

We thank the reviewer for their helpful comments and suggestions. The reviewer's main point is that we should perform simulations using a traction condition in addition to the displacement controlled boundary condition we show. Fortunately, we have already done these simulations as part of our numerical hygiene and can show that there is no distinction between results from displacement controlled and traction controlled simulations for carefully curated traction boundary conditions. However, as we show below, identifying appropriate traction boundary conditions does have a subtlety because the boundary conditions also have to obey symmetry conditions that permit closely spaced crevasses to form in the first place. Demonstrating this requires a slightly more nuanced discussion of boundary conditions and matching conditions. Let us begin.

**Prequel: Attack of the phantom boundary conditions.**

One of the reviewer's claims is that the most appropriate boundary condition to create the stress field in an ice shelf is a traction boundary condition. We believe that the situation is more complex because ice shelves are three-dimensional and our simulations only apply to a small portion of an ice shelf far from lateral margins and the calving front. In reality, in addition to the calving front, ice shelves have inflow at the grounding line and lateral margins (where a no-slip condition is often required). Because the lateral margins (and pinning points) are the physical cause of buttressing, these conditions are important. In general, simulating the dynamics of a small portion of an ice shelf, requires that we match both normal tractions and normal velocities /displacements at the boundaries with the surrounding ice shelf body. In other words, the stresses in the interior region, that corresponds to our simulation, can affect the stresses in the exterior portion of the ice shelf. Here we only seek to simulate a small portion of the ice shelf and are not able to do a full matching. Hence, we need to limit ourselves to either velocity/displacement OR traction boundary conditions. We agree with the reviewer that, in the case of an unconfined ice shelf with no buttressing, the traction boundary condition seems more appropriate. When considering ice shelves with lateral margins and the potential for no-slip conditions along portions of the domain, the choice is less obvious to us. Nonetheless, we show next that applying a symmetry preserving traction boundary conditions results in identical results to the displacement condition we showed in the initial paper.

**A New Hope (for the HFB): Traction boundary conditions:**

The reviewer makes the compelling point that a traction boundary condition could result in different results for closely spaced crevasses. Fortunately, we have already done these calculations as part of our convergence studies and can confirm that implementing a horizontal traction boundary condition does not alter our conclusions (see our modified Figure 2a, in Figure R1). For these simulations, we dispensed with the symmetry arguments that simplified earlier calculations. Focusing primarily on simulations with the elastic rheology, we compute crevasse depth using an energy method (accounting for tractions

applied along boundaries). This allows us to democratically account for all crevasses simultaneously and is more numerically stable than attempting to extract stress or stress

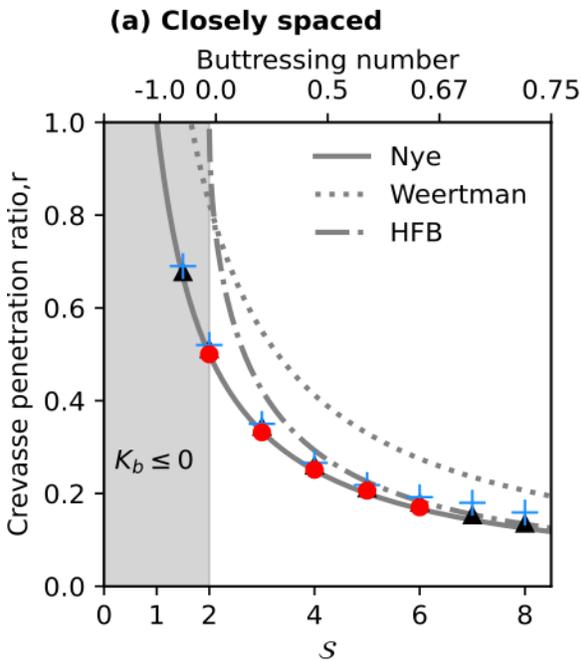


Figure R1. Modified version of Figure 2a. Here the red circles denote results for closely spaced crevassed obtained using a traction boundary condition instead of a displacement boundary condition. We see that simulations using the traction boundary condition closely match the Nye model and our initial set of displacement controlled simulation results.

domain and the final crevasse centered at the right edge of the domain). Suppose instead we consider a numerical domain with crevasses offset by half the crevasse spacing  $w/2$  so that we impose the traction boundary on a vertical column of un-crevassed ice? A comparison of the horizontal stress associated with these two sets of boundary conditions is shown in Figure R2. (Crevasses are spaced 50 m apart in all simulations, but they are difficult to see because they are closely spaced and narrow). Panel (a) shows results from our initial set of experiments with crevasses centered at the right and left edges of the domain. The horizontal stress varies vertically, but not horizontally, consistent with a field of crevasses of equal depth. By contrast, in the second experiment (panel b), crevasses are offset from the

intensity near the tips of multiple crevasses. These simulations, done as part of our numerical hygiene, were computed using coarser resolution than the results in the main manuscript, but with at least 10 crevasses in the domain. As illustrated in Figure R1 (revised Fig 2a), we see that the traction boundary condition yields nearly identical results as the displacement boundary condition (compare red circles to black triangles). Crucially, in this case at least, we see that displacement and traction conditions are consistent, which both supports earlier numerical results and hints at the possibility that we may be able to fully match displacement and traction boundary conditions for this case.

