

Authors reply to Donelick

General comments

I love this work and this approach to modeling apatite CFT data. I recommend this paper be published with minor revisions. I have attached my version of the problem of deconvolving c-axis projected CFT lengths for timetemperature modeling. There is some emphasis on calibration and on constructing a physical model. I provide this to add to the author's work but also, perhaps, to prod the author to clarify – perhaps with a few additional drawings – the physical model of deconvolution he has developed. I apologize for taking so long to review this. I hope you find this review constructive.

Author: Thanks, two new figures are added after line Line 234:

Revision: The vectors in Eq. (A24) are plotted as histograms in Fig. A1 and matrixes are mapped in Fig. A2.

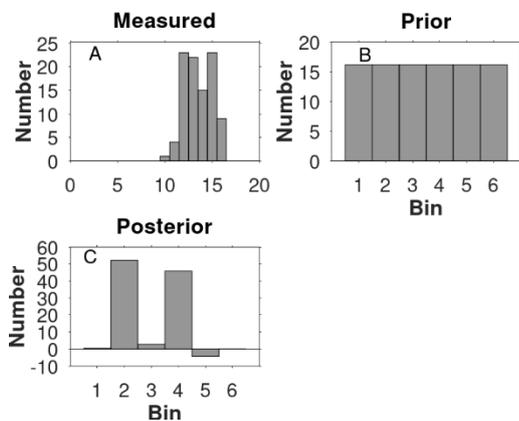


Figure A1: An example of vectors in Eq. (A24) shown as histograms. A. Fission track sample measurements h . B. Prior information h_{prior} of the result of the inversion using Eq. (A24). The expected number of tracks are equally distributed in the bins. It is possible that all tracks are placed in a single bin but with low probability. C. The result of the deconvolution is the posterior \tilde{h} calculated by Eq. (A24).

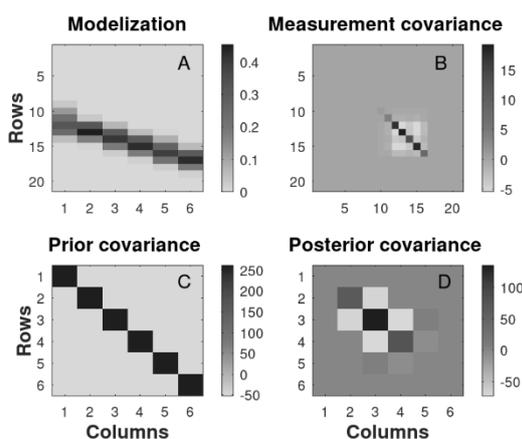


Figure A2: Maps of matrixes in Eq. (A24). A. The columns of matrix G are normalized histograms. The values of the columns are obtained by interpolation among track length distributions measured in the laboratory. The track lengths are increasing along the rows and the mean track lengths are increasing along the columns. B. Variance-covariance matrix C_H of the measurements. The diagonal values are the variances of the measurements calculated by Eq. (D1). The off-diagonal values are covariances of the measurements calculated by Eq. (D2). C. The prior variance-covariance values C_p are calculated by Eq. (D3) and Eq. (D4). The posterior variance-covariance \tilde{C}_p values are calculated by Eq. (A25).

Abstract

Lines 5-6

Reword sentence to: "It is generally accepted that the traditional fission track age equation accurately calculates the cooling age for apatite minerals when the cooling rate is fast."

Author: I agree.

Revision:

"The traditional fission track age equation accurately calculates the cooling age for apatite minerals when the cooling rate is fast."

Replaced by:

It is generally accepted that the traditional fission track age equation accurately calculates the cooling age for apatite minerals when the cooling rate is fast.

Lines 10-11

Reword sentence to: "The difficulty is that the track length–age relation is blurred by the spread in lengths due to the inherent spread in fission decay energies, crystallographic anisotropy, and observational uncertainties."

Author: I agree.

Revision:

"The difficulty is that the track length–age relation is blurred by the spread in lengths due to crystallographic anisotropy and observational uncertainties."

Replaced by

The difficulty is that the track length–age relation is blurred by the spread in lengths due to the inherent spread in fission decay energies, crystallographic anisotropy, and observational uncertainties.

Lines 15-16

Regarding "This information may be used to calculate the age of the oldest fission track among the track populations which can constrain the timing of tectonic events and provide the basis for calculation of past temperature." I agree.

Author: Thanks.

Revision: No revision needed.

1 Fission track age equations

Line 18

Reword "The traditional fission track age equation (Equation 4-6 of Fleischer et al., 1975; Kohn et al., 2024) is..." Two things: 1) Here especially, you need to include reference to the full equation, 2) Write the equation here as I encounter a paywall trying to get Kohn et al. (2024).

