

Response to reviewers

We thank the reviewers and all commenters for their helpful and insightful comments. We respond to reviews and comments below. First, however, reviewers and commenters made some excellent points about boundary conditions, which we expand on below. Both reviewers 1 and 2 are entirely correct that our boundary conditions were not especially well motivated and ad hoc. We have since expanded to a wider suite of boundary conditions, including applying ocean pressure at the calving front.

To summarize, we find that when we consider an un-buttressed ice tongue using ocean pressure as the calving front boundary condition, the stress regime is unlikely to produce an array of closely spaced crevasses. Instead, we anticipate isolated or widely spaced crevasses, which is broadly consistent with the tabular berg calving regime of most ice tongues and ice shelves.

To find conditions when closely spaced crevasses are possible, we also consider confined ice shelves where drag from lateral margins and pinning points limits the velocity of the ice shelf and contributes to the driving stress. This situation is more analogous to the situation initially envisioned by Nye. In this case, we need to approximate the lateral shear stresses and bottom drag that give rise to buttressing. Here we find conditions where it *may* be possible to form closely spaced crevasses and, in this case, the crevasse depths follow the predicted Nye depth. Together, these two loading configurations suggest that closely spaced crevasses are most likely to form near shear margins and upstream of pinning points, but as crevasses extend into unbuttressed portions of the ice shelf, we anticipate that crevasses transition to isolated rifts. The preference for isolated crevasses is broadly consistent with observations of tabular bergs that detach from ice shelves and ice tongues.

We discuss all of these issues in addition to reviewer comments below, starting with a deeper consideration of the boundary conditions and consequences of applying different boundary conditions.

Idealized stress balance and the shallow shelf approximation

Ideally we would, as reviewer 2 suggests, simply solve the relevant equations in three-dimensions across the entire ice shelf domain, applying ocean pressure at the calving front, no-slip along the margins and a grounding line velocity at the grounding line. However, this is numerically impractical for us to do in a fully three-dimensional model that resolves crevasses. Instead, our goal is insight based on idealized geometry and boundary conditions. To reduce the number of simulations and simplify the discussion, we neglect surface crevasses and concentrate exclusively on bottom crevasses in the discussion that follows.

Case 1: Water pressure boundary condition for an unconfined ice tongue with no lateral shear stress and no buttressing

At the calving front of ice shelves and ice tongues, the dynamic boundary condition corresponds to water pressure. Results from a suite of experiments using water pressure as the traction boundary condition at the calving front are shown in Figure R1 for a suite of purely elastic simulations with varying crevasse spacing. In these simulations, all crevasses are assumed to penetrate to an equal depth and the array of crevasses are placed symmetrically starting at positions $x=w/2, 3w/2, \dots$ where w is the crevasse spacing. It is evident from Figure R1 that bending stresses play a role modulating the stress up to a few ice thicknesses from the calving front and that stresses at the tips of shallow bottom crevasses increase upstream from the calving front. In this case, we anticipate that crevasses closer to the calving front will penetrate a smaller fraction of the ice thickness compared to those further from the calving front. This will result in less shielding, a situation that would seem to favor the formation of more isolated crevasses further back from the calving front. This, however, seems

