**Referee #1**

**Overall evaluation:**

This study presents a novel EnKF-like image assimilation system that integrates curvelet transform into the Common Land Model (CoLM) to improve the spatial structure of soil moisture analyses. By shifting the assimilation process from point-based magnitude corrections to multi-scale structural adjustments in the spectral domain, the proposed method addresses a critical limitation of conventional land data assimilation systems. The experimental design is sound, the use of GLDAS and in-situ observations for validation is appropriate, and the results clearly demonstrate the advantages of the approach in enhancing spatial correlation and reducing errors across different soil layers and vegetation types. The manuscript is well-written and the scientific contribution is significant. I recommend publication after major revisions.

Response:

We thank you for the positive and encouraging assessment of our manuscript. We sincerely appreciate your careful reading, constructive comments, and positive feedback on the overall study design, validation strategy, and scientific contribution of this work.

Below, we provide point-by-point responses to your comments. All corresponding revisions have been marked in blue and red in the revised manuscript, with blue indicating added text and red indicating deleted text.

**Major Comments:**

1. Methodological Clarifications

1.1 Flowchart/Schematic (Section 3): Section 3 would benefit from the inclusion of a flowchart or schematic diagram illustrating the overall structure of the EnKF-like image assimilation system. Such a visual aid would help readers better understand the integration of curvelet transform, ensemble-based error estimation, and the Kalman filter update within the spectral space. It would also clarify the sequential flow of data from observation space to image space and back.

Response:

Thanks for your comments. Following your suggestions, we have added a schematic flowchart as Figure 2 in the revised manuscript to Section 3.3, “EnKF-like image assimilation”, to illustrate the overall structure of the curvelet-transform-based EnKF-like image assimilation system. As shown in Figure R1, the flowchart presents the complete assimilation cycle, including ensemble perturbation generation in physical space, forward curvelet transformation, ensemble-based estimation of background and observation error covariances in spectral space, Kalman filter update of the curvelet coefficients, and inverse curvelet transformation back to physical space. We also added corresponding explanatory text in the manuscript to clarify the sequential data flow from physical space to spectral space and back. We have added the relevant description in the revised manuscript (**Lines 274–286**).

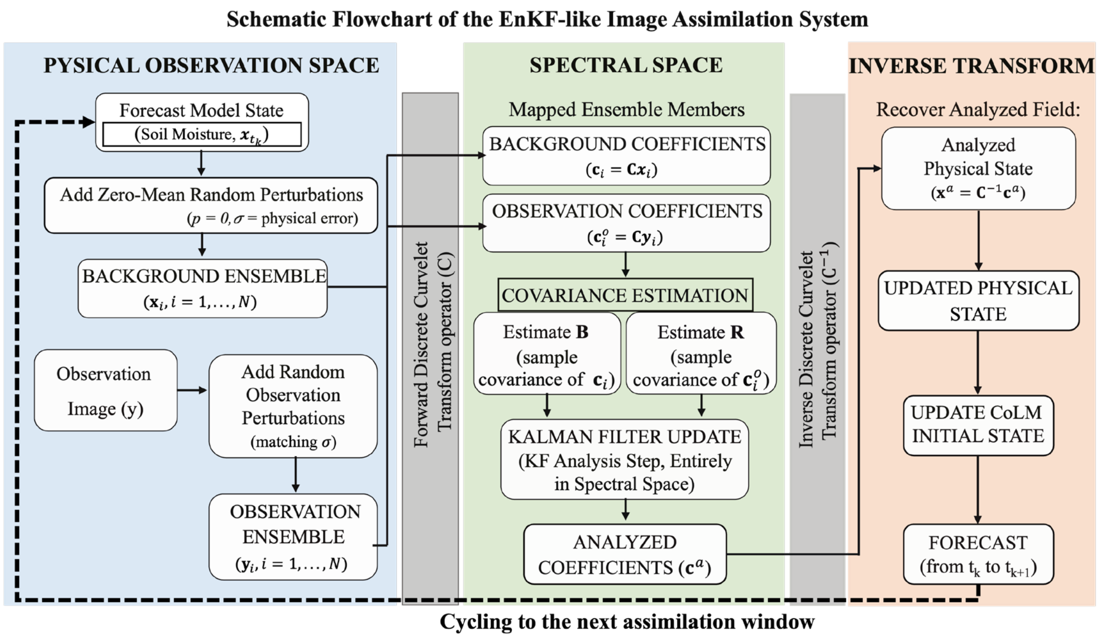


Figure R1: Schematic Flowchart of the EnKF-like Image Assimilation System.

