

Reply on RC1

Dear reviewers and editors:

Thank you very much for the suggestive comments. We reply to them one by one as follows and modify our manuscript. These comments have significantly enhanced the scientific rigor of the manuscript. Black text is reviewer's comments and blue text is the reply.

Best wishes,

The authors

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(1) The introduction lacks a clear articulation of the core research question, specifically the rationale for investigating the relationship between duration and frequency. This should be supplemented.

REPLY: Thanks a lot for the suggestion. We have added more information about the articulation of the core research question in section 1(Line 45-51).

(2) While the paper focuses primarily on establishing statistical relationships, the physical mechanisms underlying the double-logarithmic relationship remain insufficiently explained. Does this relationship reflect certain characteristics of scale invariance or self-organized criticality?

REPLY: This study utilized more refined daily indices to investigate the nonlinear characteristics during drought persistence processes, revealing a double-logarithmic linear relationship. Furthermore, through research on all China stations, we demonstrated the spatial universality of this nonlinear feature. This indicates that the double-logarithmic linear relationship exhibits certain self-organization characteristics on daily-scale drought events. In section 4, we expanded the discussion on this aspect.

(3) Regarding the results, the lowest  $R^2$  values ( $<0.4$ ) observed in Kashgar, Xinjiang—could this be attributed to sparse station coverage or particular climatic characteristics of the region?

REPLY: The spatial values of  $R^2$  in the northwest region are significantly smaller than those in the southeast region, with the core reason being the prolonged duration

and low frequency of droughts in arid zones, resulting in a low sample size for individual drought events. This is particularly evident in Xinjiang's Kashgar region. Notably, such stations are rare, and there are 98.31% stations with the value of  $R^2$  exceeding 0.6.

(4) In section 3, it states that “ the occurrence frequency of droughts in humid and semi-humid regions is much higher than that in arid and semi-arid regions ” (line155) which appears counterintuitive (as one might expect more frequent droughts in arid regions). Clarification is requested.

REPLY: This conclusion is highly intriguing. In conventional theories, more frequent droughts are observed in arid regions. In fact, when considering shorter time scales, short-duration droughts are more frequently detected, characterized by brief duration and high frequency. Consequently, from a frequency distribution perspective, they appear more pronounced.

(5) This study points out that the nonlinear relationship is present in the drought duration and the frequency. I am wondering whether this nonlinear relationship is time-dependent? This is, the nonlinear relationship exhibit interannual or interdecadal variability? In addition, whether the nonlinear characteristic of the relationship between the drought duration and the frequency could be quantified by some indices, such as the Lyapunov exponent (References are listed below) or other methods? Please clarify and give some discussion.

Reference:

Li, X., et al., 2020: Quantitative comparison of predictabilities of warm and cold events using the backward nonlinear local Lyapunov exponent method.

Li X., et al., 2023: Estimating the local predictability of heatwaves in south China using the backward nonlinear local Lyapunov exponent method.

Li X., et al., 2026: Investigating the predictability limit and forecast error dynamics of summer air temperatures over East Asia using the ensemble forecast dataset from ECMWF.

REPLY: Indeed, an intriguing question arises as to whether the nonlinear relationship between drought duration and frequency exhibits temporal stability or

evolves over longer time scales. However, robust statistical characterization of drought frequency necessitates sufficient sample sizes; consequently, shortening the observation period inevitably compromises the significance of identified patterns. Future investigations may benefit from extended observational sequences or paleoclimate reconstructions to elucidate long-term variations in this nonlinear relationship. Notably, complementary nonlinear indices — such as those capturing drought intensity dynamics or compound event characteristics — could collectively advance our understanding of these evolving patterns. We have expanded our discussion of this research frontier in Section 4 (Discussions), and more introduction about the exploration of nonlinear research in section 1(Introduction).

Additional minor issues:

- (1) Figure 9 caption: “Severe drought (a)” should be “(b)”;
- (2) Figure 10 caption: the "2" in "R2" should be formatted as a superscript ( $R^2$ );
- (3) “Extraordinary drought” is recommended to be changed to “extreme drought”.

REPLY: All above mistakes have been modified.

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Thanks very much for your suggestive comments. We reply to them one by one as follows and modify the manuscript. These comments have significantly enhanced the scientific rigor of the manuscript. Black text is reviewer's comments and blue text is the reply.

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Based on the China daily-scale meteorological drought index, this manuscript revealed a double-logarithmic relationship between drought duration and occurrence frequency, and proposed that a single parameter can characterize this nonlinear feature. The findings provide quantitative evidence for water resource management, agricultural planning, and drought disaster prevention and mitigation. The following issues need to be corrected.

