectra rises markedly as visibility decreases: Heavy fog (500–1000 m): bimodal spectra account for only 0.77 % (95 % confidence interval: 0.03 %–1.79 %), with unimodal and shoulder shapes dominating; Dense fog (200–500 m): the bimodal fraction increases to 4.22 % (95 % confidence interval: 1.45 %–7.96 %); Very dense fog (100–200 m): the bimodal fraction jumps to 24.07 % (95 % confidence interval: 9.11 %–40.04 %); The 50–100 m bin yields a point estimate of 35.90 % bimodal fraction, but only 3 events contribute to this interval with a confidence interval spanning 0–100 %, so it is not used for robust quantitative inference.

No DSD samples are available for the extremely dense fog (0–50 m) grade in this dataset. Overall, when visibility falls below 200 m, the deviation of fog DSD from a single lognormal distribution becomes pronounced, and bimodal structures emerge as a non-negligible spectral feature. Fig. 3c also presents bimodal fractions under alternative statistical frameworks (minute-weighted, event resampling), all of which show consistent trends.

3.1.3 Comparison of microphysical parameters between two visibility strata

Building on the 200 m visibility threshold identified above, samples are further divided into a low-visibility stratum ($0 < V_{\text{Mie}} \leq 200\text{ m}$) and a high-visibility stratum ($200\text{ m} < V_{\text{Mie}} \leq 1000\text{ m}$). Second, we exploit the microphysical coupling between r_m and intrinsic radar reflectivity Z to establish an empirical relationship linking these two variables. Through these two key steps, the physically constrained retrieval of LWC and the estimation of r_m are obtained for Vis inversion, while simultaneously completing the correction of radar signal attenuation caused by liquid water in fog. Based on this framework, we propose

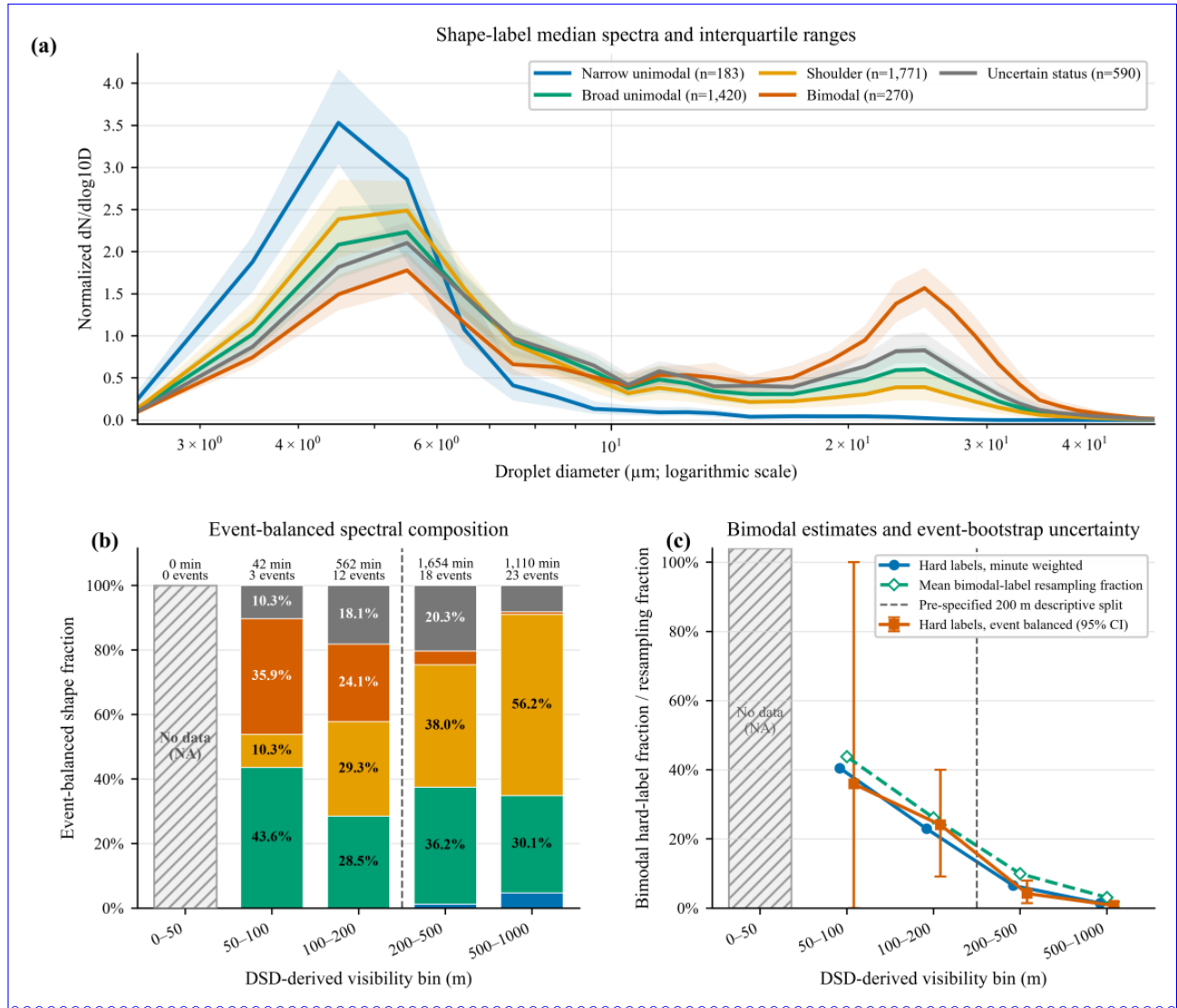


Figure 3. FM-120 spectral summaries from the spectral-regime dataset. (a) Median normalized spectra and interquartile ranges by fixed shape label. (b) Event-balanced hard-label composition in DSD-derived visibility bins; the dashed line marks the pre-specified 200 m split. (c) Minute-weighted, event-balanced, and label-resampling bimodal summaries.

270 a self-consistent visibility retrieval algorithm, with the workflow summarized in the flowchart shown in We compare the distribution of five microphysical parameters between the two strata: radar-equivalent reflectivity factor Z , total number concentration N_T , liquid water content (LWC), effective radius r_e , and geometric median radius r_m , as illustrated in Fig.2-4.

Distinct microphysical differences are observed between the two strata. The median Z in the low-visibility stratum is -2.61 dBZ, 4.92 times the value of -9.53 dBZ in the high-visibility stratum, a difference of approximately 6.92 dBZ corresponding to a 4.92-fold increase.

275 The median N_T and LWC in the low-visibility stratum are $97.31 \times 10^6 \text{ m}^{-3}$ and 0.21 g m^{-3} , 2.19 times and 3.83 times the values in the high-visibility stratum ($44.34 \times 10^6 \text{ m}^{-3}$, 0.06 g m^{-3}), respectively, also showing strong discriminative power.

Differences in radius parameters between the two strata are comparatively small. The geometric median radius r_m and effective radius r_e in the low-visibility stratum are $13.57 \mu\text{m}$ and $11.90 \mu\text{m}$, only $0.83 \mu\text{m}$ and $1.55 \mu\text{m}$ larger than in the high-visibility stratum, with far weaker separability than reflectivity and LWC.

3.2 Mass-absorption-constrained LWC retrieval and radar reflectivity attenuation correction

280 In the Vis parameterization scheme, Vis inversion is inseparable from accurate LWC. In the visibility parameterization, accurate retrieval of liquid water content (LWC) and unattenuated radar reflectivity factor Z are essential. Therefore, we adopted the self-consistent LWC-LWC retrieval algorithm developed in Ge's research (Ge et al., 2023) to calculate LWC, and correct the absorption attenuation in the measured radar reflectivity Z_m .

285 The attenuation of millimeter-wave radar signals during propagation in fog is mainly caused by mass absorption of water droplets; meanwhile, under the Rayleigh approximation, regardless of DSD, the proportional relationship between the absorption attenuation coefficient A and LWC (i.e., $A = K^* \text{LWC}$) is adopted for LWC retrieval.