Author: I agree and the traditional age equation is added to the manus:

Revision:

“The traditional fission track age equation (Kohn et al., 2024) is used to calculate the age of rapid cooling apatite minerals to temperatures below 60 °C.”

Replaced by:

The traditional fission track age equation (Equation 4-6 of Fleischer et al., 1975; Kohn et al., 2024) is used to calculate the age of rapid cooling apatite minerals to temperatures below 60 °C.

$$t = \frac{1}{\lambda} \ln \left(1 + \lambda \zeta g \frac{\rho_s}{\rho_i} \rho_d \right), \quad (1)$$

where λ is the total decay constant, ζ is a calibration constant, g is the geometrical factor (0.5 for 2pi geometry, 1.0 for 4pi geometry), ρ_s is the surface track density, ρ_i is the induced fission track density, and ρ_d is the track density in the detector.

Lines 22-23

Regarding “...partial annealing window from 120 °C to 60 °C.” it is true that significant, measurable, and useful geological **partial annealing** occurs between 0 °C and 60 °C.

Author: I agree.

Revision: No revision needed.

Lines 26-28 (27-29)

Regarding “...so that ideally the oldest tracks are the shortest ones...” We already know that some of the short tracks are younger than some of the longer tracks. This better re-worded “...so that in previous work it is assumed that the oldest tracks are the shortest ones...”

Author: I agree.

Revision:

“The tracks are generated through time, and the length is shortened as a function of time so that ideally the oldest tracks are the shortest ones and the more recent tracks are the longest ones.

Replaced by:

The tracks are generated through time, and the length is shortened as a function of time so that in previous work it is assumed that the oldest tracks are the shortest ones.

Line 29

Regarding "A given track can therefore be aged by counting the number of shorter tracks..." The word "counting" implies to the fission tracker that we are talking about counting etched fission tracks on a mineral grain surface. But, we are not. These are the random unetched tracks. Please clarify this here.

Author: I agree.

Revision:

"A given track can therefore be aged by counting the number of shorter tracks, adding one, and dividing by the track generation rate."

Replaced by

A given track can therefore be aged by integrating the number of random unetched tracks from the longest to the given track and dividing by the track generation rate.

Lines 34-35

Regarding "...there is no need for the surface track density...to derive the age of a track. Unfortunately, it is difficult to see..." So, because unetched fission tracks are difficult (actually, practically impossible) to see, we fall back on using "surface track density" for which there is "no need". Drop this sentence or flip its meaning. The math might allow something, but reality matters too.

Author: I agree.

Revision:

"As seen from Eq. (1) there is no need for the surface track density and a track length annealing model to derive the age of a track."

The sentence is removed.

Lines 41-43

Regarding "Fortunately, the blurring is known ...and can... be removed from the observed track length histogram by mathematical deconvolution (Jensen et al., 1992; Donelick et al., 1988)." No, the deconvolution accommodates the blurring, preserving it but not sweeping it under the carpet. Deconvolution does not decrease the blurring.

Author: Ok, I agree that "removed" is too strong and should be relaxed.

Revision:

"Fortunately, the blurring is known from artificial annealing experiments and can, to some extent, be removed from the observed track length histogram by mathematical deconvolution (Jensen et al., 1992; Donelick et al., 1988)."

The sentence is replaced by:

Fortunately, blurring is known from artificial annealing experiments and can therefore be deblurred by mathematical deconvolution (Jensen et al., 1992; Donelick et al., 1988).

Lines 44-45

Regarding “The track length histograms were transformed to deconvolved histograms with a tighter relation between histogram columns and time intervals.” Re-word this to “transformed to histograms of deconvolved Gaussians”. Are your CFT populations Gaussians? Defined by?

Author: The sample histograms are not transformed to histograms of deconvolved Gaussians. Instead of using explicit Gaussian distribution functions in the computer program for deconvolution, it is enough to deal with Gaussian mean values (the number of tracks in a histogram column) and std. as the data input. The output is therefore also mean values of Gaussians with std. This is possible because of the least squares method built into the inversion equation given by Tarantola. Please note that there are no iterations involved in the process.

Revision:

“The track length histograms were transformed to deconvolved histograms with a tighter relation between histogram columns and time intervals”

Is replaced by:

After deconvolution the track length histograms were transformed to histograms with a tighter relation between histogram columns and time intervals.

The following is added in line 44:

The inversion relies on an extended least squares method in which variance and prior information is considered. It is implicitly assumed that the number of tracks in each histogram column is the mean value of a Gaussian distribution with a standard deviation. Because of the use of the least squares method, we only need the mean values and their variances (squared standard deviations) of each histogram column in the equations. Gaussian distribution functions are not needed.