General comments

1. Page3: The manuscript mentions “Qinghai-Tibet Plateau (gray area in Fig. 1) region is not included in our study due to sparse station data”, and “the selected stations distribute in most regions of China except Hainan and Taiwan”. However, the impact of such exclusion on the representativeness of conclusions at the national scale was not discussed.

REPLY: The omission of observation stations on the Qinghai-Tibet Plateau has indeed constrained the investigation of drought nonlinearities in this region. However, the scarcity of high-quality, long-term observational records in these areas currently precludes their reliable inclusion; utilizing such data would likely introduce substantial uncertainty and potentially compromise the robustness of the findings. While, Taiwan and Hainan — characterized by insular climate regimes distinct from continental patterns — were excluded due to data limitations. Their exclusion is not expected to materially affect the conclusions of this study, given their climatic dissimilarity to the mainland focus. Future research may incorporate these regions as

enhanced observational capabilities and methodological advances become available.

2. Page4: Formula (2) uses  $\log_2$  instead of natural logarithm or common logarithm. There is no justification for selecting base 2. It is recommended to briefly explain the rationale for this choice in the text or to uniformly use natural logarithm.

REPLY: This is a good question. Indeed, the study by Şişman (2020) employed the natural logarithm. However, in this manuscript, we adopt base-2 logarithm. When analyzing drought duration processes, we use  $x^t$  (where  $x$  represents either  $e$  or  $2$ ). As showing in figure 3b (as follows), when using a logarithmic function with base 2, the x-axis represents duration more precisely and clearly. When using a logarithmic function with base  $e$ , the x-axis represents duration more roughly. For other quantitative relationships in this manuscript, only the parameter values change; this does not alter the nonlinear relationship derived from the quantification.

REF: Şişman E.. Self-similar characteristics of drought duration, total deficit, and intensity curves. Arabian Journal of Geosciences. 13:8, 2020. DOI:<https://doi.org/10.1007/s12517-019-4977-9>

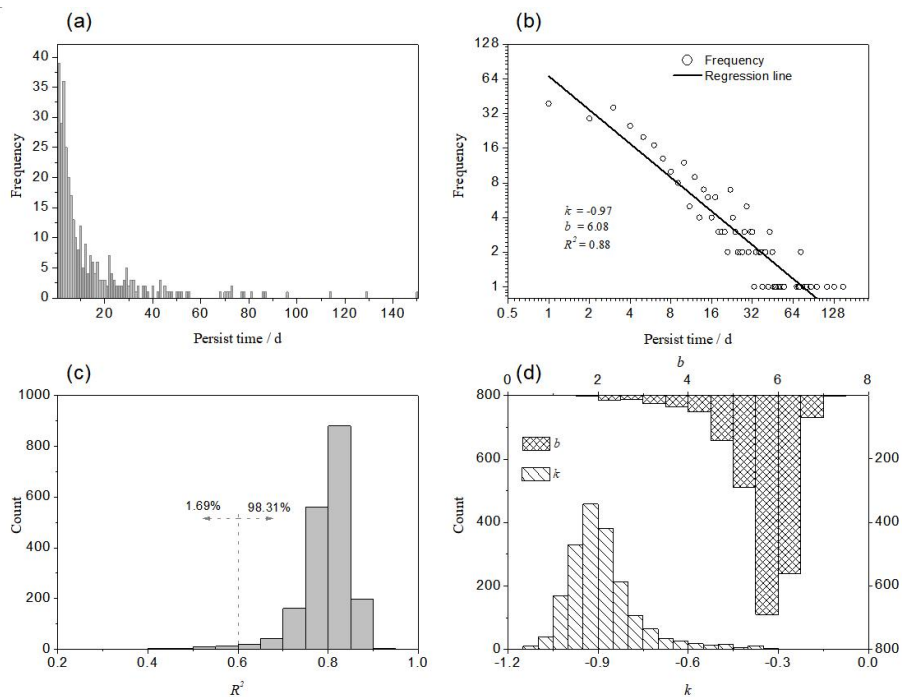


Figure 3. Linear relationship of frequency and drought duration and the PDF of parameters of relationship. (a) the frequency of drought persist time. (b) the linear relationship between the frequency and drought persist time. (c) the PDF of  $R^2$ . (d) the PDF of parameters  $k$  and  $b$ .

3. Page12: The article states that parameter  $k/b$  can independently characterize

the relationship between drought duration and occurrence frequency (page 8), but Figure 5a shows a highly linear correlation between  $k$  and  $b$  ( $R^2=0.95$ ). This does not imply that  $k$  itself contains all information, but rather indicates a strong linear relationship between the two. It is recommended to phrase more cautiously as “in practical applications,  $k$  can be used alone for regional comparisons.”