Through mathematical and physical transformation of the radar equation, a self-consistent relationship between the reconstructed measured radar reflectivity one-way liquid-water attenuation is proportional to LWC, $A = K^* \text{LWC}$, independent of the droplet size distribution. Here, K^* is determined by the radar wavelength, temperature, and complex refractive index of cloud droplets, and can be treated as a constant for a millimeter-wave radar operating at a fixed wavelength. Based on this, a self-consistent LWC retrieval framework is adopted: the radar radiative transfer equation is transformed into a Bernoulli differential equation and integrated within the cloud layer to establish a constrained relationship among the measured reflectivity factor Z_m , the true reflectivity factor Z , and LWC. To obtain a closed-form solution, a segment-wise power-law relation is employed as a numerical closure. A bounded trust-region reflective (Z_{mc}) and LWC was established as follows:-

295
$$Z_{mc,r_i} = c \text{LWC}_{r_i}^{\frac{1}{p}} \exp \left(-0.46 \int_{r_0}^{r_i} K^* \text{LWC}(s) ds \right)$$

Subsequently, the optimal parameters in the LWC expression for each radar range are obtained by minimizing the error between Z_m and the theoretically reconstructed reflectivity factor (Z_{mc}) expressed by Eq. (12). The trust-region reflection (TRR) algorithm is employed to minimize the error function. This method breaks through the prior assumption of DSD in traditional algorithms, utilizes nonlinear least-squares regression for parameter optimization, and enables accurate LWC

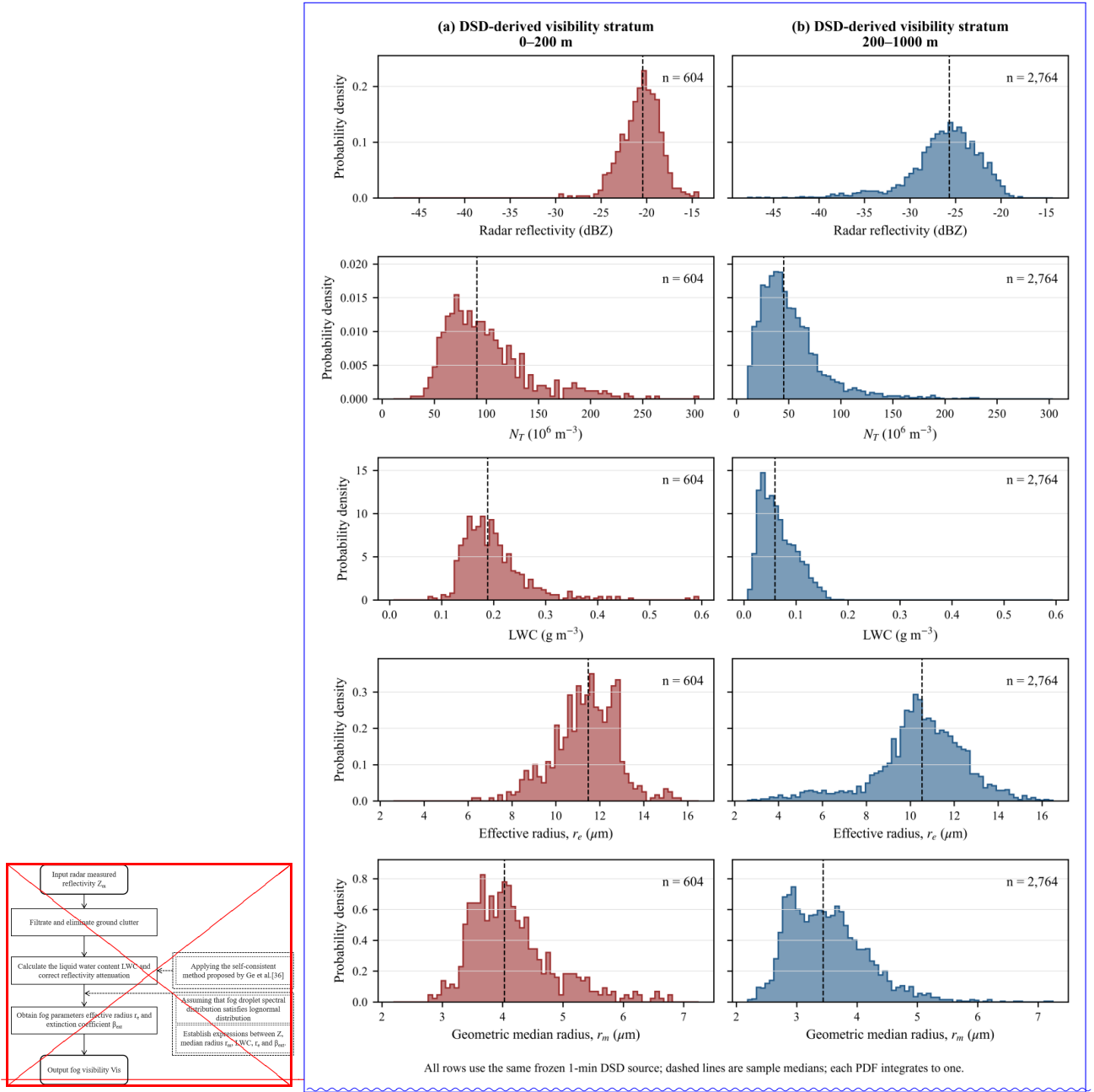


Figure 4. Flowchart of Timestamp-resolved FM-120 parameter distributions in the V_{is} retrieval by our method pre-specified strata (a) $0 < V_{\text{Mie}} \leq 200 \text{ m}$ and (b) $200 \text{ m} < V_{\text{Mie}} \leq 1000 \text{ m}$. The framework of the overall algorithm is shown on the left; the method used to establish expressions between Z Rows show FM-120/DSD-derived radar-equivalent Z (dBZ), median radius r_m (m^{-3}), LWC (g m^{-3}), r_e (μm), and β_{ext} under the lognormal droplet size distribution visibility-workflow geometric median radius proxy r_m (DSD μm) assumption. Each histogram is shown in the normalized within its stratum; dashed box on the right lines denote sample medians.

inversion for liquid water clouds and fog. The accuracy of this algorithm has been verified by comparing the retrieved LWC with in-situ aircraft data and microwave radiometer products. Notably, there are differences between sea fog and clouds: the horizontal scale of sea fog typically ranges from several kilometers to tens of kilometers, far exceeding the vertical scale of clouds. The significant fluctuation least-squares solver is used to optimize the parameter vector by minimizing the absolute dBZ residual between the measured and reconstructed reflectivity (Cartis et al., 2009). The reconstructed reflectivity is expressed as

305

$$Z_{mc,r_i} = cLWC_{r_i}^{\frac{1}{b}} \exp \left(-0.46 \int_{r_0}^{r_i} K^* LWC(s) ds \right). \quad (12)$$

where the effect of gaseous attenuation on radiative transfer is neglected. Once LWC is retrieved, the accumulated attenuation can be readily computed and applied to correct the measured reflectivity. Given the substantial variability in liquid water path (LWP) adversely affects the stability of the algorithm. To mitigate this impact, this study adopts the along fog profiles detected by radar—as noted in Sect. 3.1, bimodal fog events exhibit significant enhancements in both Z and LWC—a 10-bin segmentation method to divide continuous long-range fog echo data into several segments of 4–10 bin echo data (Zhang et al., 2024; Boers et al., 2011). This approach aligns the initial LWC range with previous studies, thereby reducing the number of iterations and calculation errors. approach is applied to partition the profile into segments such that the LWP of each segment falls within the bounds prescribed by the TRR method. The numerical branch is selected according to the maximum original reflectivity within each segment: for a maximum not exceeding -15 dBZ, the starting vector is $[0.5, 0.01, 1]$; for a maximum greater than -15 dBZ, it is $[0.5, 0.01, 0.1]$. Both branches use lower bounds $[0, 0, 0]$ and upper bounds $[1, 1, 10]$. Each segment is fitted independently, with the accumulated attenuation derived from the retrieved LWC applied to its reflectivity prior to fitting. This approach requires neither ancillary instruments nor empirical coefficients, achieving LWC retrieval and attenuation correction through the self-consistent interdependencies among cloud physical variables.

320 3.3 Relationship between r_m and Z Extinction-oriented conditional parameterization of r_m

In addition to obtaining accurate LWC, determining r_m is also accurate LWC retrieval, determining r_m is a critical step. Although some observational studies on r_m have been conducted previously, no empirical relationship between r_m and Z for fog r_m and Z has been established to date. Since both r_m and Z are closely related for fog. Given that both r_m and Z are intimately linked to DSD, a strong power-law correlation exists between them is expected. Herein, we adopt the power function form $r_m = aZ^b$ to fit their relationship. The specific approach is as follows: combining Eqs. (4)–(7) and the DSD data observed by the power-function form $r_m = aZ^b$ to parameterize this relationship.

A single-frequency reflectivity measurement cannot determine r_m uniquely because total number concentration and spectral width also affect the sixth DSD moment. We therefore compare two sample-internal conditional parameterizations. The direct radius-space fit is used only as a comparison baseline. The selected extinction-oriented relation is fitted to extinction-matching pseudo-targets in the moment-closure development dataset; the pseudo-targets are constructed from reflectivity, LWC, and size-dependent Mie extinction co-derived from the same FM-120 fog droplet spectrometer, DSD.

330

For each development row i , candidate radii from $1.00\text{ }\mu\text{m}$ to $20\text{ }\mu\text{m}$ at $0.01\text{ }\mu\text{m}$ spacing were inserted into Eq. (10). The row-wise extinction-matching pseudo-target was

$$r_{\beta,i}^* = \arg \min_{r \in \mathcal{G}} \frac{|\beta_{\text{ext,closed}}(r; Z_{\text{lin},i}, \text{LWC}_i) - \beta_{\text{ext,FM120},i}|}{\beta_{\text{ext,FM120},i}}, \quad (13)$$

where $\mathcal{G}$ is the finite candidate grid. Thus $r_{\beta,i}^*$ is a constructed radius pseudo-target, not an independently observed geometric median radius. A free power law was then fitted to these pseudo-targets by unweighted radius-space least squares:

$$\frac{r_m}{\mu\text{m}} = 21.87 \left[\frac{Z_{\text{lin}}}{\text{mm}^6/\text{m}^3} \right]^{0.20}. \quad (14)$$

Linear Z_{lin} is the predictor for both relations, whereas dBZ is used only as a plotting coordinate. Against their respective fit targets, the true values of fog microphysical parameters are calculated; based on Eq.(9), a residual formula is constructed by fitting the estimated β_{ext} corresponding to different r_m values with the true values; finally, the fitting relationship $r_m = aZ^b$ is obtained through regression analysis. The results are shown in Fig. 3, with the fitted relationship being $r_m = 21.96Z^{0.2}$. At low Z values, the fitted r_m is significantly overestimated; however, considering that the main Z range of fog observed by radar is $-40\text{ dBZ} - 10\text{ dBZ}$ (Du et al., 2024), this effect can be appropriately neglected. direct relation has same-sample Pearson $R = 0.74$ and RMSE $0.86\text{ }\mu\text{m}$, and the extinction-oriented relation has $R = 0.73$ and pseudo-target RMSE $1.64\text{ }\mu\text{m}$. Fig. 5 displays the direct comparison baseline in panel (a) and the selected extinction-oriented relation in panel (b).