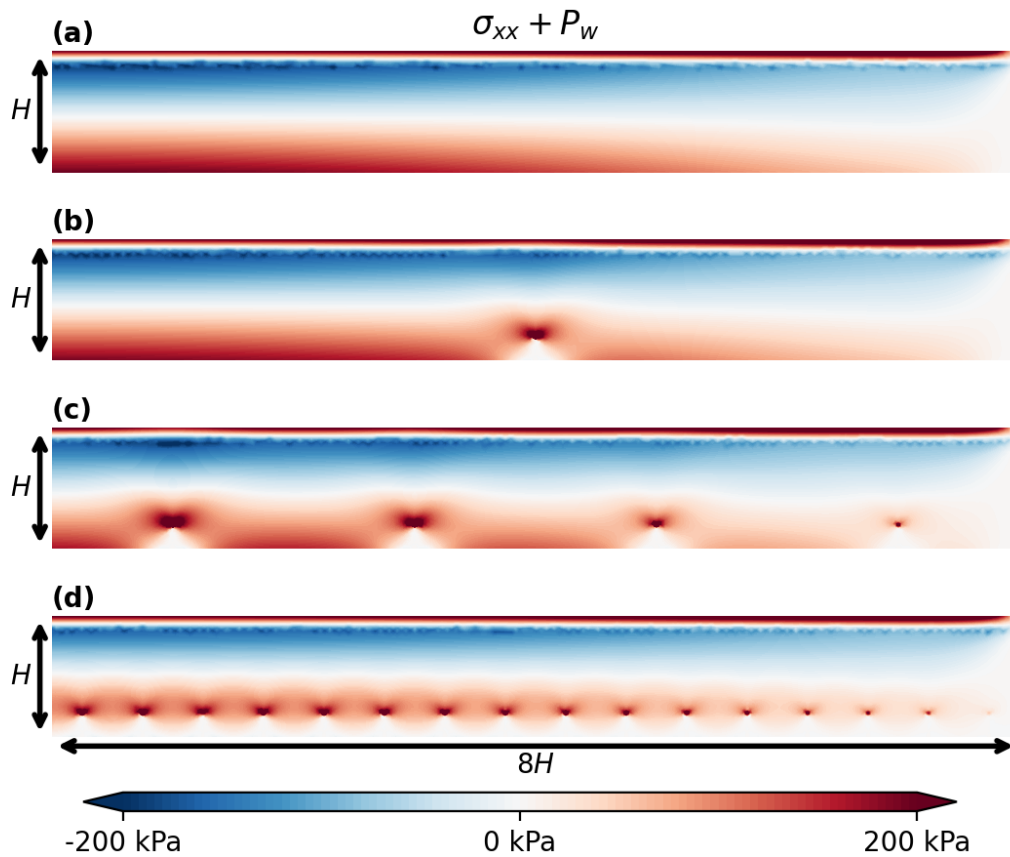


Figure R1. Suite of simulations conducted using water pressure as the traction boundary condition at the right edge of the domain. Panel (a) shows an ice shelf with no crevasses. Panel (b) show an example with crevasse spacing 8 times the ice thickness. Panel (c) shows a simulation with crevasse spacing 2 times the ice thickness. Panel (d) shows an example with crevasse spacing half of the ice thickness. Bottom crevasses penetrate 20% of the ice thickness.

consistent with the calving behavior of ice shelves and ice tongues where tabular bergs are typically favored.

Although we don't anticipate that this situation will lead to stable crevasses, we can evaluate the depth of crevasses by either looking at the stress at the tip of the crevasse furthest upstream or using the change in elastic potential energy. The change in elastic potential energy that we evaluate, however, corresponds to the penetration depth of an array of equal depth crevasses, treating all crevasses equally. These two depths are shown in Figure R2. Here we see that crevasse depths computed using the energy and stress method largely agree for large crevasse spacing, but diverge as crevasse spacing becomes small. In the small crevasse spacing limit, the depth of closely spaced crevasses appears to tend towards zero, evidence that an array of equal depth crevasses that extends to the calving front is unlikely. By contrast, depths for the crevasse furthest from the calving front asymptotes to a value that is close to Weertman's thick plate solution for an isolated bottom crevasse. Although perhaps

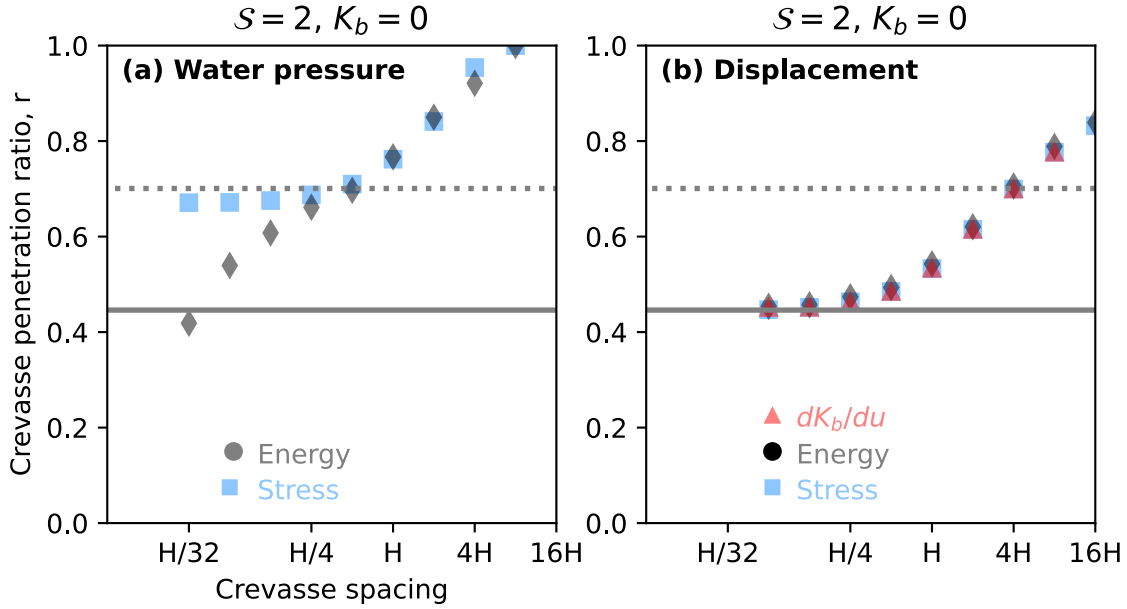


Figure R2. Panel (a) shows the depth of bottom crevasses associated with a water pressure boundary condition. We compute the depths using an energy method, that treats all crevasses equally and a stress method that only examines the stress at the crevasse located furthest from the calving front. Panel (b) shows an analogous calculation, but this time for displacement boundary conditions. Here the bottom crevasse depths computed using both the stress and energy method are numerically identical for all crevasse spacing. The red triangles show results that consider the gradient in buttressing, which we consider in Case 3.

coincidental, the crevasse penetration ratio is almost exactly the average of the Nye and Horizontal Force Balance crevasse depth. We reiterate, however, that closely spaced crevasses seems unlikely in this case.

