Responses to RC2

Neng Lu

Abstract, free surface numerical fluctuation ("drunken sailor instability") is also characteristic for true Lagrangian free surface treatment (Kaus et al., 2010).

Response: The marker fluctuations in the tractional sticky air method are primarily caused when using the PIC methods. This issue can be addressed by employing the level set method or the ALEIB method we propose here. All free surface simulation methods, including ALE and ALEIB, can experience the "drunken sailor" issue when using relatively large time steps, which needs FSSA to fix. More details are provided in the introduction section, and additional context is added in the abstract.

Line 45: Corrected to 10^{22}-10^{24}.

Line 105: Add the water-load case into the text. "The density of this layer is set close to zero for ensuring it exerts no pressure on the actual free surface (the interface between the air and rock), or it is set to 1000 kg/m^3 to approximate a water-loaded free surface (Gerya and Yuen, 2003)."

Line 125: Yes, we consider this; that's why we perform the resampling process using algebraic techniques. Generally, the new coordinates of the topography are (x2,y2), where x2 = x0+Vx*dt, y2 = Vy*dt. We then interpolate the new topography (x2,y2) onto the fixed x0 to obtain (x1,y1), where x0=x1. This strategy is similar to ALE; for more details, please refer to Thieulot (2011).

Line 235, Please specify how η_0 is defined (from the dislocation creep parameters? Empirically?).

Response: The reference viscosity is defined based on the Rayleigh number, approximately 1e6, as outlined in Crameri et al. (2017). The temperature-based Rayleigh number (Ra) can be expressed in terms of density (q), gravitational acceleration (g), temperature scale (ΔT), mantle depth (D), thermal diffusivity (κ), and reference viscosity (η_0)as: $Ra = \frac{\rho g \alpha \Delta T D^3}{\eta_0 \kappa}$. As we are using different values for temperature scale and depth from their paper, the reference viscosity used here is 10^{21} (theirs is 10^{23}).

Line 265: We added this improved strategy into the context, the volume of fluid (VOF) method as suggested. We cited the related thesis by Timothy Stephen Gray (Gray, 2025).

Line 285,the poor performance of the Eulerian MIC scheme is somewhat surprising and could be related to the inaccuracy of the applied marker advection scheme (4th order Runge-Kutta, ontinuity-based advection scheme should typically work better) and/or to the way or recovering the free surface position from markers (Eulearian material type isosurface interpolated from markers should typically work better, see above). Is this specific to the used FEM approach? Does a standard staggered Eulearian grid MIC produce similarly poor results?

Response: This issue is specifically caused by the FEM approach used in Underworld 2. In Underworld, material properties are assigned by applying the values of the closest particles to the Gauss points (9 points in each element in 2D) and then integrating these values to the mesh nodes. In Underworld 3, we address this issue by using the KD-tree method to assign values based on a distance-weighted average from the particles. And we believe that a standard staggered Eulerian grid MIC provides better results, though it is not as effective as ALE or ALE-IB unless additional interface tracing improvement strategies are used.

Line 315. "Our ALE-IB scheme results are more comparable to the free-surface case in the Eulerian scheme reported in Crameri et al. (2017), where a shape-function averaging method was employed in their modelling on all the uppermost rock tracers and the lowermost air tracers. This approach, combined with their sticky air method, yields more accurate surface representations." Would be good to show their results for comparison on your plots. Will your Eulerian results become more accurate with the use of a similar shape-function averaging method as Crameri et al. (2017) used?

Response: Thank you for your suggestion. Unfortunately, we cannot include their results directly in our plots due to slight modifications in parameter settings compared to their models. However, we have included a citation to their Figure 4 for reference. We agree that adopting a shape-function averaging method, as used by Crameri et al. (2017), could enhance accuracy. However, implementing this in Underworld 2 presents certain challenges.

Line 330, Does Underwold have these higher order elements? If so, presenting some additional tests would be very useful.

Response: In Underworld 2, the primary and secondary element types are offered as pairs (e.g., Q1/dQ0, Q2/dPc1, or Q2/dQ1). In Underworld 3, we provide a wider range of element type options. We have included a test case of topography relaxation models with different element types in the appendix.

Line 350. Advantages over the Eulerian scheme are only demonstrated for the specific scheme explored in Underworld and explored in this study. It performs notably worse than the Eulerian staggered grid MIC scheme of Cramery et. (2017) (see comments to line 315 above).

Response: We acknowledge that the advantages of the ALE-IB scheme, as presented in our study, are demonstrated specifically within the context of the Underworld framework (FEM-PIC). While our results show improvements over certain Eulerian schemes in FEM, we recognize that the Eulerian staggered grid MIC scheme using STAGYY by Crameri et al. (2017) may outperform in specific scenarios. We appreciate the opportunity to clarify these distinctions and will ensure that the limitations and specific contexts of our findings are clearly articulated.

References

Crameri, F., Lithgow-Bertelloni, C., and Tackley, P. J. (2017). The dynamical control of subduction parameters on surface topography. *Geochemistry, Geophysics, Geosystems*, 18(4):1661–1687.

Gerya, T. V. and Yuen, D. A. (2003). Rayleigh–taylor instabilities from hydration and melting propel 'cold plumes' at subduction zones. *Earth and Planetary Science Letters*, 212(1-2):47–62.

Gray, T. S. (2025). *Free surface methods applied to global scale numerical geodynamic models*. PhD thesis, ETH Zurich.

Thieulot, C. (2011). Fantom: Two-and three-dimensional numerical modelling of creeping flows for the solution of geological problems. *Physics of the Earth and Planetary Interiors*, 188(1-2):47–68.