Authors Response

Review: Review Report

This manuscript presents a useful methodological contribution to multistatic meteor radar wind retrievals by reformulating Volume Velocity Processing in spherical coordinates. The topic is timely and relevant for Atmospheric Measurement Techniques, since multistatic meteor radar networks now cover domains of several hundred kilometers, where plane-geometry assumptions may introduce projection errors. The multi-year Nordic Meteor Radar Cluster observations and the comparison among plane-geometry VVP, spherical VVP, and 3DVAR+DIV provide a valuable basis for assessing geometric effects on retrieved MLT winds and higher-order kinematic quantities.

However, I think the manuscript needs substantial revision before publication. My main concern is not the usefulness of the spherical formulation, but the interpretation and positioning of the method. The discussion sometimes gives the impression that SVVP is a more reliable alternative to 3DVAR+DIV, especially for vertical winds. This is not sufficiently justified. SVVP is an important geometric improvement of VVP, but it remains a reduced-order, first-order representation of the wind field. By contrast, 3DVAR+DIV is theoretically a more general tomographic framework that can resolve spatially heterogeneous three-dimensional winds and include physical constraints. Therefore, the differences between SVVP and 3DVAR+DIV should not be interpreted simply as evidence that SVVP is superior. They may instead indicate limitations in the present 3DVAR+DIV implementation, including geometry, continuity constraints, regularization, and sparse-sampling treatment.

I recommend major revision. The paper has clear potential, but the conclusions should be more balanced, and the retrieval uncertainty, validation, and method comparison need to be strengthened.

General Reply: *We thank the reviewers for a constructive review of the submitted manuscript. The developed SVVP complements the more advanced 3DVAR+DIV retrieval method. The main goal was to develop a computationally less expensive retrieval method that uses a multistatic geometry and infers higher-order kinematic terms that are not accessible with monostatic retrievals. The new algorithm targets climatological studies and longer-time series analysis.*

We appreciate the review and will revise the manuscript according to the suggestions, clarifying some aspects pointed out by the reviewer. We have performed a reduced run to store the condition numbers of the SVD singular values. We added some residual plots to demonstrate how planar and spherical geometry differ, and also found cases where the first-order approximation is no longer sufficient to underpin the importance of the 3DVAR+DIV method. Below, we provide point-by-point replies in the attached PDF. The revised manuscript is prepared with Latexdiff track changes.

Comments:

Review: Major Comments

Reviewer Comment: The manuscript should more clearly distinguish between the improvement of SVVP relative to plane-geometry VVP, the broader theoretical capability of 3DVAR+DIV, and the limitations of the current 3DVAR+DIV implementation. SVVP is a geometrically improved version of VVP, but it does not remove the reduced-order nature of VVP, which still represents the wind field using mean winds and first-order spatial gradients within a finite volume. In contrast, 3DVAR+DIV is designed to retrieve spatially varying three-dimensional wind fields with physical constraints. The manuscript should therefore avoid implying that SVVP supersedes 3DVAR+DIV. The larger vertical winds from 3DVAR+DIV should be discussed in terms of spherical/ellipsoidal geometry, continuity assumptions, Tikhonov regularization, and sparse meteor sampling. A better framing is that SVVP provides a geometrically self-consistent benchmark for reduced-order retrievals, while an improved 3DVAR+DIV remains the more general framework for future multistatic meteor radar tomography.

Response: *The developed SVVP provides a computationally less costly alternative to infer mean winds, including seasonal vertical wind variability and higher-order kinematic terms such as horizontal divergence, relative vorticity, stretching, and shearing deformation. The algorithm is supposed to complement the much more advanced 3DVAR+DIV retrieval, which can deal with arbitrary wind fields. We will explicitly refer to this in the revision. It was not our intention to generate the impression that the new analysis is better than the 3DVAR+DIV. This will be made clearer in the revised draft.*

Reviewer Comment: The observational comparison shows that SVVP, plane VVP, and 3DVAR+DIV produce different results, especially for vertical winds and higher-order kinematic quantities. However, this does not fully isolate the role of spherical geometry. I recommend adding a synthetic or closed-loop retrieval experiment. The authors could prescribe known wind fields, generate synthetic line-of-sight velocities using the actual NORDIC radar geometry and meteor sampling locations, and then retrieve the winds using both plane VVP and SVVP. Test cases could include uniform horizontal wind with zero vertical wind, known divergence/vorticity, prescribed vertical velocity, and simple gravity-wave-like perturbations. Such experiments would directly show whether SVVP reduces projection errors and whether plane VVP produces spurious vertical wind or divergence.