4 Algorithm sensitivity analysis Evaluation of retrieval assumptions and uncertainties

There are two key uncertainties in algorithm construction that induce retrieval errors: the simplification of $Q_{\text{ext}} \approx 2$ under Mie scattering theory, and potential deviations in retrieved LWC and empirically derived r_m . To quantify their respective impacts on β_{ext} (and thus indirect effects on Vis retrieval), we perform a systematic uncertainty analysis below.

4.1 Uncertainty from the $Q_{\text{ext}} \approx 2$ approximation

For 550-nm electromagnetic waves, fog droplets are typically treated under Mie scattering theory. In this framework, Q_{ext} of large particles tends to 2—a simplification that introduces errors in practical calculations. For this purpose, error analysis is performed by combining Mie scattering theory with DSD data: Q_{ext} values for droplets of different sizes are calculated via Mie scattering theory, and these values are integrated with DSD measurements from the FM-120 fog droplet spectrometer to derive the accurate β_{ext} . The retrieval framework constructed in Sect. 3 reduces reliance on direct visibility–reflectivity fitting, but its accuracy is ultimately governed by two algorithmic assumptions. The first is the constant extinction-efficiency approximation ($Q_{\text{ext}} = 2$), which replaces the size-dependent Mie efficiency shown in Fig. 2 with its large-particle asymptote. The second is the single-lognormal closure, which constrains the droplet size distribution to a parametric form that links reflectivity, liquid water content, and effective radius through shared moments. Both assumptions introduce structural biases whose magnitude depends on droplet size and spectral modality. Section 4.1 attributes the extinction-residual patterns revealed

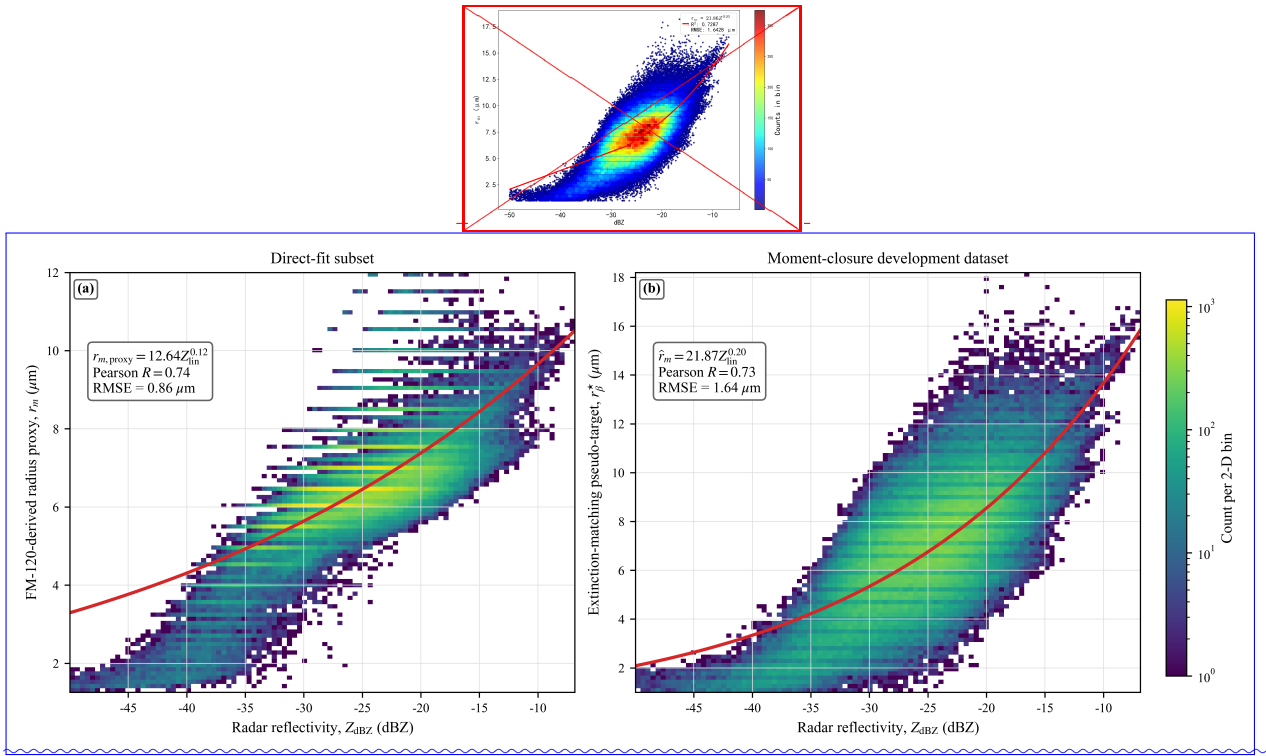


Figure 5. By minimizing the error between the estimated $\beta_{\text{ext},\text{est}}$ calculated from Eq. (10a) and its true value β_{ext} calculated from Eq. (3b) with the FM-120-derived radius proxy, the power-law fitting relationship $r_m = 21.96Z^{0.2}$ was derived. This figure illustrates the relationship between the Linear Z_{lin} is fitted r_m and Z . Herein, while dBZ is the abscissa represents Z (unit: dBZ) display coordinate; equations, same-sample Pearson R , and the ordinate denotes the fitted r_m (unit: μm) radius RMSE are shown to two decimal places. The red line denotes the fitted curve.

in Fig. 6 to these two sources: the weak-reflectivity bias is traced to the $Q_{\text{ext}} = 2$ approximation, while the broader residual reflects the lognormal closure and is partially mitigated by the extinction-oriented parameterization. Section 4.2 quantifies the analytical sensitivity of the retrieved extinction to perturbations in LWC and radius.

Figure 4 illustrates the variation of Q_{ext} with fog droplet radius ($1 \mu\text{m}$ – $25 \mu\text{m}$) based on Mie scattering theory, where red dots represent Q_{ext} values corresponding to each size bin of the FM-120. To assess errors induced by the $Q_{\text{ext}} \approx 2$ approximation, we compared two β_{ext} datasets: one approximated via the assumption ($\beta_{\text{ext},\text{appr}} = \int_0^\infty 2n(r)\pi r^2 dr$) used in our algorithm, and

4.1 DSD effects on the radius–reflectivity relation and extinction retrieval

The DSD primarily affects the retrieval through the radius term.

To quantify the response to this structural variability, the other accurately calculated from Eq. (3) via Mie scattering theory and direct and extinction-oriented relations are evaluated on the moment-closure development dataset. For each relation, the

