Revision of the manuscript egusphere-2025-6282

REVIEWER #1

We thank the Reviewer for the positive comments on the submitted manuscript. Following the concerns expressed by them and Reviewer #2, we modified the manuscript to take their comments and criticism into account.
Minor amendments to the text have been added as well, according to the suggestions of both Reviewers. All the changes are highlighted in the submitted track-changes version of the manuscript. Hereafter, we reply to the suggestions and recommendations expressed by Reviewer #1. For the sake of clarity, the comments are reported in **bold**, and our replies are reported in *italics*. Additionally, we reported the added or changed sentences in normal font after the reply.

We believe the manuscript has substantially improved, and we look forward to receiving positive feedback from this Reviewer.

Best regards,

Giulio Calvani and Paolo Perona

Is the approach similar to that used in GAMLSS (Villarini et al., 2009) when time is adopted as an explanatory variable, or, for example, to the method of Šraj et al. (2016)? Please consider adding a brief comparison among these approaches. I also suggest commenting on the results of previous works employing similar methodologies for nonstationary GEV estimation, especially concerning the range of variability in the estimated parameters.

We thank the Reviewer for pointing out these interesting works and the references therein. The work of Villarini et al. (2009) was already cited in the text regarding the change in return period. Based on the suggestion and the analysis in Šraj et al. (2016), we briefly extended the introduction on the estimation of GEV parameters. Now, some of the available models for GEV parameter estimation are mentioned in the text (e.g., MLE, GAMLSS, DE-MC) as well as the comparative analysis made in Šraj et al. (2016). However, we must point out that the proposed framework does not aim at GEV estimation, as stated in the manuscript; rather, it provides guidance on the admissible magnitude of trends (see also reply to Reviewer #2).
The added text reads (lines 126-133):

For the sake of clarity, the GEV parameter estimation and their time-dependence (i.e., non-stationary trends in $\xi(t)$, $\mu(t)$, and $\sigma(t)$) go beyond the scope of this work. Herein, we assume that they are known based on the plethora of models available in the literature, usually relying on a pre-assumed definition of the non-stationary trend (Šraj et al., 2016). In this regard, one may consider the (Generalized) Maximum Likelihood Estimation, (G)MLE (e.g., Katz et al., 2002; Obeysekera and Salas, 2014; El Adlouni et al., 2007), the Differential Evolution Markov Chain approach based on the Bayesian inference through MonteCarlo simulations, DE-MC (e.g., Cheng et al., 2014), or the GAMLSS (Generalized Additive Models for Location, Scale and Shape) tool proposed by Villarini et al. (2009).

Line 16. I assume that the process is independent in time, or, if temporal persistence is present, that the authors are referring to the marginal distribution of the process. Since non-stationarity and time persistence are distinct properties, this point should be briefly explained or at least mentioned.

According to the comment, we recognized the wrong use of time persistence as equivalent to time stationarity. Indeed, the meaning was for the process statistics (particularly the probability distribution function, p_X) to be independent in time (i.e., $\partial p_X/\partial t=0$). Therefore, we changed the text, and the term "persistence" has been turned into "steadiness", as the null time-derivative suggests.
The changed text now reads (lines 19-22):

In this regard, the term *stationarity* (also known as *strict* or *strong stationarity*) refers to the over-time steadiness of the probability distribution function (pdf) of the process (Katz, 2013). Mathematically speaking, we can write $p_X(X, t) = p_X(X, t+c)$, with $p_X(X, t)$ the pdf of the process at time t, and c a constant. Accordingly, the steadiness condition can be written as $\partial p_X(X, t)/\partial t = 0$.

Line 52. Additional relevant studies addressing nonstationary frameworks could be cited, such as Read and Vogel (2015) and Vogel et al. (2017).

The suggested references have been added to the text in line 64.