Response: *The comparison between SVVP and the planar geometry targets to demonstrate only the effect of the geometry. Both algorithms are embedded in the same script with identical data curation, non-linear error propagation, all quality control filters, and a singular value decomposition solver. Hence, the remaining differences are solely attributable to the geometric implementation of the fits. This will not change by applying it to synthetic data. The 3DVAR+DIV retrieval is designed to resolve arbitrary wind fields and small-scale gravity wave fluctuations. However, the implementation of the continuity equations resulting in vertical winds depends on a lower or upper boundary for vertical integration. This limits the applicability for* *seasonal comparisons, as the underlying assumption is already biased. Synthetic data was already used in Stober et al. (2022). The algorithm is reliable for what is presented in the above-mentioned manuscript.*

Reviewer Comment: The paper retrieves not only mean winds but also horizontal divergence, relative vorticity, stretching deformation, and shearing deformation. These quantities are sensitive to sampling geometry, station distribution, and the conditioning of the least-squares problem. The manuscript mainly presents retrieved fields and correlations, but it does not sufficiently show whether all fitted parameters are well constrained. The authors should include diagnostics such as condition numbers, singular values, parameter covariance, and correlations among retrieved coefficients. This is particularly important for vertical wind and gradient terms. The contribution of each radar station should also be quantified, since uneven spatial overlap and different meteor counts may cause the retrieval to be dominated by certain stations or viewing directions.

Response: *The reviewer mentions an important point. In the supplementary material, we present 49 Figures showing all temporal resolutions and radii and the corresponding correlations to address this aspect. The different temporal resolutions correspond to different data sparsities. As this point was brought up by other reviewers, we also performed a leave-one-station out experiment (see Figure 1, added to supplementary material as Figure S51).*

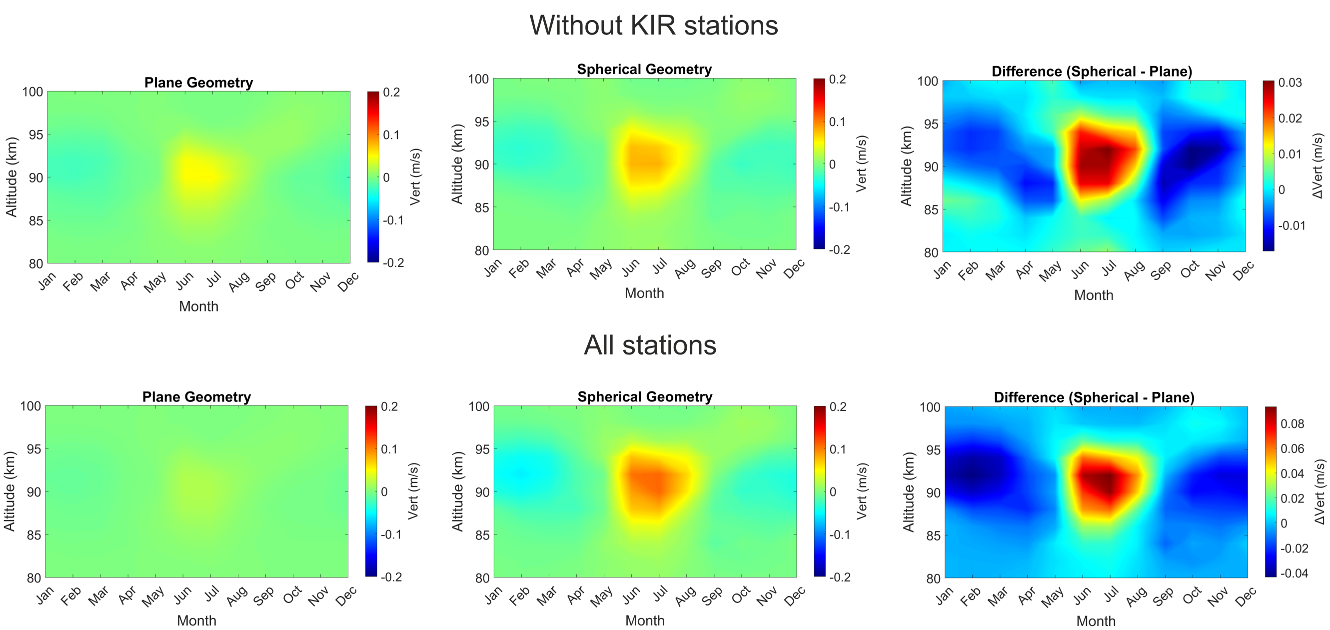

Figure 1. Monthly mean vertical wind components retrieved using the Volume Velocity Processing (VVP), showing the spherical geometry, plane geometry, and their corresponding differences for the experiment with all stations (bottom row) and without the KIR station (top row).