1.2 Curvelet mode interpretation (Lines 170–175): The description of reconstruction using different modes is informative but somewhat abstract. It would be helpful to briefly explain what is meant by "first mode," "first two modes," etc., in terms of spatial scale (e.g., largest scale features correspond to low-frequency modes). This would improve accessibility for readers unfamiliar with curvelet decomposition.

Response:

Thank you for the comments. To help readers better interpret the reconstruction results based on different curvelet modes, we have added a short explanatory paragraph at the beginning of this section. The corresponding revision can be found in the revised manuscript, **Lines 179–188**. The added text is as follows:

Similar to the PCA method commonly used in meteorological studies, curvelet analysis can also decompose the variable into several components with different spatial characteristics. PCA usually extracts leading modes according to the magnitude of variance contribution in time series, and these modes reflect the dominant spatial structures of the variable. In contrast, curvelet analysis uses predefined basis functions at different scales and orientations and projects the variable field at a given time onto these basis functions, thereby obtaining components with different scales and orientations. A single mode in curvelet decomposition can be interpreted as a spatial fluctuation feature within a certain orientation and scale range, and can therefore represent soil moisture variations at a specific spatial scale. In general, lower-order modes correspond to low-frequency signals and mainly reflect large-scale spatial distribution features, whereas higher-order modes correspond to higher-frequency components and describe smaller-scale spatial details. As subsequent modes are gradually introduced, curvelet analysis can resolve increasingly finer spatial structures.

1.3 Error covariance transformation (Lines 210–215): The explanation of how background and observation error covariances are transformed from observation space to spectral space is critical but brief. A more explicit description—perhaps with a step-by-step outline or a mathematical summary—would help readers understand how ensemble perturbations are propagated through the curvelet transform.

Response:

Thank you for your comments. To present the covariance transformation procedure more explicitly, we have provided a step-by-step description of how perturbed ensemble members are generated, how the curvelet transform operator is applied, and how the covariance matrices are estimated in spectral space. In the revised manuscript, we have added the following text in **Lines 249–270**:

Leveraging the mathematical exactness of curvelet analysis, we can transform not only the structure of variables in physical space into spectral coefficients, but also the corresponding errors into coefficients in spectral space. In this EnKF-like assimilation framework, following the error estimation strategy of the EnKF, we first estimate the background error of soil moisture at each grid point in physical space, random perturbations with zero mean and a standard deviation corresponding to the physical-space error are added to the background soil moisture field, thereby generating an ensemble of perturbed members. Let the $i$-th ensemble member in physical space be denoted as $\boldsymbol{x}_i$ ($i = 1, 2, \ldots, N$), where $N$ is the ensemble size. In this study, the ensemble size is set to $N = 50$. By applying the curvelet transform operator $C$, these members are mapped one by one into spectral space, yielding the corresponding curvelet coefficients with errors:

$$\boldsymbol{c}_i = C(\boldsymbol{x}_i) \tag{7}$$

The background error covariance matrix **B** in spectral space is dynamically estimated directly from the ensemble samples in spectral space:

$$\mathbf{B} = \frac{1}{N-1}\sum_{i=1}^{N}(\boldsymbol{c}_i - \bar{\boldsymbol{c}})(\boldsymbol{c}_i - \bar{\boldsymbol{c}})^{\mathrm{T}} \tag{8}$$

where $\bar{\boldsymbol{c}}$ denotes the ensemble mean of the curvelet coefficients, and the superscript T indicates matrix transpose. The observation error covariance matrix **R** is transformed in a similar manner. The standard deviations of the random perturbations are prescribed as 0.15 $m^3/m^3$ for the background soil moisture field and 0.1 $m^3/m^3$ for the observation field. In fact, owing to the orthogonality of different basis functions, the background error covariance **B** and observation error covariance **R** are diagonal in the spectral space. Thus, the unrealistic divergence of error impacts can be avoided, making the error localization and inflation procedure unnecessary.