REPLY: Thanks for the useful suggestion. We have modified the manuscript as follows in page 12:

*..., meaning that either parameter can be used independently for regional comparisons in studying the nonlinear association between drought duration and frequency.*

4. Page12: In practical drought management, mild and frequent short-duration droughts have significantly different socioeconomic impacts compared to rare long-duration droughts. It is recommended to supplement the conclusions or discussion with the implications of these findings in this regard.

REPLY: In section 4 (discussion) of the original manuscript, we clearly outlined the significant implications of this conclusion. In this version, we have expanded and reinforced this perspective as follows:

*Quantifying the relationship between drought duration and occurrence frequency provides critical insights for optimizing water resource management, agricultural planning, and disaster mitigation strategies, thereby enhancing societal resilience to drought impacts. In particular, the revealed nonlinear relationship not only quantifies the characteristics between drought duration and frequency, but also provides new insights for research on drought predictability and disaster severity.*

#### Text and Formatting Issues

1. Page 2, line 35: “Generally, drought events occur on various time scales, including inter-decadal(Xu et al., 2017), inter-annua(e. g. Wang et al, 2018; Zhang et al, 2020)” → “inter-annua”should be “inter-annual”.

REPLY: We have corrected the mistake.

2. Page 2, line 45: “As it can be seen, droughts occurring on different time

scales...” → is recommended change to be “Thus, droughts occurring on different time scales...”.

REPLY: We modify the description about this part as follows.

*As it can be seen, This demonstrates that droughts can occur across different time scales, with shorter drought events expected to occur more frequently, while prolonged droughts should be relatively rare.*

3. Page 9, Figure 7: The title is written as “Distribution of parameter k (a) and the relationship... (a)” → The second "(a)" should be corrected to "(b)", and Figure 7b is described as Figure 7b in the main text but lacks a title. It is recommended to maintain consistency.

REPLY: We have corrected the mistake.

4. Page13-14: Some references include DOI, while others do not. It is recommended to standardize the format or complete the DOI information for all references.

REPLY: We have supplemented the DOI numbers for all references except The People's Republic of China National Standard.

## Reply on RC4

Dear reviewer and editors,

We sincerely appreciate the reviewer for the insightful and constructive comments, which have significantly enhanced the quality of our manuscript. We have carefully revised the manuscript to address all the concerns raised. Below, the reviewer comments are shown in black, with our responses in blue.

**Q1:** The authors base their study on a daily index that they briefly discuss the derivation and discusses the concept of flash droughts which seems like an impossibility to me. The main result of the study is the fitting of a double-logarithmic function for the combination of frequency and duration of the drought periods. As one of the motivations are these flash droughts, how can this research help with that?

**REPLY:** As you rightly pointed out, the distribution of parameter  $k$  in our study indeed reflects the distribution of droughts. In arid regions, the absolute value of  $k$  is larger, and the maximum drought duration tends to be longer. In contrast, in humid regions, the absolute value of  $k$  is smaller, the maximum drought duration is shorter, and flash droughts also frequently occur there. We have incorporated this information in section 4 discussion.

*This finding also facilitates the identification of flash drought events with greater precision. As evidenced by the spatial distribution of parameter  $k$ , flash droughts persisting for several days to several tens of days occur more frequently in humid southern regions, where higher absolute values of  $k$  correspond to elevated occurrence frequencies of short-duration droughts.*

Additionally, we have made the following improvements to the description of Figure 7b.

*It is indicated that although the drought occurrence frequency in humid and semi-humid regions is relatively higher, the corresponding drought duration is generally shorter, whereas drought duration in arid and semi-arid regions is generally longer.*

**Q2:** Is the derived function well-constrained and in what range is it valid? In Figure 7b, the frequency curve seems to not approach zero rather flatten at a constant value, what does that do to the overall distribution?

**REPLY:** We understand your concern. In fact, according to the equation, as drought duration  $\tau \rightarrow \infty$ , frequency  $f$  should approach 0. Table 1 below presents the numerical values of  $\tau$  and  $f$  when  $k > -0.6$  in Figure 7b ( $125 \leq \tau \leq 150$ ) marked as red line in Figure 7b(as follows) . It can be seen that  $f$  indeed decreases with increasing  $\tau$  and approaches 0 value slowly. Tables 2 and 3 present the results for  $-0.8 < k \leq -0.6$  and  $k \leq -0.8$ .