### Symmetry Strikes Back

At first glance, our simulations with the traction boundary condition appears to foreclose options for the horizontal force balance, but . . . no, there is another. There is a subtle, but important symmetry assumption in our simulations. The traction boundary condition in our numerical domain was applied to a crevassed vertical column of the ice shelf (i.e., our domain consists of evenly spaced crevasses with the first crevasse centered at the left edge of our

left and right edges of the domain. This results in *horizontal gradients* in the stress  $\sigma_{xx}$ . Here, it would seem, there is a disturbance in the force (balance). Crevasses near the right edge of the domain will penetrate deeper into the ice shelf than those closer to the left side of the domain and this, ultimately, results in unequal crevasse depths. This situation appears to be inconsistent with a field of closely spaced crevasses of uniform depth. Although the notation is different and we considered different parameter regimes, our results for crevasse depths appear to be

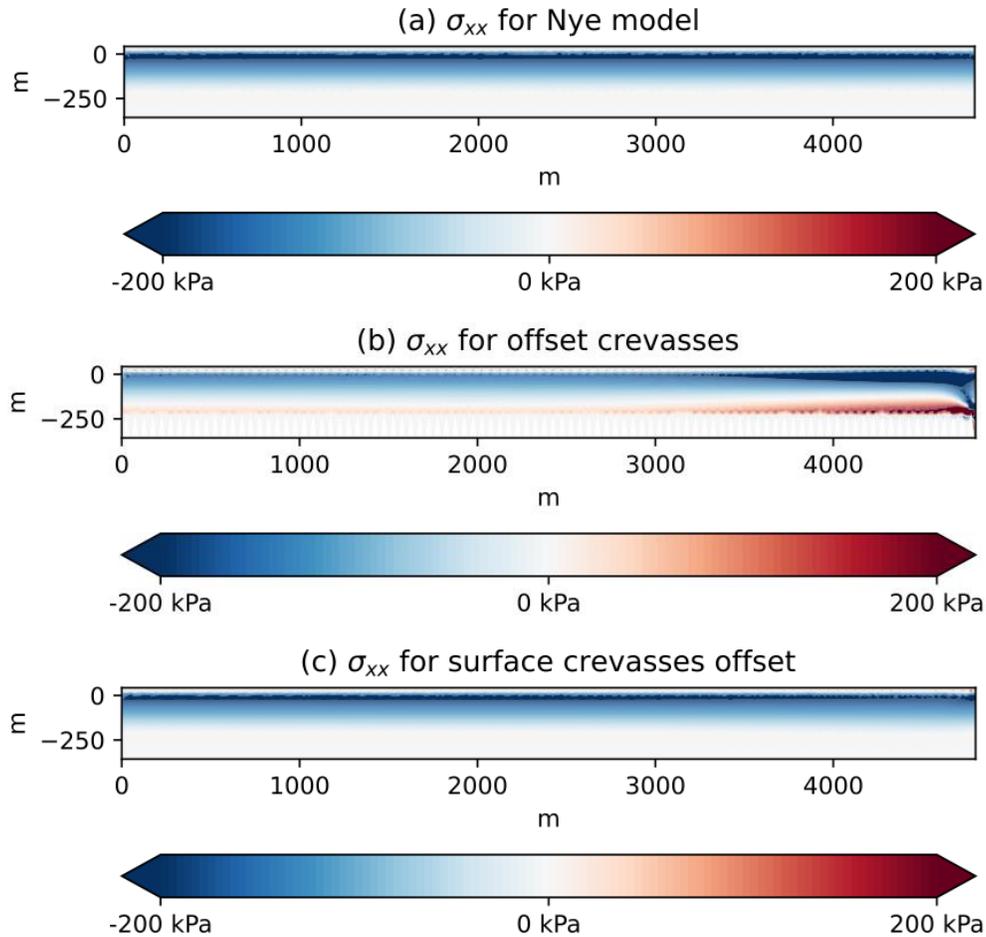


Figure R2. Stress field associated with three different configurations of crevasses relative to the right edge, where the traction boundary condition is applied. Panel (a) shows the default location where the ice thickness at the location of the traction boundary condition is reduced by surface and bottom crevasses. Panel (b) shows results for surface and bottom crevasses offset by half of the crevasse spacing from the calving front. Panel (c) shows surface and bottom crevasses offset by  $w/2$  from each other with bottom crevasses located at the right edge and surface crevasses offset by  $w/2$ .

broadly consistent with results from the elegant boundary element model of Zarrinderakht et al., (2024). As a final check, we also considered a field of crevasses where the surface and bottom crevasses are offset from each other (panel c) such that the bottom crevasses are

centered at the left and right edges, but surface crevasses are offset by  $w/2$ . Once again, in this situation we find Nye's model for crevasse depth is correct.