Case 2: Traction boundary condition for an unconfined ice tongue with no lateral shear stress and no buttressing

Although less realistic, we can write the shallow shelf approximation for the x-component of conservation of momentum in the form:

$$(1) \quad 2 \frac{\partial (H\tau_{xx})}{\partial x} + \frac{\partial (H\tau_{xy})}{\partial y} - \tau_b = \frac{\rho_i g'}{2} \frac{\partial H^2}{\partial x},$$

where H denotes the ice thickness and we have defined the reduced gravity $g' = (1 - \rho_i/\rho_w)$ where ρ_w is the density of seawater. The second term on the left hand side denotes lateral shear stress gradients whereas the term τ_b denotes any resistance associated with sliding over grounded portions of the ice sheet. At the calving front $x = x_c$, continuity of traction with the ocean water requires that the deviatoric stress is given by:

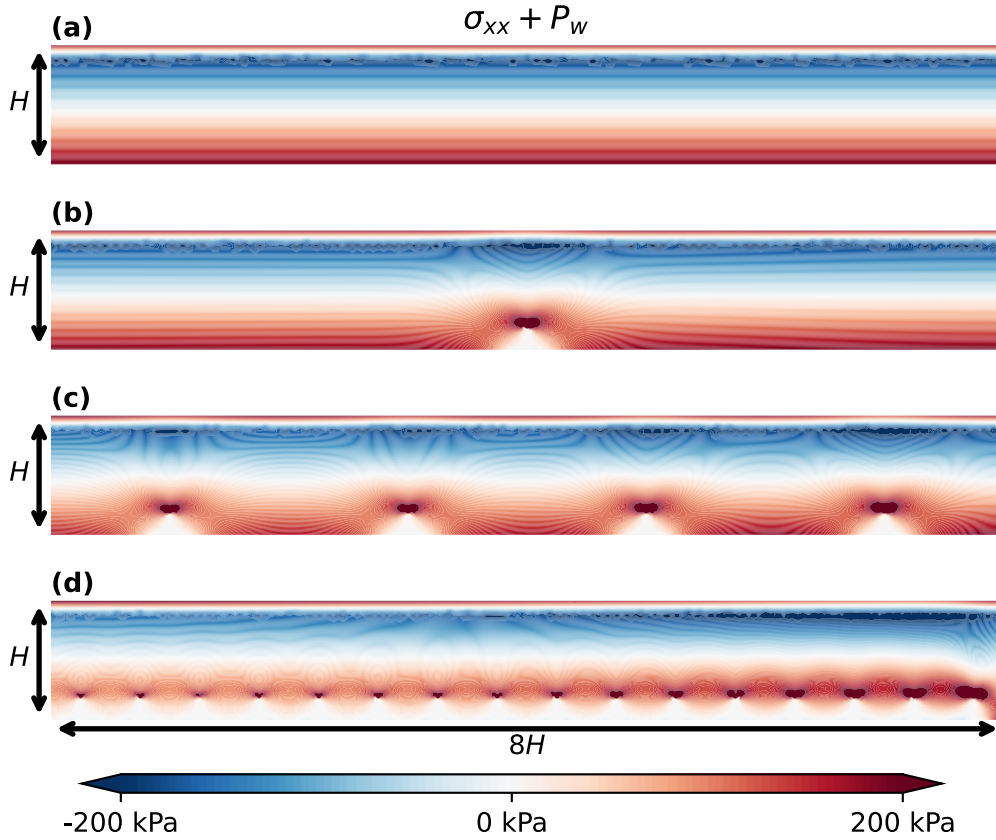


Figure R3. Suite of simulations analogous to those in Figure R1 except these simulation used Equation 3 as the traction boundary condition at the right edge of the domain.

$$(2) \quad \tau_{xx} = \frac{\rho_i g' H}{4}, \text{ at } x = x_c.$$

Dropping lateral and bottom drag, we can immediately integrate Equation (1) and apply the calving front boundary condition to find that the horizontal stress is given by:

$$(3) \quad \sigma_{xx}(x) = 2\tau_{xx}(x) - \rho_i g(s(x) - z) = \frac{\rho_i g' H(x)}{4} - \rho_i g(s(x) - z).$$

Here we have explicitly written the horizontal stress varies as a function of position, x , because Equation (3) holds locally for any thickness. Equation (3), however, neglects the bending stresses associated with the water pressure boundary condition considered previously. We can try to apply Equation (3) with deviatoric stress at the calving front given by Equation (2) as a boundary condition. Results for these simulations are shown in Figure R3. Here, bottom crevasses penetration is obviously contaminated by proximity to the traction boundary condition applied at the right edge of the domain. The stress at crevasses far upstream converge, however, to the same value as that obtained using the more physical water pressure boundary condition. We abandon this boundary condition as both unphysical and unlikely to generate closely spaced crevasses, but include it here because it is analogous to the boundary condition frequently assumed in previous studies.