Lines 48-49

Regarding “In this paper the deconvolution technique presented takes advantage of the c-axis projection method by Donelick et al. (1999).” Presumably, the model referred to is shown as Figure B1 in this paper. Figure B1 here is far from the model of Donelick et al. (1999) as shown in Fig R3 of the ATTACHMENT.

Author: I am not referring to Figure B1 here. Nevertheless, the remark is relevant.

In this paper the deconvolution technique presented takes advantage of the c-axis projection method by Donelick et al. (1999).

Revision: This sentence is replaced by

In this paper the deconvolution technique presented use c-axis projected tracks (Donelick et al., 1999).

Lines 52 and 146

The term “exact expression” is used to suggest that the equations provided are exact needing no further refinement. None of our mathematical descriptions of fission tracks is exact, and certainly not an expression that is built upon in-exact equations such as the c-axis projection model of Donelick et al. (1999). I am not sure how to reword this...replace “exact” with “more precise”?

Author: I agree.

Revision:

“An approximation in the equations given in Jensen and Hansen (2021) is replaced by exact expressions.”

Is in line 52 replaced by

An approximation in the equations given in Jensen and Hansen (2021) is replaced by a more precise expression.

and in line 146

“The approximation used to derive Eq. (38) in Jensen and Hansen (2021) is here replaced by an exact expression leading to updated equations.”

is replaced by

The approximation used to derive Eq. (38) in Jensen and Hansen (2021) is here replaced by a more precise expression leading to updated equations.

Lines 52-55

Regarding “The inversion technique...the priors dominating the histograms.” I have trouble with negative-data added to known-error-prone-human-derived-data. Just because the math allows it, does not mean that it should be allowed. We need to know where measured data mis-match

Author: I am not adding negative-data to human-derived data. The measurements are not touched. The priors are forcing the deconvolved histogram columns to be mainly positive as they should be. The weight of the prior can be diminished by increasing the variance (or std.) of the prior. This is a user input option to the computer code.

Revision: The following text will be included in the manus after “This is of course unrealistic, but the problem can be resolved by the choice of positive priors with small variances but large enough to avoid the priors dominating the histograms.”

That is, the priors are forcing the deconvolved histogram columns to be mainly positive. The weight of the prior can be relaxed by the user of the computer code by increasing the variance of the prior.

Line 55

Re-word “An overview of symbols is...”

Revision:

“And overview of symbols is shown in Appendix C.”

Is replaced by

A list of symbols is given in Appendix C.