We also performed an additional run and stored condition numbers and singular value thresholds of 10^{-4} *and* 10^{-6}*. This had basically no impact on the significant digits given by the non-linear error propagation. The spatial sampling appears to be less critical as long as at least three systems form a triangle shape. Tromso and Alta have higher data rates than Sodankylä or Kiruna, but due to the long distance, a few measurements there are sufficient to mitigate most sampling biases in the mean quantities. A binary system of meteor radars suffers from sampling bias much more than tristatic or higher-order networks. The results are robust within the total statistics. When the problem becomes too sparse, our solver tends to produce zero vertical winds. This is the case at the upper and lower edges of the meteor layer. We added in the revision a meteor count density plot to illustrate the distribution of the number of detections. We also performed a reduced run and added the condition number to the stored data* $k(A) = \frac{\max \sigma_A}{\min \sigma_A}$*. A potential ill-conditioning is cured due to the spatio-temporal Laplace filter, which estimates a certain smoothness between neighbouring grid cells in altitude and time. For good least square fits (well-conditioned), the impact is negligible. The condition number ranges from log10(k(A)) 2.4 to 3.8, indicating a sufficient numerical stability for double precision. The minimum number of meteors for a fit is 15, but we only use data with more than 50 meteors.*

Reviewer Comment: The manuscript subtracts a long-term or seasonal vertical-wind bias based on the assumption that the long-term mean vertical wind should vanish. This correction is critical because its magnitude is comparable to the retrieved vertical winds. The authors should describe how the bias is estimated in much greater detail. It should be clarified whether the correction is derived separately by altitude, radius, temporal resolution, season, station combination, or from the full dataset. The authors should also test whether the seasonal upwelling/downwelling pattern remains under different bias-correction strategies. The uncertainty associated with this correction should be propagated into the final vertical-wind climatology.

Response: *We revised the manuscript and added a detailed description of the bias removal. The bias is estimated for each altitude using the full 5-years or the climatology. Both methods result in almost identical bias values for the Doppler-based vertical winds. The largest biases are observed at the edges of the meteor layer, and there is a scaling behaviour with the magnitude of the Doppler-derived vertical winds. The order of the bias ranges from a few cm/s to about 10 cm/s. For very large radii and long averaging, the assumption of the linear wind field approximation seems to be no longer valid and results in even larger biases.*

Reviewer Comment: The comparison between SVVP and 3DVAR+DIV needs a more careful interpretation. Since 3DVAR+DIV is a more general framework, the comparison should not simply emphasize that SVVP gives smaller or more physically plausible vertical winds. The authors should analyze why the two methods differ. First, the current 3DVAR+DIV implementation may not include the same spherical geometric treatment as SVVP. Second, vertical wind in 3DVAR+DIV is constrained differently from the Doppler-derived vertical wind in SVVP. Third, the influence of Tikhonov regularization should be quantified more systematically, rather than illustrated mainly with selected cases. This would help clarify whether the discrepancy reflects method hierarchy, geometry, physical assumptions, sampling sparsity, or regularization choices.

Response: *This is an important comment, and we will revise the discussion accordingly. The first two points will be discussed as recommended. We already expanded the discussion about the Tikhonov regularization and its impact on the results. We found a rather wide plateau ranging from 0.4 to about 5 for the regularization strength, which led to very similar wind field solutions. Our best choice, so far, has been optimized with the Hunga Tonga gravity wave. The Lagrange multiplier of 150 was used to illustrate that, in principle, the smoothness constraint could enforce solutions very similar to the VVP or SVVP. However, this results in the loss of small-scale features of the arbitrary wind fields and, thus, minimizes the benefits of the 3DVAR+DIV retrieval. Furthermore, the divergence-derived vertical wind starts to be systematically deflated.*

Reviewer Comment: The manuscript discusses the relation between horizontal divergence and vertical velocity using a simplified continuity equation. However, the MLT is strongly density-stratified, and an incompressible continuity equation may not be appropriate without qualification. An anelastic form involving density stratification may be more relevant. Even if the main SVVP climatology is based on Doppler-derived vertical velocity, divergence, and 3DVAR+DIV results are still used to support the interpretation of vertical motion. The authors should clarify how the continuity constraint is used in 3DVAR+DIV, whether density stratification is included, and how this affects the comparison with SVVP vertical winds.