Case 3: Crevasses near margins and/or upstream from pinning points in confined confined ice shelves

The third and final case we consider corresponds to the stress regime of an ice shelf that has some combination of lateral shear and pinning points that contribute to buttressing. Here it is useful to recognize that crevasse penetration in a lateral drag dominated ice shelf is closest to Nye's original treatment of crevasses in an ice sheet where deformation is dominated by vertical shear stresses.

In this case, the horizontal stress upstream of the calving front needs to account for lateral and bottom drag. We can integrate Equation (1) from some position x_0 to the calving front located at x_c to find:

$$(4) \quad 2H^c \tau_{xx}^c - 2H^0 \tau_{xx}^0 + \int_{x_0}^{x_c} \left[\frac{\partial(H\tau_{xy})}{\partial y} - \tau_b \right] dx^* = \frac{\rho_i g'}{2} (H^{c2} - H^{02}),$$

where x^* denotes a dummy variation of integration and we use the superscripts to denote whether quantities are evaluated at x_c or x_0 . Using the calving front boundary condition defined by Equation (3) we can rewrite Equation (4) in the form:

$$(5) \quad 2\tau_{xx}^0 = \frac{\rho_i g'}{2} H^0 + \frac{1}{H^0} \int_{x_0}^{x_c} \left[\frac{\partial(H\tau_{xy})}{\partial y} - \tau_b \right] dx^*.$$

The last term on the right of Equation (5) corresponds to the reduction in deviatoric stress associated with drag at the margins and pinning points that gives rise to buttressing. A situation analogous to Equation (5) was treated by Coffey and Lai (2025). We, however, diverge from the treatment in Coffey and Lai (2025) in one key respect: we assume that placement of crevasses can alter the displacement/velocity and, hence, the integrated drag. In other words, crevasses can alter both lateral shear stress and longitudinal extension stresses.

To make progress, we recall that both the lateral shear stress and bottom drag depend on geometry and choice of sliding law. For example, bottom drag is often written as a function of the horizontal speed u with the simplest expression a linear-drag law $\tau_b = \beta u$. Similarly, the lateral shear near the centerline of a confined ice tongue is often approximated in the form $\tau_{xy} \sim -y\eta u/W^2$ with η the viscosity of ice, y the transverse distance from the centerline, W the half-width and u representing the the centerline velocity (see e.g., Hindmarsh et al., 2012 for the viscous case). Motivated by this, we consider a general form for an integrated drag coefficient:

$$(6) \quad B(u^0) = -\frac{1}{H^0} \int_{x_0}^{x_c} \left[\frac{\partial(H\tau_{xy})}{\partial y} - \tau_b \right] dx^*,$$

which we have written explicitly as a function of the horizontal velocity at position x_0 , which we denote by u^0 . Using (6), we can now write Equation (5) in the form:

$$(7) \quad 2\tau_{xx}^0 = \frac{\rho_i g'}{2} H^0 - B(u^0).$$

or, equivalently,

$$(8) \quad \tau_{xx}^0 = \frac{\rho_i g'}{4} H^0 (1 - K_b(u^0)),$$

where

$$(9) \quad K_b = 4 \frac{B(u^0)}{\rho_i g' H^0},$$

denotes a “buttressing number”.

Recalling that in the shallow shelf approximation, velocity is approximately independent of depth, we write the horizontal velocity (or displacement for an elastic model) as $u^0 = U^0 + \delta u^0$ where U^0 corresponds to the velocity at x_0 before crevasses are emplaced and δu^0 corresponds to the additional displacement at x^0 after crevasses are added.

In general, crevasse opening will change the integrated drag (and possibly upstream crevasse depths). Nonetheless, we can write the deviatoric stress at position x_0 after crevasses are added as:

$$(10) \quad \tau_{xx}^0 = \frac{\rho_i g'}{4} H^0 (1 - K_b(U^0 + \delta u^0)).$$

with the full horizontal stress then given by:

$$(11) \quad \sigma_{xx}^0 = \frac{\rho_i g'}{4} H^0 (1 - K_b(U^0 + \delta u^0)) - \rho_i g(s - z).$$

If δu^0 is small compared to U^0 , we can Taylor expand Equation (11) to find:

$$(12) \quad \sigma_{xx}^0 = \frac{\rho_i g'}{4} H^0 \left[1 - K_b(U^0) - \frac{\partial K_b}{\partial u^0} \delta u^0 \right] - \rho_i g(s - z).$$

The gradient term on the right hand side of Equation (12) couples the solution for the horizontal stress with the velocity upstream and represents a restoring force: crevasse opening increases drag and, hence, the buttressing number. The problem with Equation (12) is, of course, that we do not know the gradient of integrated drag or buttressing.