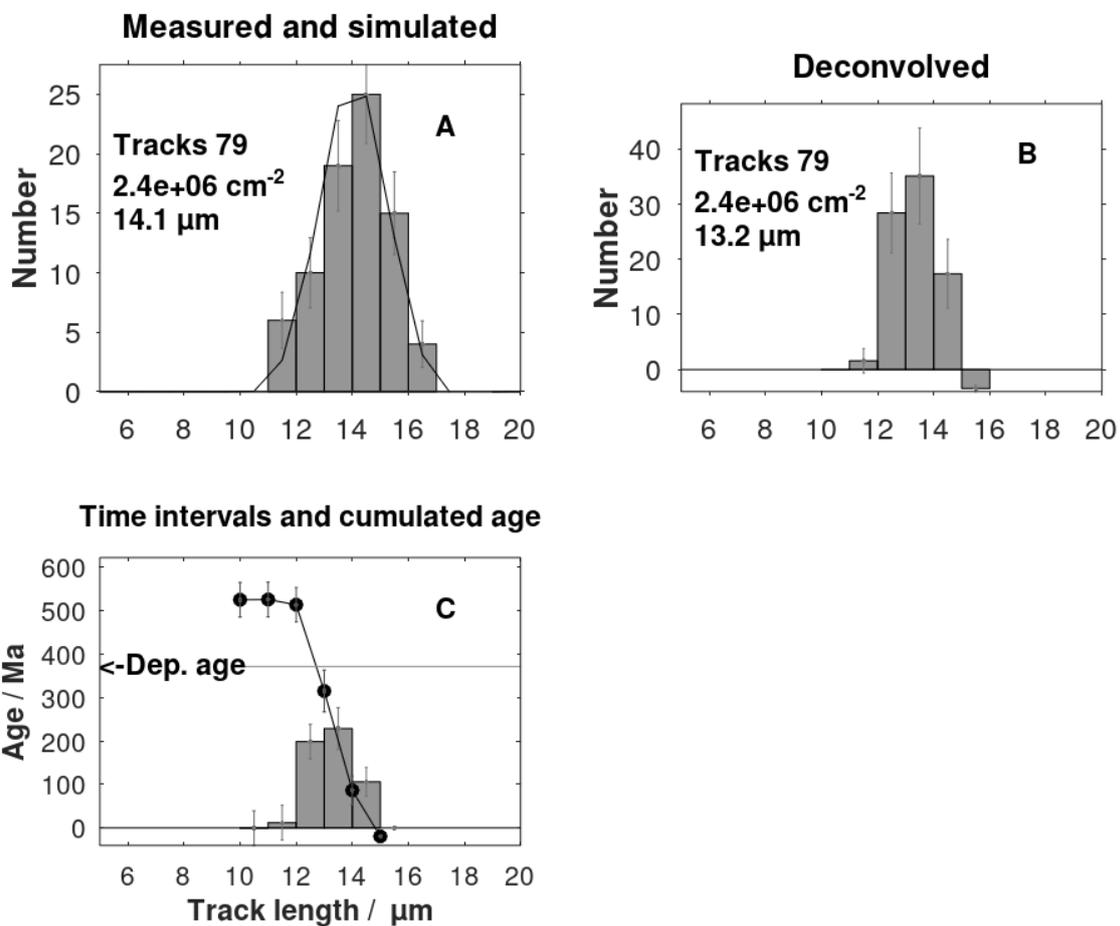
2 Application of age nodes

Lines 59-60

Regarding “...to illustrate the calculation of age nodes.” Please indicate in the captions for Figures 1,2,3 that the “age nodes” are the y-values plotted in plot C of each figure and tabulated in Table 1.

Author: I agree and: “in a” → in A and “age. D.” → age.

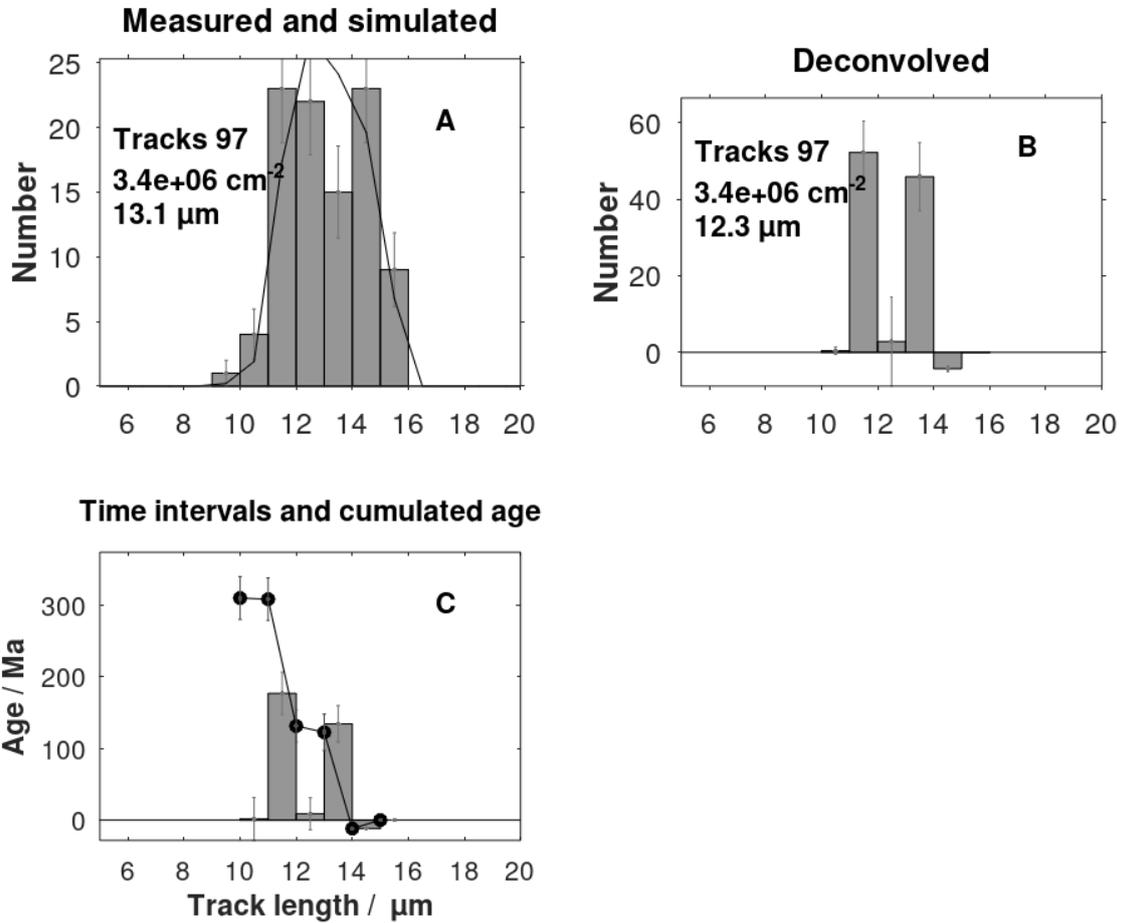
Revision:



“Figure 1: A. c-axis projected track histogram of a sandstone sample CXII-46, Spiegel et al., (2023b) together with distribution. B. Deconvolution of the histogram in a. C. Histogram of time intervals and cumulated track ages. The horizontal line shows the deposition age. D.”