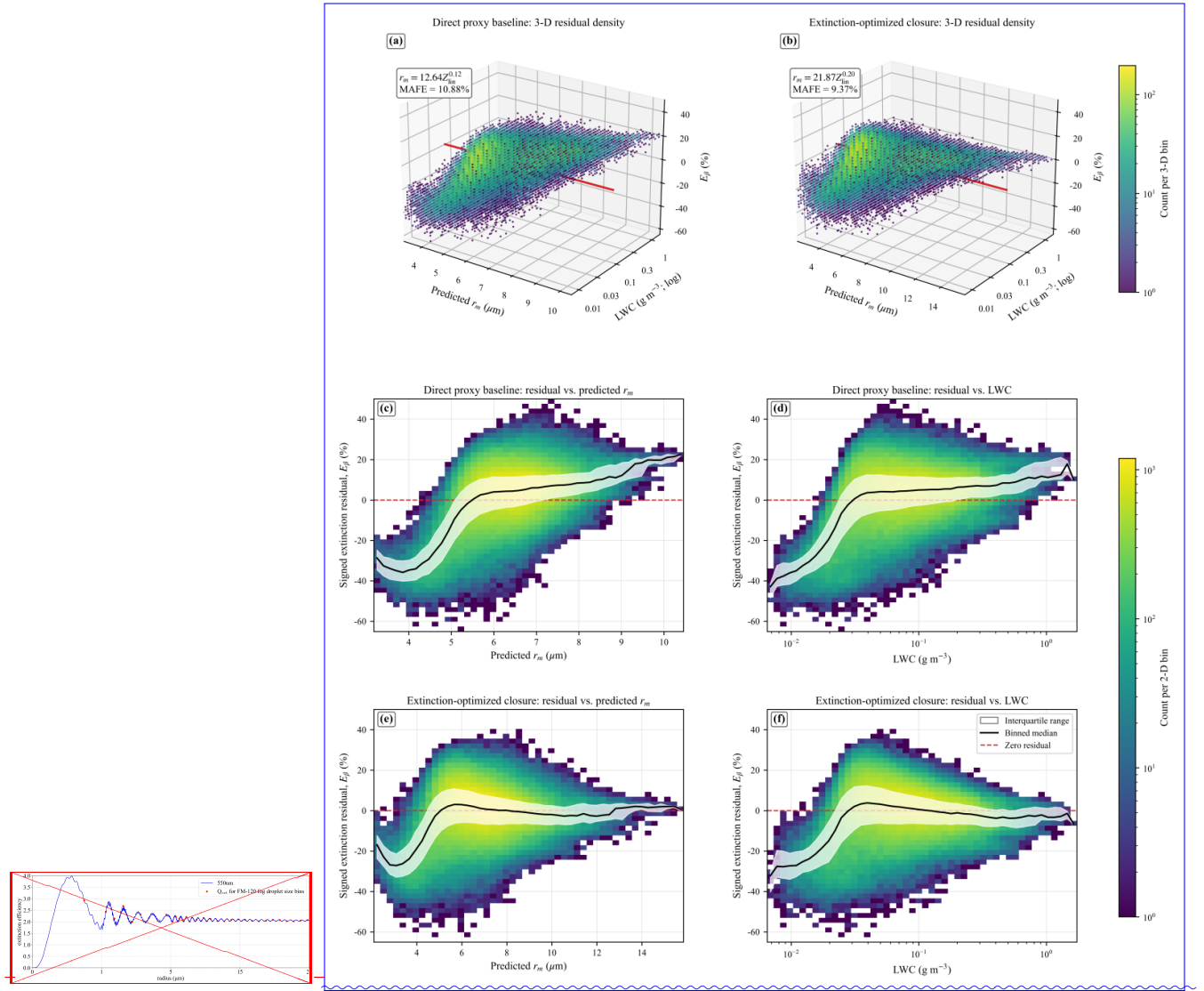


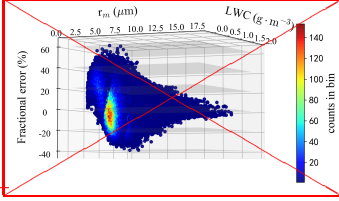
Figure 6. Relationship between Q_{ext} and particle size. Same-sample extinction-residual distributions for the moment-closure development dataset. (a, b) Three-dimensional residual density for the direct and fog-droplet-radius-extinction-oriented relations. Red dots represent Q_{ext} . (c, d) Direct-relation residual versus predicted radius and LWC; (e, f) corresponding to each particle size extinction-oriented views. In (c–f), colour denotes count per bin of, white bands the FM-120 spectrometer interquartile range, and black curves the binned median; red references mark zero residual, and both methods share the residual scale.

predicted radius is inserted into Eq. (10) with the same row's Z_{lin} and LWC and compared with the same size-dependent Mie extinction reference. All quantities are same-sample and deterministic; there is no held-out subset or independent physical-radius observation. Accordingly, all R , RMSE, signed-residual, and MAFE values in Sects. 3.3 and 4.1 are resubstitution diagnostics that measure numerical consistency with the development targets. The signed fractional residual and mean absolute fractional error (MAFE) are

$$E_{\beta,i} = \frac{\hat{\beta}_{\text{ext},i} - \beta_{\text{ext,FM120},i}}{\beta_{\text{ext,FM120},i}} \times 100\%, \quad (15)$$

$$\text{MAFE} = \frac{1}{N} \sum_{i=1}^N |E_{\beta,i}|. \quad (16)$$

where N is the total number of rows in the moment-closure development dataset. For the $Z_{\text{dBZ}} < -40$ subset, the signed mean of E_{β} is -25.32% . The negative sign means that the closed-form extinction is lower on average than the same-row FM-120DSD data. Vis retrieved from these two β_{ext} datasets was compared to evaluate errors, using metrics including mean error (ME), mean absolute error (MAE)/Mie extinction; because visibility is inversely proportional to extinction, the closed-form result therefore tends toward visibility overestimation in this subset. This same-row, mean squared error (MSE), and root mean squared error (RMSE). The results show an MAE of 18.12 m, an RMSE of 25.74 m, and a fractional error of 8.35%, indicating that the $Q_{\text{ext}} \approx 2$ approximation introduces a certain degree of error and thus affects visibility retrieval outcomes.



3D histogram illustrating the relationship between fractional error of β_{ext} and two key fog microphysical parameters: r_m and LWC. All data points are derived from the 2019–2023 fog droplet spectrum observations collected by the FM-120 spectrometer at DOB. After incorporating fog microphysical parameters into β_{ext} parameterization, we obtained its parameterization equation same-source signed mean is not an independent error estimate and is distinct from the signed mean and 22.01 % MAFE for the broader $-50 \leq Z_{\text{dBZ}} < -35$ interval. The bimodal error structure visible in Fig. 6 can be attributed to the two algorithmic assumptions identified above. For weak-reflectivity cases, the predicted radii fall predominantly below $5 \mu\text{m}$, a range in which the actual Mie extinction efficiency at 550 nm deviates most strongly from the assumed constant value of 2 (Fig. 2). Because the constant- Q_{ext} approximation enters the analytical closure linking extinction to LWC and radius (Eq.(10)). We conducted an in-depth comparative analysis between this equation and the original β_{ext} calculation equation (Eq. (3)) based on Mie scattering theory (Fig.5). The results show an average fractional error of 8.5% with no significant improvement, as the metric exhibits minimal variation. Given the strong correlation between LWC and r_m , the fractional errors of β_{ext} plotted against LWC and r_m on the x - y plane cluster around a single curve, with error values decreasing as LWC and r_m increase. Notably, for $r_m < 5 \mu\text{m}$, the $Q_{\text{ext}} \approx 2$ approximation underestimates the extinction contribution of small fog droplets (actual $Q_{\text{ext}} > 2$), thereby increasing the relative deviation of the parameterized β_{ext} from

10), this deviation systematically biases the closed-form extinction downward relative to the size-dependent Mie reference in the small-radius regime. The resulting extinction underestimation is consistent with the pronounced negative signed residual and elevated MAFE (22.01 %) observed in the weak-reflectivity bin of Fig. 6. The residual that persists across the remaining reflectivity range arises primarily from the single-lognormal assumption: when the observed DSD departs from lognormality, as occurs increasingly at visibilities below 200 m where bimodal spectra become prevalent (Sect. 3.1), the true value calculated via Mie scattering theory—this deviation further propagates to Vis retrieval. As fog droplet size increases, most β_{ext} errors fall within $\pm 20\%$, and the fractional error decreases progressively, confirming that small fog droplets are the primary error source for β_{ext} retrieval using Eq. (10). The MMCR’s detection range covers $Z > -40$ dBZ, corresponding to a fitted r_m of 4.4 μm . Although the radar blind zone overlaps with the range of large errors, moment-based closure relating reflectivity, LWC, and radius no longer holds exactly. The extinction-oriented parameterization was designed to reduce precisely this distribution-shape error: by calibrating the radius-reflectivity relation against extinction-matching pseudo-targets rather than direct geometric radius proxies, it absorbs part of the departure from lognormality into the empirical power-law coefficients. The reduction in overall MAFE from 10.88 % to 9.37 %, and most notably from 12.25 % to 4.96 % at intermediate reflectivities, confirms the effectiveness of this strategy. The weak-reflectivity residual, however, is dominated by the Q_{ext} approximation and is not substantially improved by the extinction-oriented fitting alone.

Across the moment-closure development dataset, the algorithm maintains stable performance. Specifically, MMCR is inherently less sensitive to fog droplets corresponding to geometric mean radii smaller than this reference value. Given that small droplets are the primary source of large β_{ext} retrieval errors, their weak influence on radar measurements effectively minimizes the propagation of such errors to the final retrieval results. For this reason, the algorithm will maintain robust performance in practical Vis retrieval applications. The direct relation yields an extinction MAFE of 10.88 %, whereas the extinction-oriented relation yields 9.37 %. The largest decrease occurs for $-15 \leq Z_{\text{dBZ}} < 0$, from 12.25 % to 4.96 %.

4.2 Uncertainty from Analytical sensitivity to LWC and r_m deviations

In practical application of the algorithm, reliable retrievals of LWC, Z , and r_e are essential for accurate Vis calculation via Eq. (10). On one hand, the fog DSD does not strictly follow the lognormal distribution and may exhibit bimodal or multimodal characteristics, which introduces uncertainties into the calculation. On the other hand, both LWC and r_m are derived from Z —while Z is simultaneously corrected during LWC retrieval via the self-consistent algorithm, residual deviations between the retrieved LWC and its true value still exist. Additionally, the corrected Z is influenced by LWC, which further propagates uncertainties to r_m . Furthermore, r_m itself is an estimated parameter, with a RMSE of 1.64 μm as shown in the validation results (Fig. 3).