Equation (12), however, approximates an upstream boundary condition. Conceptually, in the limit that buttressing and gradients in buttressing are small, the upstream boundary condition is equivalent to the ocean pressure and/or traction boundary conditions that we previously considered and is unlikely to give rise to closely spaced crevasses. By contrast, if gradients in buttressing are large, Equation (12) approximates a displacement controlled boundary condition, which is the situation that we considered in our initial manuscript. To summarize: the buttressing gradient controls whether introduction of crevasses results primarily in a change in the horizontal stress σ_{xx} (as assumed in the horizontal force balance), lateral shear stress τ_{xy} or some combination of both.

Of course, a full solution requires the more numerically expensive 3D solution for the full ice shelf, including lateral shear. This is an important situation that we postpone to a future study. We instead sketch out a simple scaling argument. Suppose we consider a linear elastic material with a lateral shear drag that scales roughly by $\tau_{xy} \sim -y\mu u/W^2$. (A similar calculation applies to a viscous ice shelf with shear modulus replaced by viscosity). We can roughly estimate the magnitude of the integrated drag as:

$$(13) \quad B(u^0) \sim \frac{\mu u_0}{W^2} (x_c - x_0),$$

As a crude scaling argument, let's denote the distance to the calving front by $\lambda = (x_c - x_0)$ and assume that the change in displacement scales like ice thickness H^0 (or crevasse depth).

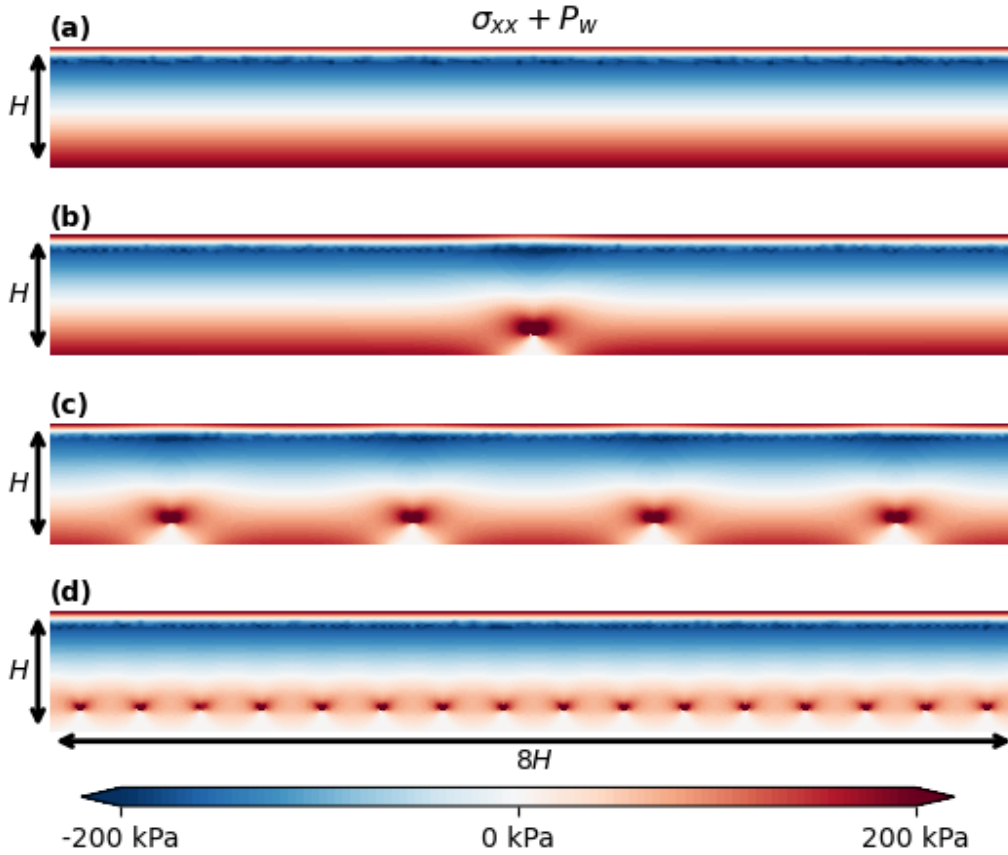


Figure R4. Suite of simulations analogous to those in Figure R1 except these simulation used Equation 12 as the traction boundary condition at the right edge of the domain with the drag gradient set to μ .

This implies that the integrated drag scales like $\lambda H^0/W^2$. Or, to explain this more conceptually, the gradient in buttressing is large close to the shear margins and decreases away from the shear margins.