After **B** and **R** are constructed, the assimilation system performs the EnKF analysis step in spectral space to compute the updated curvelet coefficients $\boldsymbol{c}^a$. Finally, the inverse curvelet transform operator $C^{-1}$ is applied to map the updated curvelet coefficients back to physical space, thereby obtaining the final analyzed field:

$$\boldsymbol{x}^a = C^{-1}(\boldsymbol{c}^a) \tag{9}$$

1.4 Sensitivity to GLDAS biases (Section 3.4): The decision to omit bias correction to preserve spatial structure is reasonable given the study's focus. However, the potential sensitivity of results to systematic biases in GLDAS is not addressed. A brief discussion or a sensitivity test (even in a supplement) would strengthen the robustness of the conclusions.

Response:

Thank you for your comments. To strengthen the robustness of our conclusions, we have included an additional discussion on the potential influence of interpolation in the Discussion section of the revised manuscript. The added text is as follows, and can be found in **Lines 590–596**:

Bias correction represents another source of uncertainty. A spatially uniform bias may have a limited impact on image-based assimilation because it does not substantially alter spatial patterns. Conversely, regionally varying systematic biases in observations can be incorrectly incorporated as spatial structures during assimilation, thus degrade assimilation performance. But because estimating such regional bias characteristics requires long-term observational records, and because this study focuses on evaluating the ability of image assimilation to capture and reconstruct spatial structures, systematic bias correction is not considered here. Future work will assess the impact of bias correction using long-term observational datasets.

2. Data and Validation

2.1 Interpolation effects (Lines 100–105): Bilinear interpolation of GLDAS data from 0.25° to 1.4° is mentioned, but the potential impact of this resampling on spatial structure—especially in heterogeneous regions—is not discussed. A short comment on how this might affect the comparison would be valuable.

Response:

Thanks for your suggestions. We have included additional discussion of the potential impact of bilinear interpolation on the spatial structure of the GLDAS data. The revised text clarifies that resampling the 0.25° GLDAS data to the 1.4° model grid may introduce a smoothing effect, especially in heterogeneous regions, where local soil moisture extremes and small-scale spatial gradients may be weakened. We also explain that this preprocessing step is necessary to ensure spatial-scale consistency between the observation field and the model background field, and that the smoothing effect is expected to be reduced in future applications with higher-resolution land surface models. The corresponding discussion has been added to the revised manuscript from **Lines 99 to 104**.

2.2 Scale mismatch in in-situ validation (Lines 380–385): The independent validation using in-situ data is a strong point. However, station density in some regions (e.g., western China) is low, and the representativeness of point-scale measurements for grid-scale (1.4°) soil moisture is not addressed. A brief note on this scale mismatch and its implications for interpreting correlation and ubRMSE values would improve transparency.

Response:

Thank you for the comments. To provide additional context for the interpretation of these metrics, we have added a brief discussion of this issue in the Discussion section. The added discussion can be found in the revised manuscript from **Lines 584 to 589**. The added text is provided below:

It should be noted that the in-situ validation is affected by a point-to-grid scale mismatch. A single station may not fully represent the mean soil moisture condition of a 1.4° grid cell, particularly in heterogeneous or sparsely instrumented regions such as western China. Therefore, the correlation and ubRMSE values should be interpreted as indicators of temporal consistency with independent observations rather than exact point-scale accuracy. In particular, ubRMSE may include both retrieval/model errors and representativeness errors associated with spatial scaling.

3. Figures and Visual Presentation

Figure 2 caption (Line 225): As noted in the initial review, the caption should be corrected to read: "Figure 2: (a) Soil moisture background field and (b) difference between the background field and the reconstructed field after adding background error perturbations and applying first-mode curvelet inverse transformation."

Response:

Thank you for your comments. We have revised the caption accordingly. Because a new flowchart has been added to the revised manuscript, the original Figure 2 has been renumbered as Figure 3. The revised caption is as follows:

Figure 3: (a) Soil moisture background field and (b) difference between the background field and the reconstructed field after adding background error perturbations and applying first-mode curvelet inverse transformation.

4. Terminology and Consistency

uRMSE vs. ubRMSE (Line 370): The term "uRMSE" appears here, while "ubRMSE" is used elsewhere. Please ensure consistent terminology throughout (preferably "ubRMSE" as defined earlier).