To quantify the impact of these uncertainties on algorithm performance, we conducted perturbation simulations on both the retrieved LWC and empirically derived r_m , with a focus on analyzing the resulting characteristics of retrieval errors. We define a sensitivity metric f relative extinction response f that characterizes how retrieval extinction errors respond

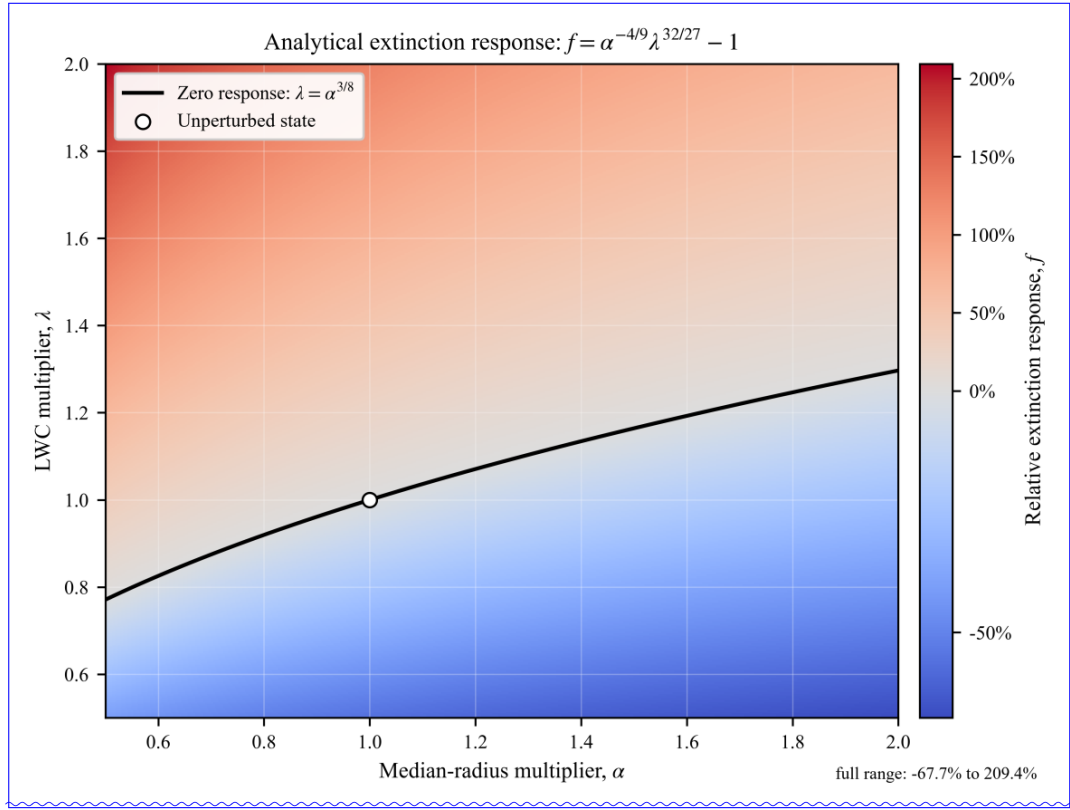


Figure 7. Analytical relative extinction response to r_m multiplier α and LWC multiplier λ . The black curve marks $f = 0$, the open circle marks the unperturbed state, and the diverging colour scale spans the complete calculated range.

to perturbations in these two parameters. The perturbed extinction coefficient ($\beta_{\text{ext,pert}}$) is calculated as follows:-

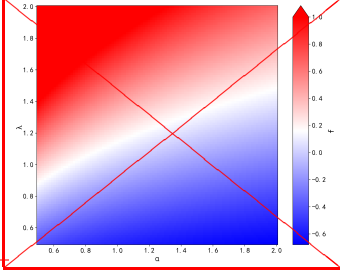
$$\beta_{\text{ext,pert}} = \frac{3}{2} \alpha r_m^{-\frac{4}{9}} \left(\frac{\pi \rho_w Z}{48} \right)^{-\frac{5}{27}} \lambda \text{LWC}^{\frac{22}{27}}$$

where α and λ denote the perturbation multipliers for parameter scaling—values greater than 1 indicate parameter amplification, while values between 0 and 1 indicate parameter reduction. Specifically, α corresponds to the perturbation multiplier for r_m , and λ corresponds to the perturbation multiplier for LWC, with both multipliers ranging from 0.5 to 2.0. The range of α (for r_m) is derived from 5 years of empirical statistical analysis of DSD data collected by the FM-120 fog droplet spectrometer, while the range of λ (for LWC) is adopted from the findings of Ge et al. (2023). We conduct the subsequent sensitivity analysis using DSD data acquired from the FM-120 fog droplet spectrometer, with the sensitivity metric f (relative error of β_{ext}) defined as:-

$$f = \frac{\beta_{\text{ext,pert}} - \beta_{\text{ext}}}{\beta_{\text{ext}}}$$

λ be multiplicative perturbations to r_m and LWC. From Eq. (10),

$$\beta_{\text{ext,pert}} = \beta_{\text{ext}} \alpha^{-\frac{4}{9}} \lambda^{\frac{32}{27}}, \quad (17)$$



Sensitivity comparison of the algorithm to variations in LWC and r_m . The α represents the scaling factor for r_m (where $\alpha=1$ denotes no scaling of r_m), and λ represents the scaling factor for LWC (where $\lambda=1$ denotes no scaling of LWC). The f values correspond to the relative errors associated with different combinations of α (for r_m) and λ (for LWC), as visualized in Fig.6. Specifically, a value of $0 < f < 1$ indicates that the retrieved β_{ext} (when LWC is scaled by λ and r_m is scaled by α) is systematically underestimated, whereas $f > 1$ indicates a systematic overestimation of β_{ext} . As shown in Fig. 6, the algorithm exhibits higher sensitivity to LWC variations: retrieval errors (quantified by f) exhibit an exponential dependence on LWC changes, but a linear dependence on r_m perturbations. When both LWC and r_m are either simultaneously overestimated or underestimated—i.e., when they increase or decrease proportionally in tandem—the f values remain within a stable range. This confirms that the algorithm maintains reliable and stable performance even under coordinated variations of these two key microphysical parameters so the relative extinction response is

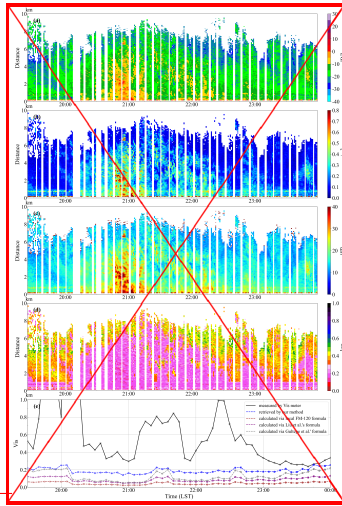
$$f = \frac{\beta_{\text{ext,pert}} - \beta_{\text{ext}}}{\beta_{\text{ext}}} = \alpha^{-\frac{4}{9}} \lambda^{\frac{32}{27}} - 1. \quad (18)$$

Ge et al. (2023) reported that retrieved LWC can deviate from its true value by 50 % to 200 % under the self-consistent mass-absorption algorithm. In this study, the same fractional range applies to the empirically derived r_m . Both multipliers (α and λ) are therefore varied from 0.50 to 2.00, spanning the physically plausible uncertainty range of these two parameters. Figure 7 displays the complete response range with a zero-centred normalization. The zero-response curve is $\lambda = \alpha^{3/8}$; same-signed perturbations partly compensate because radius and LWC enter with opposite-signed exponents. The larger LWC exponent produces a stronger analytical response to LWC perturbations. The ratio of the exponent magnitudes is $(22/27)/(4/9) = 11/6 \approx 1.83$, meaning that for any given fractional perturbation, the extinction response to LWC is approximately twice that of r_m . For instance, a 10 % increase in LWC raises the extinction by 8.1 %, whereas a 10 % increase in r_m lowers it by only 4.1 %. In practice, drizzle-sized droplets (diameter $> 100 \mu\text{m}$) may appear near the fog top or during the mature-to-dissipation stage. Because the mass-absorption retrieval of LWC depends on the integrated liquid water mass (third DSD moment) rather than the sixth-moment reflectivity, it is intrinsically less sensitive to sparse large droplets; the attenuation coefficient k remains closely tied to the bulk LWC, and no pronounced overestimation is expected. This is consistent with Fig. 7, which shows that LWC perturbations dominate the extinction response. By contrast, the extinction-oriented r_m - Z relation (Sect. 3.3) is calibrated against the FM-120 dataset, whose upper measurement limit is $50 \mu\text{m}$. When drizzle

droplets dominate the radar sampling volume, they disproportionately increase reflectivity while contributing modestly to the geometric median radius inferred from the FM-120-constrained power law. Consequently, the parameterized r_m may be systematically biased high. Because r_m enters the visibility retrieval with a negative exponent (Eq. 10), an overestimated r_m reduces the inferred extinction, partially offsetting the enhanced extinction from the larger droplets. The net retrieval error under drizzle contamination therefore depends on the competing effects of LWC (well constrained by mass absorption) and r_m (poorly constrained by the FM-120-limited closure). A rigorous quantification of this trade-off requires dedicated observations extending into the drizzle regime.