Figure R4 shows an example of a simulation for the scaling $\frac{1}{\mu} \frac{\partial B(u^0)}{\partial u_0} \sim \delta u_0$. Here we finally find a situation where the stress field is uniform and closely spaced crevasses appear possible.

More generally, approximating buttressing gradients as $\frac{1}{\mu} \frac{\partial B(u^0)}{\partial u_0} \sim \alpha \delta u_0$, Figure R5 shows how the choice of the magnitude of the buttressing gradient modulates the transition from changes in horizontal stress to changes in lateral stress. Here we have set τ_{xx} using $\tau_{xx} = \frac{\rho_i g' H}{2S_0}$. Focusing on the depth of crevasses furthest from the calving front (i.e., the

stress based criterion), we see that so long as α is of the order of 10^{-3} or larger, the depth of closely spaced crevasses approximated the Nye model. As α decreases, there is a transition to a horizontal stress controlled regime that is analogous to the results we showed in Figure R2. Panel b in Figure R2 also shows the depth of bottom crevasses computed using the boundary Equation in 12 with $\alpha = 1$.

Conceptually, these results indicate that close to the shear margins and behind pinning points, closely spaced crevasses might be possible and, in this case, crevasse depths will follow the Nye model. However, as crevasse penetrate further from the margins, it seems more likely that isolated crevasses will become favored, leading to a transition to more widely spaced or isolated crevasses as crevasses penetrate into more central portions of the ice shelf as crevasses evolve into rifts. It is possible that these crevasses follow the depth of the horizontal force balance, but it seems unlikely crevasses would be closely spaced.

Although these simulations show that we can find boundary conditions that result depth of closely spaced crevasses that align with Nye, we concur with reviewer 2 that our simulation results remain ad hoc and beg for a more systematic treatment of boundary conditions in a fully three-dimensional model. However, we still believe that our simulations provide insight, caution and stimulus for future studies that are able to explore the stability and three-dimensional nature of crevasses.

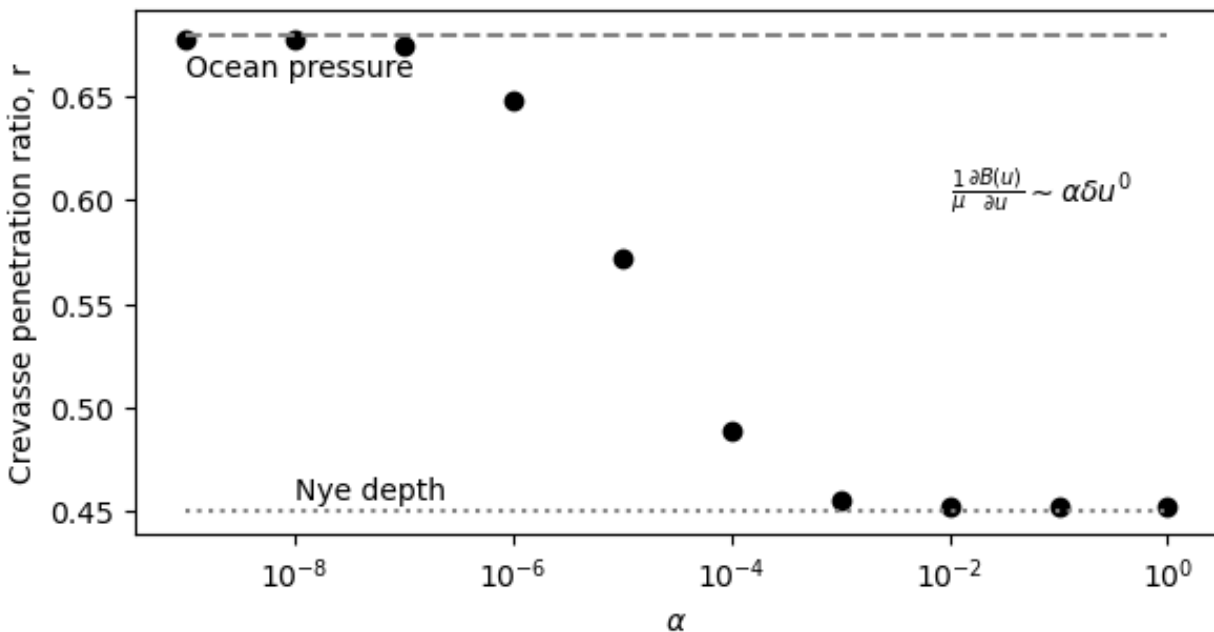


Figure R5. Bottom crevasse penetration depth for closely spaced crevassed using $S_0 = 2$ and varying the gradient in integrated drag. We see that for small buttressing gradient, we recover the crevasse depths from cases 1 and 2.

Reviewer #1

We thank reviewer #1 for their comments. In addition to our previous comment, the discussion above is largely designed to address the issue of boundary conditions, which we increasingly view as both nuanced and important.