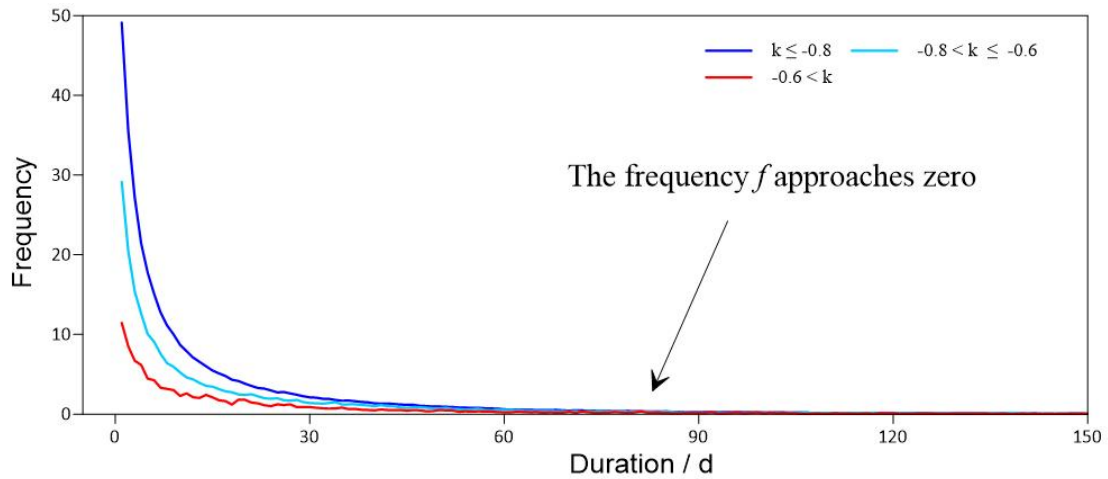


Figure 7b The relationship between the drought duration and its frequency with different parameters

Table 1 Parameter  $f$  value with  $\tau$  when  $k > -0.6$

$\tau$ value	$f$ value	$\tau$ value	$f$ value	$\tau$ value	$f$ value
125	0.0615	134	0.0923	143	0.0154
126	0.0923	135	0.0462	144	0.0615
127	0.0308	136	0.0154	145	0.0154
128	0.0308	137	0.0769	146	0.0615
129	0.0615	138	0.0000	147	0.0462
130	0.0923	139	0.0308	148	0.0615
131	0.0615	140	0.0769	149	0.0769
132	0.0769	141	0.0154	150	0.0462
133	0.0615	142	0.0769		

Table 2 Parameter  $f$  value with  $\tau$  when  $-0.8 < k \leq -0.6$

$\tau$ value	$f$ value	$\tau$ value	$f$ value	$\tau$ value	$f$ value
125	0.0866	134	0.0909	143	0.0563
126	0.1169	135	0.1169	144	0.0519
127	0.0693	136	0.0476	145	0.0346
128	0.0866	137	0.0519	146	0.0649
129	0.0606	138	0.1212	147	0.0303
130	0.0736	139	0.1429	148	0.0433
131	0.0779	140	0.0693	149	0.0519
132	0.0823	141	0.0866	150	0.0303
133	0.0693	142	0.0606		

Table 3 Parameter  $f$  value with  $\tau$  when  $k \leq -0.8$

$\tau$ value	$f$ value	$\tau$ value	$f$ value	$\tau$ value	$f$ value
125	0.1006	134	0.0793	143	0.055
126	0.0974	135	0.0575	144	0.0612
127	0.0874	136	0.0681	145	0.0500
128	0.0874	137	0.0593	146	0.0518
129	0.0843	138	0.0662	147	0.0587
130	0.0675	139	0.0562	148	0.0587
131	0.0868	140	0.0581	149	0.0487
132	0.075	141	0.0643	150	0.0575
133	0.0787	142	0.06		

Q3: The discussion section of the manuscript is extremely short and does not enlighten the reader on how more specifically this can be used, in what way, uncertainties, limitations, differences to other index used in other areas of the world etc.

REPLY: In section 4 discussion, we have added more information about this as follows:

*Several limitations should be acknowledged. First, the exclusion of the Qinghai-Tibet Plateau due to sparse station data may affect the completeness of the spatial pattern in western China. Second, MCI's climate-adaptive coefficients are calibrated for China and may not be directly applicable to other regions without recalibration. Third, the double-logarithmic fitting assumes stationarity in drought statistics; climate change may alter these relationships over time, requiring dynamic updating of parameters. Finally, the study focuses on meteorological drought; propagation to hydrological and agricultural droughts involves additional complexities not captured here. The use of MCI rather than SPI or SPEI introduces both advantages and limitations. MCI's multi-timescale design (incorporating 30-150 day precipitation anomalies) captures drought evolution better than SPI alone, but the*

*index is region-specific. Future research will employ complementary drought indices and independent observational networks to validate the robustness of the duration – frequency relationship across diverse hydroclimatic regimes.*