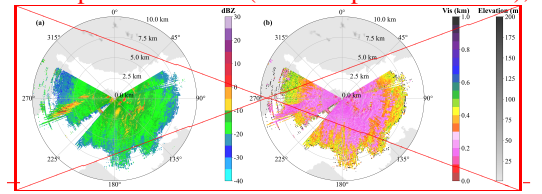
5 Application of the algorithm in Vis to visibility retrieval

This algorithm was applied to the Ka-band radar installed at the DOB, with observations conducted in spring 2025 and subsequent analysis of retrieval results. A fog event observed at the Atmospheric Exploration Base Figures 8 and 9 illustrate the retrieved temporal and spatial evolution of fog microphysical parameters and visibility during a criterion-selected case on 2 March 2025, was selected as the research case. Influenced by the southern branch trough, this event formed when March 2025. Influenced by a southern-branch trough, warm and humid southerly airflows passed through advected over the observation site, classifying it as producing a typical advection sea fog. This advection sea fog developed with high density: it expanded. The fog developed rapidly after 18:00 local time standard time (LST) and reached maximum intensity around 21:00. At a 00 LST. At the radar elevation angle of 2° , fog data could be detected up to 2° , fog returns were detectable to a maximum distance of 10 km, with the fog top height exceeding range of approximately 10 km, and the fog-top height exceeded 350 m. Between 18:00 and 24:00, Z LST, the radar reflectivity factor Z generally decreased with increasing distance range, indicating a stable fog structure with density decreasing with height whose density decreased with altitude. During 20:00 to 22:00, numerous data points with $Z > -10$ dBZ were observed (peak values up to 0 dBZ). As illustrated LST, numerous range gates with Z exceeding -10 dBZ were observed, with peak values approaching 0 dBZ. As shown in Fig. 7, the 8, the polar scan at 20:29 LST reveals that high- Z regions are Z regions were concentrated to the south and southwest of the observation site—areas, where topographical undulations (e.g., hills) and man-made structures (e.g., buildings) are distributed. This indicates the high Z values are induced by the inhomogeneous underlying surface: the uneven surface generates turbulence, which in turn. The uneven underlying surface generates turbulence through surface friction, which promotes droplet coalescence. This phenomenon is attributed to turbulence generated by friction between the fog and the nearby underlying surface, which induces droplet coalescence to



495 form large droplets. Application of the proposed algorithm to the advection fog case on 2 March 2025 (Local Standard Time, LST): (a) Reflectivity factor measured by the YLUKA2 MMCR; (b) LWC retrieved via the proposed algorithm; (c) r_e retrieved via the proposed algorithm; (d) Vis retrieved via the proposed algorithm; (e) Temporal variation of visibility during this event: the abscissa denotes time (LST), the ordinate represents visibility (unit: m); the black line corresponds to DNQ3 Vis meter measurements, while the blue line denotes visibility retrieved by the proposed algorithm.

500 For reference, visibility results calculated via three empirical formulas are also plotted: brown (local empirical formula);



purple (Liu's empirical formula), and gray (Gultepe's empirical formula). Polar radar scans at 20:29 (LST) on 2 March 2025: (a) Z_m , unit: dBZ; (b) Retrieved visibility (Vis, unit: m; color represents Vis value, corresponding to right-side color bars); and the formation of larger droplets, thereby enhancing Z .

To verify the rationality of algorithm parameter selection and estimate retrieval errors, the proposed assess retrieval performance.

505 the present method was cross-validated against Vis empirical algorithms from different regions and DNQ3 observations. To ensure the diversity of the comparison, three traditional Vis-Z empirical formulas derived from different regions were selected, which are listed as follows: Local empirical formula (derived from FM-120 observations): $Vis = 20Z^{0.07}$; Liu-Guangpu et al.'s formula (derived from synchronous radar-visibility meter data in Pingtan, Fujian): $Vis = 40Z^{0.069}$ (Liu et al., 2019); Gultepe et al.'s relationship (derived from FMD-FM100 observations in Lunenburg): $Vis = 40Z^{0.098}$ (Gultepe et al., 2006).

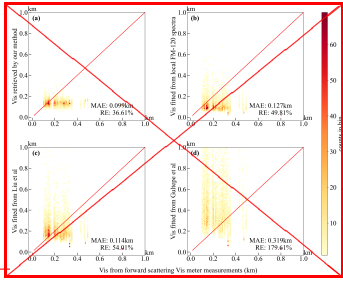
510 As illustrated in Fig. 7(c), all methods exhibited consistent Vis variation trends. However, Vis retrieved from millimeter-wave radar data using different methods were lower than forward-scattering visibility-meter observations and three direct empirical visibility-reflectivity relations (Table 1). Near-range radar data are susceptible to ground-clutter contamination, and the minimum detection range is approximately 150 m; the comparison point was therefore selected at 300 m along the 150° azimuth, where the temporal variation correlated most closely with DNQ3.

515 with smaller temporal variability sensor at 1.5 m above ground level and the radar detection altitude of approximately 30 m at

this range, arising from the 25 m antenna height and the beam elevation geometry. Notably, at 21:00 LST, the radar-detected signal intensified while the DNQ3-measured Vis increased. This seemingly contradictory phenomenon stems from the combined effects of fog microphysical processes³⁰ and instrument-specific limitations. On the one hand, droplet coalescence in the fog generates large particles, which reduces^{22:30 LST, DNQ3 visibility increased markedly while the radar retrievals did not} show a corresponding rise. The intense radar echo around 21:00 LST indicates vigorous fog processes that induced droplet sedimentation; the resulting larger particles reduced the total droplet number concentration and directly contributes to higher surface Vis. The DNQ3 is deployed at 1.5 m above the ground, whereas the radar operates at an approximate detection height of 30 m—this difference in observation heights causes large raised near-surface droplets to more easily adhere to surface objects (e.g., vegetation, structures), further enhancing the Vis measured by the ground-based visibility as measured by DNQ3. On the other hand, as fog droplet diameter increases to values far exceeding the at 1.5 m, while the radar, sampling at a higher altitude, was less affected. Furthermore, the present retrieval is based on backscatter whereas DNQ3's operating wavelength of 0.87 μm , the scattering behavior of the droplets transitions from Mie scattering to geometric optics scattering—resulting in a sharp decline in the exploits forward scattering, so inherent principle-related differences also contribute to the observed discrepancies. Comparison of FM-120-derived Mie-scattering visibility against DNQ3's sensitivity to these large droplets and an overestimation of surface Vis. In the presence of large fog droplets, there exists a systematic bias whereby MMCR-retrieved visibility is consistently lower than that measured by the shows the same tendency: the retrieval is lower than DNQ3 during the early-to-mature fog stage and converges during dissipation. It should also be noted that the FM-120 fog spectrometer has a limited observation range for large droplets, so variations in the median radius cannot be fully captured in the fitting process when large fog droplets are present, leading to larger biases in the fitted median-radius values.

Statistical analysis was performed using ten sea fog events in spring 2025. Fig. 9 presents a comparison of Vis retrieved by the proposed algorithm (Fig. 9a) and three traditional empirical Vis-Z methods (Figs. 9b–d) against in-situ measurements from the DNQ3 forward-scattering Vis meter. The proposed algorithm yields a fractional error of 36.61%, versus 46.96% for the local FM-120 empirical formula, 38.61% for Liu et al.'s formula, and 77.08% for Gultepe et al.'s formula. From the perspective of retrieval accuracy, the Vis derived by our proposed algorithm shows the closest agreement with the in-situ DNQ3 observations. It is followed by the local FM-120 formula and Liu et al.'s formula—both fitted based on observational data in Fujian Province—while Gultepe et al.'s formula (established from Lunenburg datasets) performs the poorest with the largest deviation. It is noteworthy that limited by the observation range of the FM-120 fog droplet spectrometer, variations in r_m cannot be fully captured in the fitting process when large fog droplets are present—leading to larger biases in the fitted r_m values. Although Sect. 4 identifies LWC as the primary factor influencing the algorithm, this r_m -related limitation still introduces a certain degree of impact. Despite the existence of systematic errors and limitations in formula fitting, the proposed algorithm still demonstrates superior stability and smaller retrieval errors compared to the traditional empirical methods—validating the excellent applicability of our method for Vis retrieval.

6 Conclusions



—Evaluation of visibility retrieval results from different methods versus in-situ measurements from the DNQ3 forward-scattering visibility meter, for (a)–(d) distinct retrieval approaches: (a) Results from the proposed algorithm; (b) Results from the local empirical formula; (c) Results from the method of Liu et al.; (d) Results from the method of Gultepe et al. In all panels, the abscissa represents Vis from DNQ3 (unit: m), the ordinate represents Vis retrieved by the corresponding method (unit: m); and the grey dash-line represents the same retrieved and in-situ Vis.