Reviewer #2

This paper presents numerical simulation to show that Nye's crevasse depth penetration model is correct as long as crevasses are closely spaced. The paper is well written and the figures are of good qualities. I have nevertheless some concern about the modeling and the way the boundary conditions are applied.

Regarding the lateral boundary condition, Figure 1 is unclear about what the two horizontal arrows really represent. From the text, the reader understands that different boundary conditions (BC) have been used. The first type of BC is a Dirichlet type BC (displacement for the elastic simulations and velocity for the viscous ones). It is unclear how the displacement/velocity varies with depth along these boundaries and how one can be sure that it replicates the expected stress for a flat ice shelf. The second way of applying this BC is to fix the left boundary and to apply the sea water pressure. Here also, I am not sure how the equation line 74 can really be applied as the deviatoric stress τ_{xx} is unknown? Why not applying a more natural setup for which the left boundary is fixed (zero horizontal displacement) and the sea water pressure $\rho_w g (s - z)$ is applied as a normal stress to the right boundary of the domain? All this ad-hoc treatments of the lateral boundary conditions, whereas there is a simple way of applying the real condition, render the results from the simulations questionable.

Great question. As described above, we apply the water pressure condition and find that this is unlikely to result in closely spaced results. However, it does result in exactly the same results for isolated crevasses.

Still on the BC, I don't understand why the second term in Equation (5) is proportional to $(\rho_w - \rho_i)$ and not ρ_w only. Is it in link with the BC that is applied on the top surface which states that the normal stress is not zero, as it should be for a stress free surface? Again, why not applying on the top surface which is a stress free surface the natural BC $\sigma_{nn} = 0$ (or atmospheric pressure)? Again, for these two BC, this ad-hoc treatments of the BC render the results even more questionable.

This is a good question. The technical reason we apply the boundary condition in the form presented here is because we evaluate the traction boundary condition at the ice-ocean interface *before* the ice deforms. As a consequence, we need to evaluate the change in traction associated with deformation applied at the initial interface and this is related to buoyancy forces. The most convincing way to demonstrate that this results in a more accurate boundary condition is the convergence study noted in our initial manuscript.

Starting with an initially flat ice shelf, the shelf thins with uniform strain rate E_{xx} results in the evolution equation $\frac{dH}{dt} = -E_{xx}H$, with the analytic solution $H = H_0 \exp(-E_{xx}t)$ with H_0 the initial ice thickness. Because the ice shelf is in hydrostatic equilibrium the surface, s , and bottom, b , evolve according to $\frac{ds}{dt} = -\left(1 - \frac{\rho_i}{\rho_w}\right)E_{xx}H$ and; $\frac{db}{dt} = \frac{\rho_i}{\rho_w}E_{xx}H$. We examine the error in our numerical solution (with evolution computed using a forward Euler time step) and the analytic solutions with (1) the boundary condition used in our study and (2) the more traditional sea spring. This is shown in Figure R6. The top panel of Figure R6 shows that the