Replaced by:

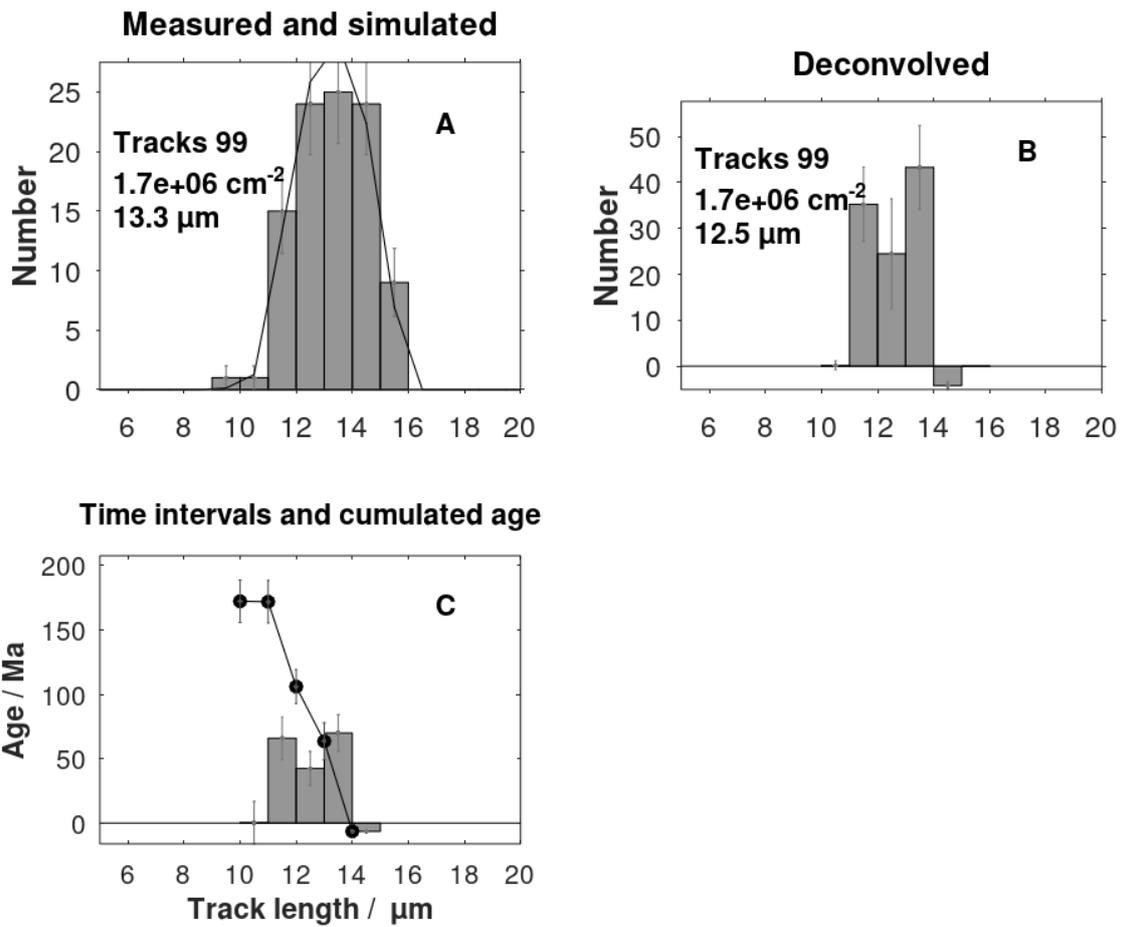
Figure 1: A. c-axis projected track histogram of a sandstone sample CXII-46, Spiegel et al., (2023b) together with distribution. B. Deconvolution of the histogram in A. C. Histogram of time intervals and cumulated track ages. The age nodes (black dots) are the y-values plotted as a function of the c-axis projected track lengths and tabulated in Table 1. The horizontal line shows the deposition age.