Response:

Thank you for your comments. We have checked all references to this metric throughout the manuscript and ensured that the abbreviation is consistently written as “ubRMSE”, following its earlier definition.

5. Interpretation and Discussion

5.1 Vertical propagation mechanisms (Lines 325–335): The discussion of how assimilation increments propagate under different vegetation types is insightful. It could be strengthened by linking these patterns to known hydrological processes (e.g., root water uptake, preferential flow) and noting whether the model’s parameterizations are capable of representing such processes.

Response:

Thank you for your suggestions. We have revised this paragraph to strengthen the physical linkage between the vegetation-dependent vertical propagation patterns and relevant hydrological processes. The revised text has been added in the revised manuscript (**Lines 414–424**) as follows:

The more rapid propagation at the Non-arctic Grass and Corn sites is likely related to the shallower effective rooting depth of low-stature vegetation. In a Richards-equation-based matrix-flow framework, positive surface increments raise soil moisture and local hydraulic conductivity, favoring downward redistribution and percolation. After the anomaly moves below the main root zone, the constraint from root water uptake becomes weaker, and the signal can continue downward through gravity drainage and matric-potential gradients (Zeng, 2001). At the NET Temperate site, the slower but more persistent deep-layer response is consistent with the deeper rooting profile of forest ecosystems and the longer memory of soil moisture in the deeper root zone. Deep roots interact with a broader soil column through sustained water uptake, which can slow the downward transfer of assimilation increments while maintaining their influence at depth. The current CoLM2014 configuration, however, does not explicitly represent macropore preferential flow. Simulated downward propagation, particularly in forested or structurally heterogeneous soils, may therefore be smoother and slower than actual field responses when preferential flow is active (Beven and Germann, 2013; Fatichi et al., 2020).

The following references have been added to the reference part of the revised manuscript:

Beven, K. and Germann, P.: Macropores and water flow in soils revisited, Water Resour. Res., 49, 3071–3092, https://doi.org/10.1002/wrcr.20156, 2013.
Fatichi, S., Or, D., Walko, R., Vereecken, H., Young, M. H., Ghezzehei, T. A., Hengl, T., Kollet, S., Agam, N., and Avissar, R.: Soil structure is an important omission in Earth system models, Nat. Commun., 11, 522, https://doi.org/10.1038/s41467-020-14411-z, 2020.
Zeng, X.: Global vegetation root distribution for land modeling, J. Hydrometeorol., 2, 525–530, https://doi.org/10.1175/1525-7541(2001)002<0525:GVRDFL>2.0.CO;2, 2001.

5.2 Hybrid assimilation outlook (Lines 475–490): The conclusion states that image assimilation and point-based assimilation are complementary. This is an important point, but the manuscript does not explore how such a hybrid system might be implemented. A brief outlook on potential hybrid approaches (e.g., scale-dependent assimilation weights) would add value.

Response:

Thank you for the comment. We have added a brief discussion on the potential technical pathway for developing a hybrid system in the Discussion section. The added text can be found in the revised manuscript (**Lines 615–620**), as follows:

For example, one potential implementation would be to use image assimilation to efficiently adjust the spatial structure of soil moisture in the background field, while also identifying regions with coherent change patterns. Conventional high-accuracy point-based assimilation could then be applied within these specific regions. This strategy would allow observational data to precisely constrain local states and anomalies without disrupting the large-scale structural continuity maintained by image assimilation, and may therefore provide a promising pathway for future hybrid assimilation.

5.3 Acknowledgement of ChatGPT: The acknowledgment of ChatGPT for language polishing (Lines 490–495) is appreciated. However, it might be more appropriate to place this in the Acknowledgements section rather than the main text.

Response:

Thank you for your suggestions. We have moved this content to the Acknowledgements section of the revised manuscript.

**Minor Comments:**

1. Minor Technical Issues

Reference formatting: Line 625: The reference for Shen et al. (2023) is listed as "Numerical methods," which appears incomplete. Please update with the full journal or conference name.

Response:

Thank you for your comments. We have updated the incomplete reference to the final published version of Shen et al. (2024), and revised the corresponding in-text citation from “Shen et al., 2023” to “Shen et al., 2024” in **Line 317** of the revised manuscript.