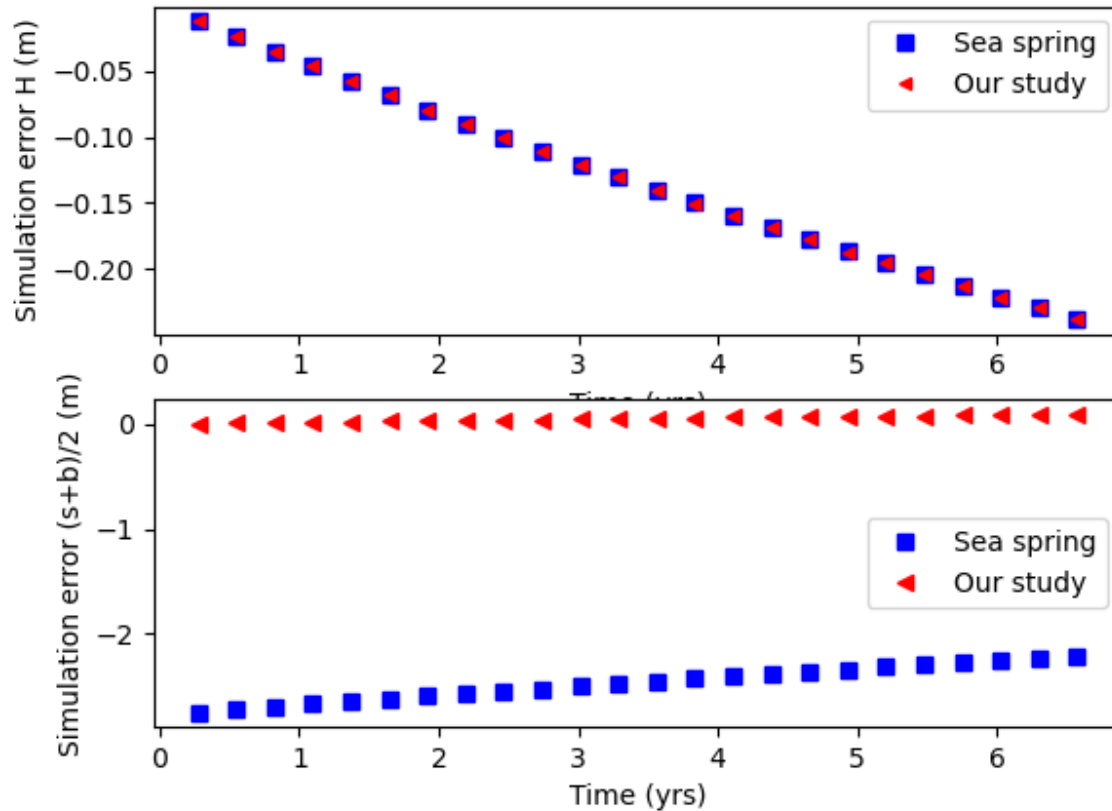


Figure R6. Error for sea spring and our stress boundary condition. Panel a shows the error as a function of time in ice thickness. Panel b shows the error as a function of time of the ice shelf midpoint.

error in ice thickness, H , for both boundary conditions are identical. This is expected because this is limited by the accuracy of the Euler time step.

The reason why we prefer our boundary condition is better illustrated in the bottom panel of Figure R4, which shows the error in the midpoint $(s + b)/2$ of the ice shelf. Here, it is clear that the “sea spring” boundary conditions result in a solution where the simulated ice shelf is displaced a few meters down relative to the analytic solution. Our preferred boundary conditions, by contrast, results in an error that is only a few centimeters. The magnitude of the difference between the two boundary conditions decreases as the time step size decreases so that these two boundary conditions converge for small time step size.

We do not show the error in stress because we don't time in our solutions and both boundary conditions yield identical stress states for all simulations we performed. We decided to use the boundary condition as it appears in our initial draft simply because it performed better in our convergence studies for a time-evolving ice shelf. For this study, we could, if the reviewer prefers, swap in the sea spring boundary condition, because it makes no difference to the stress state or any of our calculations of crevasse depths. We don't know why the sea spring approach is more popular in glaciology compared to the form that we use that is widely apply in fluid mechanics.

My third and last main comment concerns the conclusion drawn about damage-based approaches in large-scale models (lines 5-6 and 143-147). If I understand the authors' message correctly, the fact that their model shows that the stress in the intact part of ice is not affected by the presence of crevasses would disqualify the use of the damage-based approach in large-scale models. I do not think this conclusion can be drawn from the present study. The fact that the state of stress between crevasses is not affected by them does not mean that, at a larger scale, the overall ice flow is not affected by the crevasses. And this is precisely what a damage model would take into account, i.e. the presence of crevasses that would induce a change in the ice viscosity on a much larger scale than an individual crevasse. On the other hand, your modeling approach could be used to calibrate a damage model, knowing the fraction of crevasses relative to the total initial ice volume, and computing the resulting altered macroscopic viscosity.

This is a good point and we agree with the reviewers comment. We will remove this section, but try to keep a few sentences pointing out that we could potentially use this type of simulation to calibrate damage models. Based on our more nuanced results with different boundary conditions, we agree that this is even more complex than we had initially envisioned.

Minor remarks:

line 38-39: "to better understand compare predictions". understand and compare?

Fixed. Thanks!

line 97: two time "typically"

Fixed.

line 102: crevasses should penetrate?

Thanks for catching that. Fixed.

line 141-142: what would be the effect of turbulent flow in crevasses? Are you thinking in term of thermal exchange and crevasse growth by melt or in term of force balance?

Both. We added a short statement clarifying that we should treat the thermodynamics and it is possible that crevasses could freeze shut. However, we should also consider the dynamic interaction between the flow of water into crevasses. For bottom crevasses, closely spaced crevasses are very narrow and this would require a more in depth treatment of the flow of fluid in crevasses (potentially including surface tension and capillary forces) along with the potential for crevasses to freeze shut before they penetrate to their theoretical depth.

Comments from Donald Slater

Donald had some excellent comments that pointed out some sloppy word choices in our initial manuscript. Based on reviewer comments, we have decided to primarily use an energy method to compute crevasse penetration depths. This eliminates the need to consider water pressure and is a cleaner way of computing crevasse penetration depths for elastic materials. Donald also expressed interest in seeing bottom and surface crevasse depths independently. Because we have radically expanded our simulations, we decided to focus exclusively on bottom crevasses. This reduces the number of simulations we need to do and also simplifies the presentation because we need only show a single depth. Because the goal of this study is a short communication, we decided to postpone a presentation of both surface and bottom

crevasses to a future study where we could more fully explore stability and the question of whether surface crevasses are ever likely to form on top of bottom crevasses or if they are more likely to be offset.

Comments from Yao Lai

Yao also had some interesting questions. We have sought to address those questions in an earlier response, but also believe that our expanded discussion of boundary conditions above are relevant to those questions.