“Figure 2: A. c-axis projected track histogram for sample GRÖ64 (Spiegel et al., 2023b) and the calculated distribution. B. Deconvolution of the c-axis projected tracks. C. Histogram of time intervals and cumulated track ages.”

Replaced by

Figure 2: A. c-axis projected track histogram for sample GRÖ64 (Spiegel et al., 2023b) and the calculated distribution. B. Deconvolution of the c-axis projected tracks. C. Histogram of time intervals and cumulated track ages. The age nodes (black dots) are the y-values plotted as a function of the c-axis projected track lengths and tabulated in Table 1.



“Figure 3: A. c-axis projected track histogram of the gneiss sample GRÖ21 (Spiegel et al. 2023b) and the calculated distribution. B. Deconvolution. C. Histogram of time intervals and cumulated track ages.”

Replaced by

Figure 3: A. c-axis projected track histogram of the gneiss sample GRÖ21 (Spiegel et al. 2023b) and the calculated distribution. B. Deconvolution. C. Histogram of time intervals and cumulated track ages. The age nodes (black dots) are the y-values plotted as a function of the c-axis projected track lengths and tabulated in Table 1.

Line 74

Correct spelling “....of the total number of tracks.”

Revision:of the total number of tracs.” →of the total number of tracks.”

Line 83

Table 1 • Please re-order that Samples, from top to bottom CXII-46, GRO64, GRO21, to match their order of presentation in the text. • Please show where “Calibration factor” fits into an equation provided or cited. •

How do you calculate “Oldest age”? • How do you calculate errors of your “Age nodes”? • 1 sigma or 2 sigma errors?

Author: I agree.

Revision: The order is changed accordingly. The table text:

“Table 1. Age nodes related to deconvolved track length, calibration factors, rock ages (Spiegel et al., 2023b), age of oldest track, fission track central ages (Spiegel et al., 2023b).”

is replaced by

Table 1. Age nodes related to deconvolved track length, calibration factors (Eq. A30), rock ages (Spiegel et al., 2023b), age of oldest track (Eq. A30) and error (1 sigma, Eq. A25), fission track central ages (Spiegel et al., 2023b).

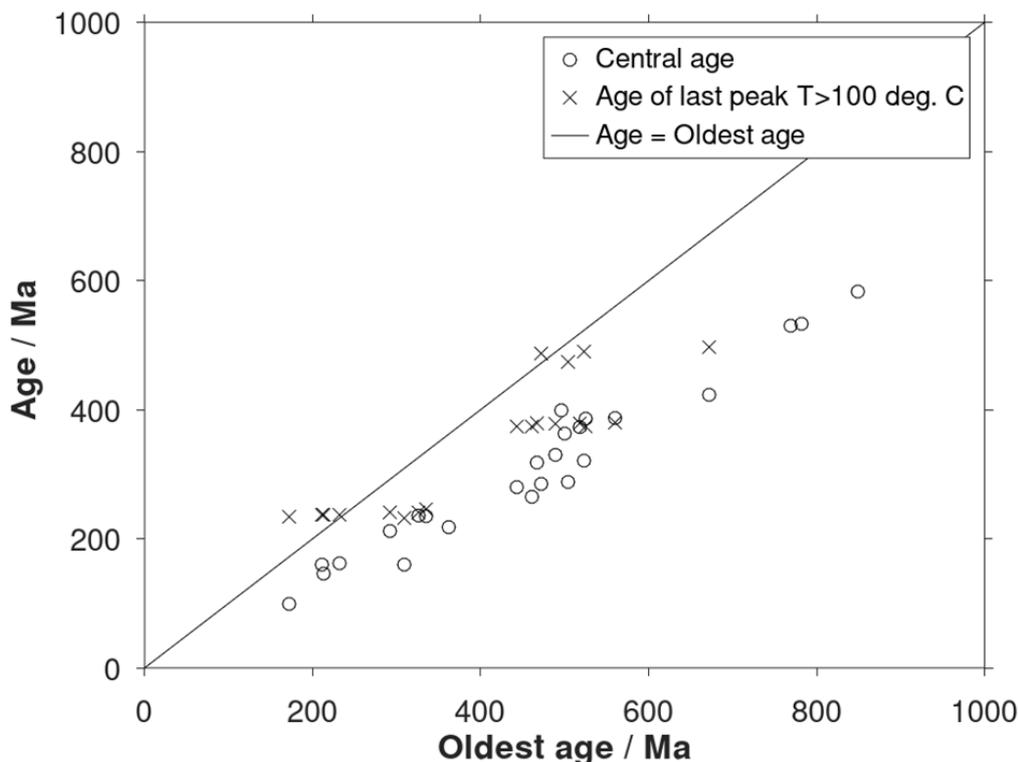
Sample Lithology	Rock age	Deconvolved track length	Age nodes	Calibration factor
	Oldest age Central age Ma			
CXII-46 Sandstone	359-383	14	86 (33)	1.59
	524 (40)	13	315 (48)	
	386 (28)	12	513 (40)	
GRÖ21 Gneiss	2500-4000	13	64 (14)	0.980
	172(17)	12	105(13)	
	99 (9)	11	172 (17)	
GRÖ64 Gneiss	2500-4000	13	122 (25)	0.962
	309 (30)	11	307 (30)	
	160 (11)			

Line 85

Regarding “...age of the oldest track is 193 (16) Ma.” Where does this come from?