Lines 56-58. I suggest explicitly stating the definition of the return period adopted here under non-stationarity (currently given only at lines 83–84). Indeed, only under the i.i.d. assumption do different definitions coincide with the formula in Eq. (1). At line 63, the authors state that the return period corresponds to the first-order moment of the distribution of extreme events; it would be clearer to state from the beginning that the return period is the mean of the inter-arrival time distribution.

According to the comment, the text has been rephrased to put forward the definition of return period. In this regard, the whole paragraph has been amended for consistency.

The changed text now reads (lines 42-46):

The traditional/historical procedure to estimate the extreme-event magnitude and the average occurrence frequency is based on the return period concept, which is defined as the average intertime between extreme events greater (lower) than, or equal to, a specific magnitude (threshold). Such a definition, usually implied in the design of engineering applications, is inherently built on the assumption of independent and identically distributed (iid) events above a specific value, x (i.e., the *threshold*).

Line 66. The infinite summation in Eq. (2) arises because the return period is the expected value of a probability mass function. In practice, this issue is often addressed by using its empirical counterpart—the sample mean—whose computational cost and estimation uncertainty depend on sample length. Note also that, in some cases, the sum is limited to a finite bound due to non-stationarity. This is closely related to the temporal evolution of the GEV parameters, which is examined in this work.

The definition of return period as the expected value (first moment) of the pdf is clarified in the text (see previous reply). According to the comment, the simplified approach to the calculation of Eq. (2) as expressed by the Reviewer has been added to the text, with a brief discussion on some potential issues: for instance, the use of the sample mean strongly depends on the sample length (as highlighted in the comment), and relies on the availability of (past) data. Therefore, the method is more useful for calculating how the return period changed in the past than for forecasting future changes (as no data are available). This is clearly analyzed in the reference suggested by the Reviewer, which has been added to the text (Šraj et al., 2016).
The added text reads (lines 80-83):

Simplified approaches to Eq. (2) consider the sample mean of the probability distribution function, but the estimation error strongly depends on the sample length, and, as a matter of fact, the method can be satisfactorily applied to determine the non-stationary return period of past time-series (e.g., Šraj et al., 2016).

Lines 107-112. In Eq. (1), $\Delta\tau$ corresponds to one year for annual maxima or to the sampling interval in a POT framework (the inverse of the average number of events per year). In Eq. (5), the authors assume that the process is stationary within $\Delta\tau$, i.e., within one year in the case of annual maxima or within a generally shorter period in POT applications. Assuming for simplicity that we are working with annual maxima, the GEV parameters are estimated across multiple years (as in the numerical example). Thus, t is much larger than $\Delta\tau$, within which the process is assumed stationary. Consequently, one must assess whether the process remains weakly nonstationary within time t so that Eq. (5) can be used instead of Eq. (2). Is this interpretation correct?

Correct. The sampling interval $\Delta\tau$ represents the duration for which the process is supposed to be stationary, and for which the GEV parameters are estimated according to the known trends in the process variables. In the shown results, it is always taken equal to 1 year, thus resembling the common Annual Maxima or Peak Over Threshold return period analyses. Accordingly, in the case of the decreasing return period, the future timeframe t cannot lead to a return period lower than $\Delta\tau$, and that is the reason behind the condition $t \leq \frac{T_0 - \Delta\tau}{a}$ derived from Eq.(10).

Eq. (10). The term $\Delta\tau$ appears to be missing from the expression. The mathematical reason is not clear. Under stationarity (a=1), Eq. (12) should correspond to Eq. (1) (or Eq. (5) under stationarity), where $\Delta\tau$ is still present. Please verify or add a brief explanation (e.g., assuming annual maxima with $\Delta\tau$=1 year).

Thanks for pointing out the issue regarding $\Delta\tau$. Indeed, the term $\Delta\tau$ should appear in all the relationships for consistency. The error came out because we considered $\Delta\tau$=1 year, but indeed the term should be kept as it is to make equations more general. According to the comment, the error has been amended in Eq. (10) and other equations (e.g., Eq.s (11) and (22)).
For the sake of clarity, stationary conditions refer to $a = 0$ (i.e., time derivative of T_0 null in Eq. (8)), not $a = 1$.