Response: *We thank the reviewer for bringing this up. This is indeed a major difference in the vertical wind retrievals between SVVP (Doppler) and 3DVAR+DIV. We have estimated how compressible small-scale gravity waves could become using linear theory (Stober et al., 2022). We have estimated isentropic Mach numbers and found often values above Mach 0.3. Flows exceeding this threshold are considered compressible (the transition is not a step function). However, Mach numbers beyond Mach 0.5 and occasionally Mach 0.7 have been observed (Qiao et al., 2025). The stratification comes on top of that. The vertical contribution of $u\nabla\rho$ with $\rho = \rho_0 \exp -z/H$ is proportional to $-w/H$. Considering the magnitude difference between the scale height H and a typical velocity w, it contributes, but is not yet a leading-order effect compared to the horizontal compressibility effects. We plan to revise the retrievals to account for all compressibility effects. The horizontal gradients in the density also depend on temperature, which is not derivable from the data with the current systems without further assumptions. Compressibility effects are the main reason why we did not include the divergence-based wind in this manuscript, which represents more of an upper limit for vertical velocities.*

Reviewer Comment: The SVVP method still assumes that the wind field within the analysis volume can be represented by a first-order expansion. This may be restrictive over 200–400 km domains, where gravity waves, tides, planetary waves, and polar-region dynamics can produce nonlinear spatial structures. The manuscript should better quantify the representation error associated with this first-order approximation. For example, the authors could show line-of-sight residuals as a function of radius, altitude, local time, or season. Synthetic tests using sinusoidal or gravity-wave-like wind fields would also help identify the spatial scales over which SVVP remains valid. This is important because some SVVP–VVP differences may arise from model truncation errors, not only from geometric effects.

Response: *Thanks for the suggestion with the residuals. We have looked at thousands of residual plots to check proper fitting. Below you find some examples. We show some representative examples and one case where the first-order approximation starts to break down. We have added a few examples below. We did not find a case that shows anomalous distributions of the residuals indicating a misfit. Concerning the differences between VVP and SVVP, this will have no impact as both analysis algorithms have the same data curation, including the circles of influence.*