Revision: the sentence “The age of the oldest track is 193 (16) Ma.” is removed.

Line 111: Since ages are recalculated due to a revised relation sigma/mean track length recommended by the reviewer. Here is the new implemented figure at line 111:



3 Discussion

Lines 119-120

Regarding “The inversion set up applied here assumes the track length histogram columns to be normally distributed.” This is the first mention of this assumption of normal distribution. See my comments for Lines 44-45, Lines 224-225, and Lines 227-228 where this information would be helpful to know.

Author: I agree.

Revision: More information on normal distribution is now added in Lines 44-45. The following is added in line 226:

The number of tracks in a histogram column is regarded as the mean values of a Gaussian. This is the case for histograms h and \hat{h} in Eq. (A24) and the histograms of each column of the matrix \mathbf{G} . The squared standard deviations (1 sigma) of the Gaussians are implemented in the diagonals of \mathbf{C}_H and \mathbf{C}_p in Eq. (A24).

Line 120

Regarding “A better distribution is Poisson’s distribution.” Why? You throw this out there with no basis. Is it theoretical based on energy distribution of fission yields and/or Poissonian etching behavior? Or, is this because Poissonian slightly better matches observation vs Gaussian? Explain or drop this comment.

Author: The Poisson distribution is the function describing the probability density of radioactive decay. The number of tracks in a histogram column is the result of radioactive decay of U-238. The Gaussian distribution is considered an approximation to the Poissonian. The Poisson distribution does not comply with the least squares principle which is the basis for the inversion given in the manus. I prefer to remove the sentence:

Revision:

The following sentence is removed:

“A better distribution is Poisson’s distribution. Fortunately, the normal distribution is a reasonable approximation for columns of more than five tracks (Bardsley, 2018).”

Line 129

Conclusion Re-word “4 Conclusions”

Replacement: **4 Conclusion** → **4 Conclusions**

Line 130

Regarding “Equations used to calculate multiple age nodes with deviations for apatite fission track length histograms are updated.” This work is great and interesting and I hope the author continues to pursue this interesting approach to apatite fission track time-temperature modeling.

Author: Thanks, I will.

Appendix A

Line 144

Regarding “They are derived from laboratory annealing experiments (Barbarand et al., 2003 and Ravenhurst et al., 2003)...” Really? You did not use Carlson et al. (1999)? If you did not, explain why not. Green et al. (1986) deserves to be cited here as does the Carlson et al. (1999) data set upon which Donelick et al. (1999) is based.

Author: It is correct that I used the laboratory annealing experiments (Barbarand et al., 2003 and Ravenhurst et al., 2003) in kernel G (Eq. A23).

Revision: the following is added at line 145:

The c-axis projection method is based on laboratory annealing experiments by Carlson et al. (1999). In future the experimental data performed by Carlson et al. (1999) and presented in Fig. 3 in Donelick et al. (1999) should be considered as well.

Lines 52 and 146

The term “exact expression” is used to suggest that the equations provided are exact needing no further refinement. None of our mathematical descriptions of fission tracks is exact, and certainly not an expression that is built upon in-exact equations such as the c-axis projection model of Donelick et al. (1999). I am not sure how to reword this...replace “exact” with “more precise”?

Revision: The approximation used to derive Eq. (38) in Jensen and Hansen (2021) is here replaced by a more precise expression leading to updated equations.

Line 154

Is this really “mathematical transposition”? Please verify and clarify if needed.

Answer: Yes it is.

Revision: T indicates mathematical transposition.

Replaced by T indicates mathematical transposition meaning that a row vector is transformed to a column vector.

Lines 172-175

Regarding “Thus, the short tracks are more likely to be accepted as being horizontal than the long ones when measured in translucent light... Tracks measured in reflected light are not biased in this respect...” I don’t agree with either of these statements, but I cannot access Jensen et al. (1992) to see the root argument/equations for these biases. Thus, I allow these statements. My first thought is that you should not mix translucent (or transmitted) and reflected light measurements. Using reflected light to click on a tip is lazy and not compatible with most published annealing experiments. I use reflected light to help me find the track tip when I return to transmitted light to measure.