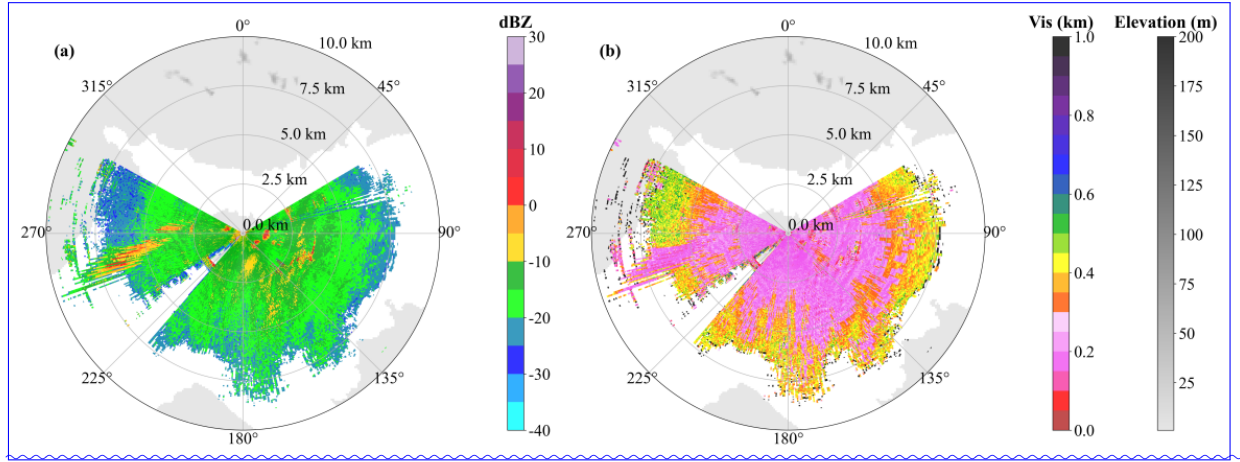


Figure 8. Polar radar scans at 20:29 LST on 2 March 2025: (a) measured radar reflectivity factor (dBZ) and (b) retrieved visibility (km). Range rings are in kilometres, and grey shading gives the displayed elevation context.

This study proposes a novel fog Vis retrieval algorithm based on single-band MMCR data, addressing the limitations of existing empirical methods and expanding the application of MMCR in fog remote sensing. The algorithm integrates radar transmission theory with fog microphysical properties that establishes a physically constrained framework to avoid reliance on empirical coefficients. By leveraging the intrinsic link between Z and β_{ext} through intermediate parameters LWC and r_m , the algorithm achieves superior adaptability across diverse fog regimes compared to traditional Vis- Z empirical methods. The algorithm also exhibits higher sensitivity to LWC variations than to r_m perturbations, while coordinated proportional changes in LWC and r_m maintain retrieval stability by mitigating cumulative uncertainties. This method requires no auxiliary instruments, applies to diverse fog conditions, and has a fractional error of 8.35% induced by the $Q_{\text{ext}} \approx 2$ approximation (with small fog droplets as the primary error source); its overall retrieval fractional error (36.61%) outperforms traditional empirical algorithms.

Application to a typical advection sea fog event on 2 March 2025 demonstrates the algorithm's ability to capture the temporal and spatial variations of fog Vis. During the retrieval process, the algorithm's retrieval error is mainly influenced by LWC

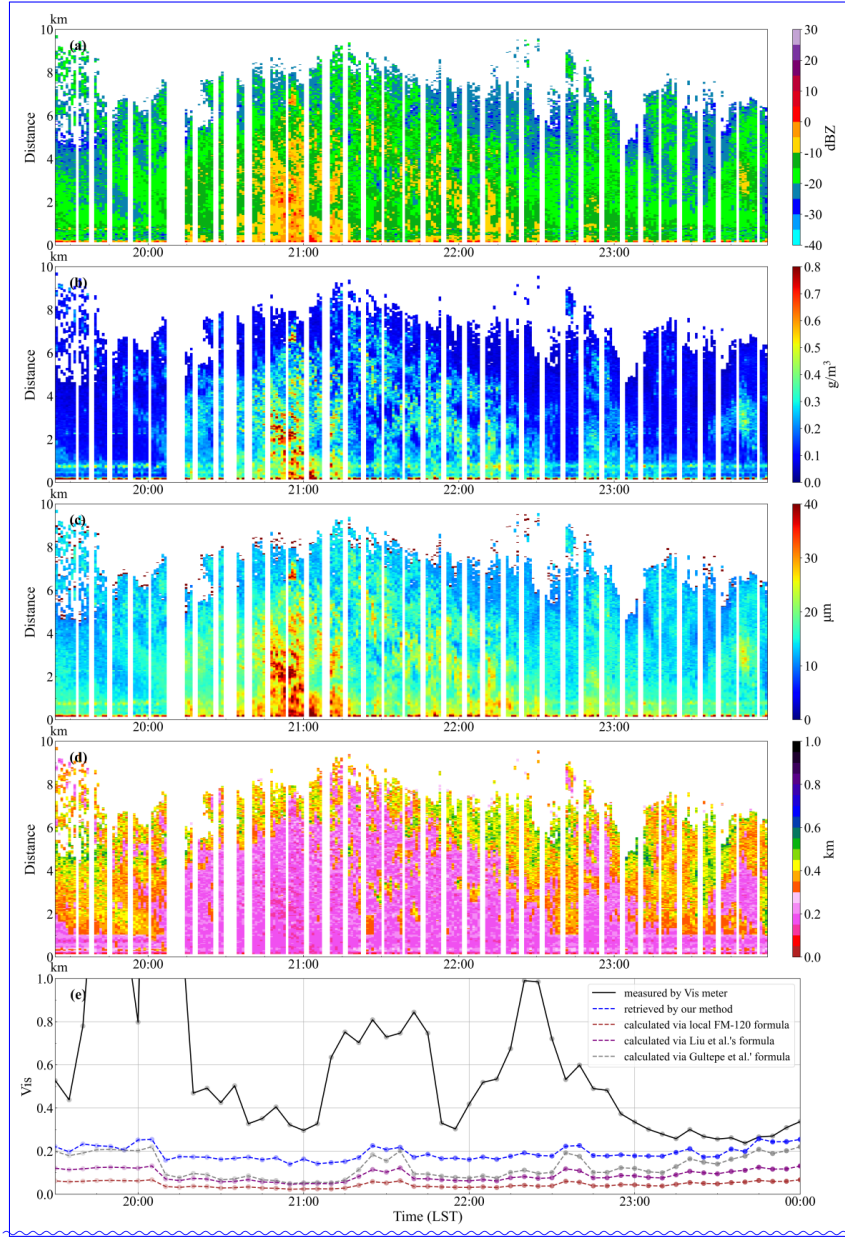


Figure 9. Application to the criterion-selected low-visibility case on 2 March 2025 (Local Standard Time, LST): (a) radar reflectivity (dBZ), (b) retrieved LWC (g m^{-3}), (c) retrieved geometric median radius r_m (μm), and (d) retrieved visibility (km), shown versus LST and range (km); (e) visibility time series (km) comparing the DNQ3 visibility meter, the present retrieval, the local site-specific FM-120 relation, Liu et al. (2019, Eq. 7), and Gultepe et al. (2009, Fig. 10b), with provenance and equations in Table 1.

Table 1. Direct visibility–reflectivity relations used in Figures 9e and 10.

<u>Relation</u>	<u>Equation</u>	<u>Provenance and comparison role</u>
<u>Local FM-120</u>	<u>$\text{Vis}[\text{m}] = 20.15 e^{-0.07 z}$</u>	<u>Site-specific fit: 732 finite pairs on 23 dates to 882 five-minute records. Local baseline only.</u>
<u>Liu et al. (2019)</u>	<u>$\text{Vis}[\text{m}] = 40 e^{-0.069 z}$</u>	<u>Eq. (7) of Liu et al. (2019): 33.44-GHz Ka-band radar and a forward-scatter visibility meter at Ningbo Beilun Port, 15–16 March 2015; reported correlation coefficient 0.732.</u>
<u>Gultepe et al. (2009)</u>	<u>$\text{Vis}[\text{km}] = 0.04 e^{-0.098836 z}$, equivalently $40 e^{-0.098836 z} \text{ m}$</u>	<u>Fig. 10b of Gultepe et al. (2009): 1-s FRAM-L/Lunenburg data on 27 June 2006; Z was calculated from FMD/FM-100 DSDs rather than observed directly by radar; reported RMSE 0.2367 km and correlation -0.7118.</u>

560 The local and r_m ; notably, since both LWC–Liu et al. (2019) prefactors are 0.02015 and r_m are derived from Z , they
tend to co-vary proportionally with changes in Z . This coordinated variation effectively mitigates the cumulative impact of
individual parameter uncertainties, ensuring the algorithm maintains good stability even under fluctuating fog microphysical
conditions. Statistical validation using 10 sea fog events shows 0.040 km, respectively, for the kilometre axes in Figures 9e and
565 pairs from FM-120 DSD observations, with elevated weight applied to the retrieval results exhibit good consistency and
accuracy when compared to both classical empirical formulas and in-situ visibility meter observations, further confirming
the algorithm’s superior accuracy compared to three traditional empirical formulas—with its retrieved Vis values being closest
to low-reflectivity regime where visibility sensitivity is highest (Table 1). Because the radar sampling volume and the DNQ3
in-situ observations point measurement differ in geometry and representativeness, and DNQ3 data also conditioned the event
570 selection and range-gate matching, the comparison in Fig. 10 assesses local consistency and reasonable range rather than
ranking methods.

~~Overall, compared to traditional radar empirical coefficient algorithms,~~

Taken together, Figs. 8–10 present the algorithm outputs used only for a DNQ3-conditioned reasonable-range and local-consistency
comparison in the criterion-selected coastal low-visibility cases. Owing to systematic differences between the radar sampling
575 volume and the DNQ3 point measurement, the comparison is a consistency assessment rather than an independent method-ranking
test. A stricter quantitative evaluation requires more completely collocated observations over additional fog regimes and explicit
characterization of inter-instrument systematic differences.