We are now in a position to understand why the original set of simulations we conducted are able to reproduce Nye's solution. Nye's (1955) argument, which was elaborated on by Weertman (1977) is that when crevasses are closely spaced, crevasse opening results in compression and bending moments on neighboring crevasses. Adding up the contributions of all of the crevasses causes the contributions to cancel by symmetry. Nye, irritatingly, viewed this as sufficiently obvious that he provided little mathematical signposting. Our initial simulations with displacement and traction boundary conditions were constructed with the symmetry assumed by Nye. By contrast, imposing a traction boundary condition to a column of intact ice (e.g., results in Fig R2 panel b) breaks the symmetry (instead of the ice) and, as a consequence, the existence of closely spaced crevasses is not possible. To preserve the symmetry we need to account for the stress exerted by the crevasses on the portion of ice *upstream* from the traction boundary. Doing this results in the traction boundary conditions we had already imposed and we find ourselves back at the Nye model. There is, however, another issue with the horizontal force balance. The assumptions made balancing forces in the vertical direction in the horizontal force balance are unclear.

### **The Return of the Vertical Force Balance**

It is possible that alternative boundary conditions or experimental design will still result in crevasse depths consistent with the horizontal force balance. However, at this point we are just guessing and we find little guidance for how (or where) the far-field traction should be imposed in the derivation of the horizontal force balance nor any discussion of the symmetry conditions necessary to yield an array of equal depth crevasses. We suspect that a more systematic treatment of the assumptions and boundary conditions necessary for the horizontal force balance would be insightful in better identifying those situations where the horizontal force balance is most likely to apply. This is especially true of the vertical stress that the horizontal force balance relies on.

For example, in the horizontal force balance, the horizontal stress in the region beneath surface crevasse and above bottom crevasses is written as (e.g., Equation 20 in Buck, 2023 with a slight change of notation):

$$(1) \quad \sigma_{xx} = R_{xx} - \rho_i g(s - z),$$

where  $s$  denotes the surface elevation of the ice shelf,  $\rho_i$  is the density of ice and  $g$  is the acceleration of gravity. Although not stated in Buck (2023), it seems that an implicit assumption of Equation (1) is that the vertical stress is given by:

$$(2) \quad \sigma_{zz} = -\rho_i g(s - z),$$

such that the stress difference  $\Delta\sigma = \sigma_{xx} - \sigma_{zz} = R_{xx}$ . The role of the far field boundary condition in formulating Equations (1) and (2) is unclear: Why are Equations (1) and (2) only

appropriate for far-field traction boundary conditions and not displacement conditions?  
How are the necessary symmetry conditions enforced?

Moreover, a key assumption underlying Equations (1) and (2) seems to be that the vertical stress remains unchanged after crevasses are emplaced and the only component of stress that changes is the deviatoric component. This is not an obvious assumption, especially because Equation (2), appears to be inconsistent with the traction boundary condition that normal stress vanishes at the atmosphere-ice interface (neglecting atmospheric pressure). If we, for example, assumed the vertical stress was given by the usual hydrostatic equilibrium relationship, we find a radically different crevasse depth associated with a horizontal force balance. To be clear, in our simulations,  $\sigma_{zz}$  is not in hydrostatic equilibrium but is altered beneath surface crevasses, although this effect is most pronounced in the region near the surface crevasse tips and for wide crevasses. Equation (2) seems like it must only be valid as an approximation for sufficiently narrow crevasses, but the remaining conditions when it applies remains shrouded in mystery (to us). We suspect that identifying the assumptions in the *vertical* force balance that leads to the horizontal force balance along with appropriate symmetry conditions needed for Equations (1) and (2) would allow us to better identify conditions when the horizontal force balance is appropriate that we can then verify with our numerical solutions. Our suspicion is that the horizontal force balance is correct under certain conditions, but we have yet to identify the assumptions and boundary conditions where it applies.

The simulations we have conducted so far all confirm Nye's intuition. However, there is one way that we have found we may be able to recover crevasse depths that are possibly consistent with the horizontal force balance. We have found that, if instead of computing the stability number  $S$  (or buttressing number) using the stresses (or displacements) imposed at the beginning of the simulation, if we recompute  $S$  using the average strain in the domain *after* crevasses open and the ice deforms, we do recover the crevasse depths associated with the horizontal force balance! This approach, however, is a recipe that only works in limited circumstances, requires very narrow crevasses and was only tested with an elastic rheology. In this case, the horizontal force balance is just the Nye model in disguise, but the relevance to crevasse penetration depths and the horizontal force balance is less unclear.

Our results so far, of course, do not say that the horizontal force balance is wrong, but that for the simulations we have conducted so far, Nye is still right.

*Disclosure: All writing, figures, puns and bad taste (cinematic and otherwise) in this response are solely the result of bad choices by the lead author. No AI, generative or otherwise, was used in the analysis, plotting or writing of this response.*