Author:

Thanks for informing me about the practice of measuring tracks in translucent/reflected light. I see that it is confusing to discuss reflected light here.

“It is expected that the number of etched tracks e_j^i is proportional to their lengths”

is replaced by

It is expected that the number of etched tracks e_j^i is proportional to their lengths. The divisor $1/L_j$ is therefore appearing in Eq. (A10). On the other hand, short tracks are more likely than long tracks to be accepted as being horizontal. The factor L_j is therefore introduced in Eq. (A8).

The following sentence is added:

In this way the tracks measured in translucent light are treated as being unbiased concerning track length.

The following sentence is removed:

“Tracks measured in reflected light are not biased in this respect, therefore $\kappa_j = 1$ in this case.”

Line 196

Re-word “where g is the geometrical factor (0.5 for 2pi geometry, 1.0 for 4 pi geometry)”

Revision: “where g is the geometrical factor.”

is replaced by

where g is the geometrical factor (0.5 for 2pi geometry, 1.0 for 4 pi geometry).

Line 198

Regarding “Appendix B”, the model show in Figure B1 is no where near matching Donelick et al. (1999) or Green (1988). See comments below for Line 281, Figure B1.

Author: Thanks, the model suggested by the reviewer is implemented and the computer program run again. A new figure is given in line 281.

Line 281

Figure B1 Figure B1 offers an unrealistic reduced fission track density vs reduced c-axis-projected fission track length model (r_c). See Fig R3 in ATTACHMENT to how poorly Figure B1 matches Donelick et al. (1999) and Green (1988). For starters, the minimum reduced c-axis-projected fission track length reported in Carlson et al. (1999) was 0.55 for run HS-21 (Appendix 2). This is a practical minimum r_c . As shown in Fig R2 in ATTACHMENT, Donelick et al. (1999) ellipses collapse quickly for r_c between 0.55 and 0.40, the latter a likely true minimum r_c .

The figure text should state that the x-axis is reduced c-axis projected lengths.

Author: A new figure is implemented in line 281 with updated figure text:

Revision:

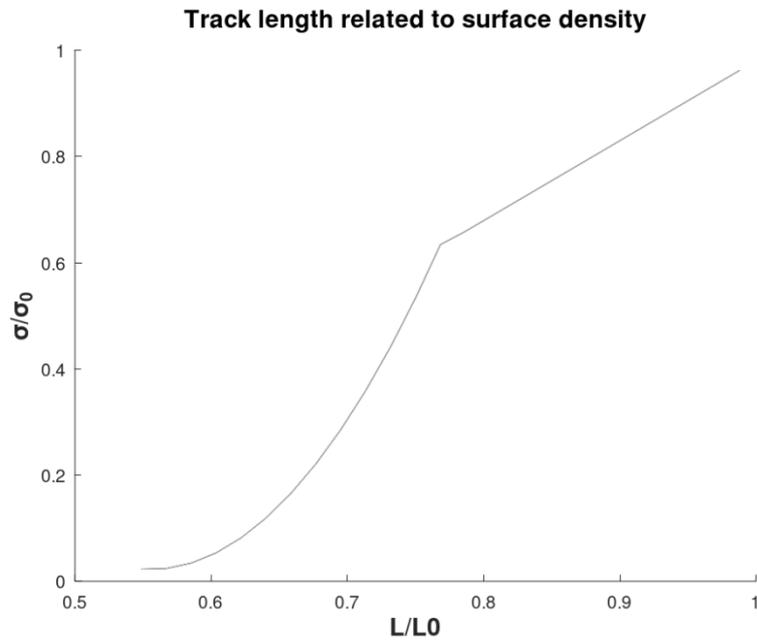


Figure B1: Reduced surface track density $\frac{\sigma^i}{\sigma_0}$ as a function of the reduced c-axis projected track length $\frac{l^i}{L_0}$. The track lengths are c-axis projected. The relations Eq. (B2) and (B3) are approximations to the relation given by Donelick (pers. com).

Appendix C No comment

Appendix D No comment

Code and data availability

I could not access this information for this review.

Author: <https://doi.org/10.5281/zenodo.18062642>

Supplement

I could not access this information for this review

Author: <https://doi.org/10.5281/zenodo.18063819>

References

Author: Line 351 reference is added:

Fleischer, R. L., Price, P. B., and Walker, R. M.: Nuclear Tracks in Solids: Principles and Applications, Berkeley and London (Univ. California Press), 1975, <https://doi.org/10.1180/minmag.1978.042.322.40>