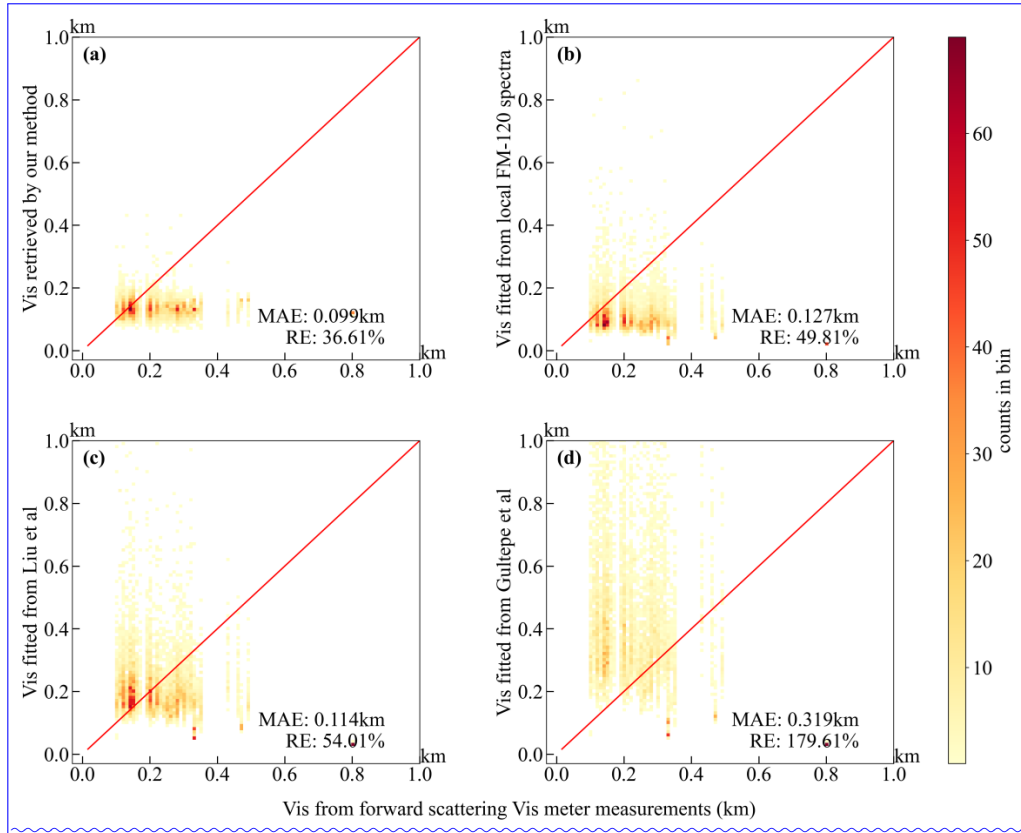


Figure 10. DNQ3-conditioned local-consistency comparison: DNQ3 visibility-meter observations versus (a) the present retrieval, (b) the local site-specific FM-120 relation, (c) Liu et al. (2019, Eq. 7), and (d) Gultepe et al. (2009, Fig. 10b); equations and provenance are given in Table 1. Axes are in kilometres; red lines show 1:1 agreement and colour denotes counts per displayed bin.

6 Conclusions

This study presents a physically constrained, observation-calibrated visibility retrieval algorithm for single-wavelength Ka-band millimeter-wave cloud radar. DSD moment analysis confirms that single-wavelength reflectivity cannot uniquely determine both droplet number concentration and geometric median radius, motivating the introduction of two intermediate microphysical constraints. The first is a segment-wise mass-absorption LWC power law that operates on contiguous 4–10-gate segments with reflectivity-dependent starting vectors, a strong-branch retry, and branch-specific attenuation accumulation. The second is an extinction-oriented radius–reflectivity parameterization calibrated against same-source FM-120 DSD observations, enabling closed-form calculation of the extinction coefficient and hence visibility. Compared with purely empirical visibility–reflectivity relations, the proposed method provides clearer physical underpinnings and better internal self-consistency. The main findings are summarized as follows.

During algorithm development, the dominant error sources are identified as the constant extinction efficiency ($Q_{\text{ext}} = 2$) approximation and the single-lognormal DSD assumption. Multi-year FM-120 observations reveal high morphological diversity in coastal fog spectra: broad-unimodal spectra dominate, whereas bimodal spectra become increasingly prevalent below 200 m visibility. Both LWC and Z exhibit strong discriminability across fog-intensity regimes, and this contrast is exploited to optimize the mass-absorption retrieval framework following Ge et al. (2023).

The single-lognormal closure introduces inherent uncertainty into the radius–reflectivity relation. The extinction-oriented parameterization mitigates systematic extinction retrieval errors without additional observational inputs, with more pronounced improvement in dense fog characterized by relatively high reflectivity, achieving a mean absolute fractional extinction error of 9.37 % on the development dataset. Both intermediate quantities (r_m and LWC) carry retrieval uncertainties, and LWC perturbations exert a stronger influence on the proposed Vis retrieval method—built on the self-consistent LWC algorithm, attenuation-corrected reflectivity, and fog microphysical parameters—exhibits better adaptability and higher accuracy, final visibility estimate than median-radius perturbations. Within the perturbation range of 0.50–2.00 times the baseline values, the relative change in extinction spans from -67.7% to $+209.4\%$. Nevertheless, the covariation of r_m and LWC can partly compensate for individual errors, as reflected in the analytical sensitivity response.

Evaluation on ten criterion-selected spring low-visibility fog events shows that the near-surface visibility retrieved by the proposed method achieves the lowest error statistics among the four compared relations, with a mean absolute error (MAE) of 0.092 km and a relative error (RE) of 42.86 %, followed by the local site-specific FM-120 relation (MAE = 0.123 km, RE = 50.53 %) and Liu et al. (2019) (MAE = 0.160 km, RE = 85.30 %), whereas the Gultepe et al. (2009) relation departs most strongly (MAE = 0.533 km, RE = 310.63 %). The present retrieval, the local FM-120 relation, and holds potential for Vis retrieval in various fog types Liu et al. (2019) yield broadly comparable visibility ranges, consistent with their shared origin in Fujian Province coastal-fog observations, while the larger departure of Gultepe et al. (2009) reflects regional and latitudinal differences in fog microphysics. However, we also note due to certain assumptions in owing to the spatial mismatch between the radar sampling volume and the ground point measurement, as well as the conditioning of event selection and range-gate

matching on DNQ3 data, this evaluation reflects local consistency under conditioned samples rather than independent retrieval validation.

Several limitations of this study must be acknowledged. First, independent in situ LWC measurements and standardized hydrometeor phase data are unavailable, so the retrieved microphysical parameters cannot be validated against reference truth.

615 Second, near-range retrieval results may still be affected by residual ground clutter, as the full quantitative performance of the clutter-filtering algorithm cannot be completely verified. Third, the study samples are limited to spring warm fog at a single southeast coastal site, so the generalizability of the algorithm ~~construction, previous studies have shown that fog often exhibits a bimodal DSD (Liu et al., 2025), and the specific impact of this phenomenon on the algorithm was not discussed here.~~ to other fog types (e.g., radiation fog, orographic fog) and other climate regions remains to be tested.

620 Future work will ~~further analyze the impact of bimodal DSD on the algorithm.~~ pursue three directions. First, dual-wavelength cloud radar observations will be incorporated to further constrain LWC retrieval using differential attenuation characteristics, reducing the non-uniqueness of microphysical parameter estimation. Second, a multi-source synergistic visibility retrieval scheme will be developed by combining lidar, microwave radiometer, and fog-spectrometer observations to add phase identification, vertical structure, and microphysical constraints. Third, observational samples from diverse climate zones and fog types will
625 be accumulated to improve parameterization robustness and generalizability, and to carry out targeted error evaluation for transportation and civil aviation operational applications.

Code availability. The instrumental data used in this work are provided by the Key Innovation Team of Marine Severe Weather Mechanism and Forecasting, Fujian Meteorological Bureau. The intermediate variable data (including liquid water content, LWC; effective droplet radius, r_e) and the final visibility (Vis) retrieval program calculated from these variables, which are presented in this paper, are available
630 upon reasonable request to the corresponding author(s).

Author contributions. Xiaojie Li developed the research concepts and methodology for fog visibility retrieval, performed formal statistical and computational analyses of radar data, conducted field investigations at the observation base, created data visualizations, and drafted the original manuscript. Jinming Ge contributed to the conceptualization of the study and refinement of the retrieval methodology, supervised the overall research process, acquired financial support for the project, managed project administration, provided key research resources,
635 and reviewed and edited the manuscript (corresponding author). Yucheng Qiu curated and processed fog observation data, validated the retrieval algorithm results, and participated in field investigations. Qiaoyi Lv provided research resources, conducted field investigations, curated experimental data, and reviewed and edited the manuscript (corresponding author). Wei Song performed formal analysis of fog microphysical parameters and refined the retrieval methodology. Jie Zhang, Chi Zhang, Bochun Liu, Jiapeng Wang, Hongwei Xia, and Yuhang Zhu participated in field investigations, created visualizations of radar and visibility data, and curated the collected observation data.

640 All authors read and approved the final version of the manuscript.

Competing interests. The authors declare that they have no conflict of interest.

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