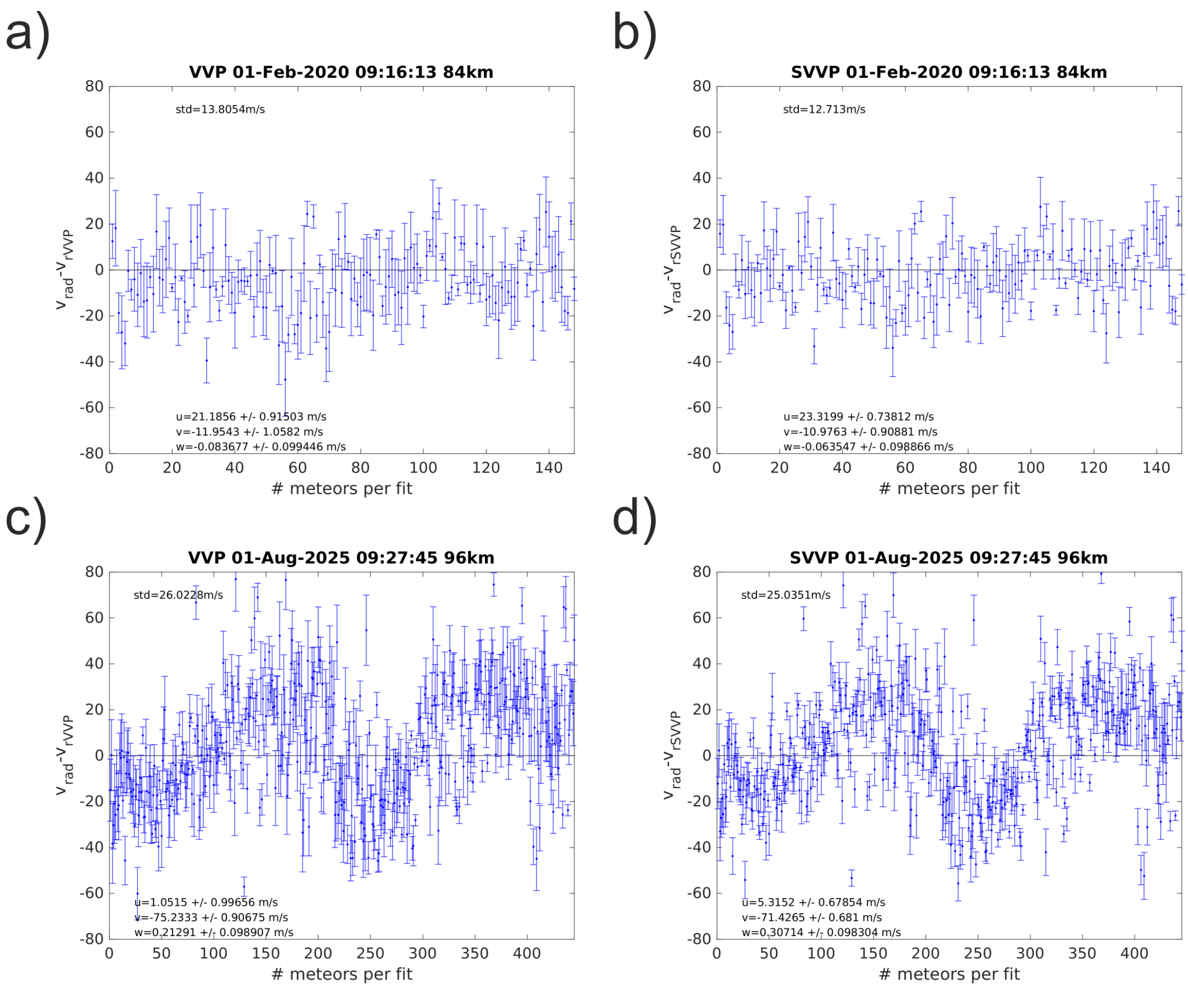

Figure 2. Fitting residuals for planar and spherical geometries for individual fits. The x-axis shows the number of meteor detections for this time-altitude bin. This is equivalent to time, but does not represent constant time steps. Panels a) and b) compare planar and spherical geometry for a typical scenario. Panels c) and d) highlight a case where the first-order approximation fails, and the residuals indicate an almost sinusoidal oscillation around the zero line.

We added Figure 2 to the revised manuscript to showcase some examples outlining the mentioned effects. Panels a) and b) show a more typical example of the residuals obtained with the VVP and SVVP for the same time-altitude bin. Panels c) and d) illustrate a case where the assumption of the first-order approximation is no longer valid, in the sense that the residuals show an oscillation. This oscillation is due to the long averaging and is indicative of a transient event moving through the domain (e.g., GWs).

Reviewer Comment: The manuscript concludes that reliable vertical winds are obtained for 15–30 min temporal resolution and a 200–250 km spatial radius. This is plausible, but the selection criterion is not sufficiently quantitative. The authors should state whether the optimal range is based on meteor counts, radar overlap, retrieval uncertainty, condition number, residual error, comparison with 3DVAR+DIV, or physical plausibility. These criteria are not equivalent. A larger radius may improve sampling but worsen the first-order approximation, while a smaller radius may reduce representation error but degrade conditioning. A summary table or figure showing these metrics as a function of radius and temporal resolution would make the conclusion more convincing.

Response: *We have added Figure 13, presenting measurement response, and Figure 14, presenting daily meteor count (30-minute resolution) to illustrate optimal coverage for the selected reference point in the revised manuscript. These plots show the ring coverage (radius 200:50:400), justifying the circle of influence. We have added similar meteor count density Figures for both 15-minute and 60-minute intervals in the supplementary material as Figure S50.*

Reviewer Comment: Because the paper focuses on geometric implementation, the definition of the vertical direction must be precise. The manuscript should state whether the retrieved vertical wind is relative to the geocentric radial direction, the geodetic normal, the local ENU up direction at each meteor location, or the reference point. Over a 200–400 km domain, local vertical directions are not parallel, and small projection differences may matter for cm s^1 vertical winds. The authors should explain how line-of-sight velocities measured in local radar coordinates are transformed into the common retrieval coordinate system, and whether residual differences in local vertical directions can project horizontal wind into the retrieved vertical component.

Response: *We will revise the definition of the vertical coordinate in the manuscript to explicitly define the directions. The vertical winds are representative of the volume segment spanned around the reference point following a constant altitude surface provided by the WGS84 ellipsoid. The value is given in ENU for the reference coordinate. Each meteor enters the retrieval corrected for its local ENU projection. This was already implemented in Zeng et al. (2024). We added that the volume quantities are now also represented with the spherical volume correction relative to the reference point. This also represents a limitation for very large domains, as the approximations start to become imprecise, resulting in an inflation of the vertical velocity. Considering that the volume quantities are derived using the mean WGS84 parameters of the reference point, when* *the domain becomes too large, they are again affected by the projection residuals arising between the Earth ellipsoid shape and a sphere. Generalized elliptical coordinates might be more universal, but it's not clear whether it would be desirable to describe very large domains with just means and 4 kinematic terms. The internal variability might be too large, and the linear approximation is no longer holding.*

References

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