Line 142. While the interpretation of Eq. (13) is straightforward, its derivation should be

explained more thoroughly. Further, the parameter a plays a key role here; what is its acceptable order of magnitude? For example, a=1, 0.1, 0.01?

Thanks for the suggestion to clarify Eq. (13). Accordingly, the derivation of Eq. (13) has been added to the text.
Regarding the parameter a, which defines a sort of degree of non-stationarity, we showed some results with different values ranging from 0.05 (Figures 5 and 6) to 1 (Figures 2, 3, and 4). Besides the non-stationary degree, the parameter a determines the estimation error in the return period, in comparison to the complete formulation (Eq. (2)) by Salas and Obeysekera (2014). As a matter of fact, the lower the a, the lower the estimation error, with the limit of a=0 corresponding to stationary conditions. From a practical point of view, the minimum acceptable value of a can be determined once the trends in the GEV parameters are known and the maximum future timeframe t is set. The associated estimation error can then be calculated from graphical plots similar to Figure 6, where we considered the simplified case of a linear trend in the GEV parameters, separately. Based on this and another comment, the explanation of how the proposed framework may be applied is now provided in the conclusions.
The added text on the derivation of Eq. (13) reads (lines 161-169):

Based on Eq.s (3) and (4), one may retrieve $G_{\pm}(x,t)$ as $G_{\pm}(x,t)^{-\xi} - 1 = \xi \frac{x-\mu_M(x,t)}{\sigma_M(x,t)}$, where $\mu_M(x,t)$ and $\sigma_M(x,t)$ are the maximum/minimum (plus/minus sign) values of the time-varying mean and variance of the process, respectively, that satisfy the hypothesis of weak non-stationarity (Eq. (8))). Similarly, one may find $G_0(x,t)^{-\xi} - 1 = \xi \frac{x-\mu_0}{\sigma_0}$, where μ_0 and σ_0 are the values of the mean and the variance, respectively, at $t = 0$. In the two relationships, it is useful to keep the -1 term on the left-hand side, such that when dividing the first one by the second one, the ξ term on the right-hand side cancels out. As a result, we obtain the solution to Eq. (11)) in the cases of the Weibull and Fréchet types in a compact way as:

$$\frac{x-\mu_M(x,t)}{\sigma_M(x,t)} = \frac{x-\mu_0}{\sigma_0} \frac{G_{\pm}^{-\xi}(x,\, t) - 1}{G_0^{-\xi}(x) - 1} \tag{1}$$

where the terms $G_{\pm}(x,t)$ and G_0 can now be written in terms of the stationary return period $\overline{T}_0(x)$, the parameter a, and the timeframe t by using Eq.s (11) and (12), respectively.

Regarding Figs. 2 and 3: as far as I understand, they depict the acceptable range of variability of the GEV parameters over given time frames for specific return period values. In other words, they represent isolines of T_0 and $T_0 + at$ — under the weak non-stationarity assumption and for a=1 — for a specific value of x (and thus a specific T_0), as functions of the parameter values. In this sense, these figures could be used as an abacus to evaluate estimated time-varying GEV parameters, for instance, using the approach of Salas and Obeysekera (2014).

Absolutely, yes. Figures 2 (for Weibull and Fréchet types) and Figure 3 (for the Gumbel distribution) can be used as an abacus to retrieve the allowed trends, provided the a value used in the figures (i.e., a=1) satisfies the weakly non-stationary conditions. However, the simple math behind the approach facilitates the calculation of the maximum values for the GEV parameters, thus preventing any potential mistakes.

Figure 4. Is a=1?

Yes indeed, we used a=1 throughout the entire Results section, except when not explicitly stated (e.g., Figure 5). For the sake of clarity and based on the comment, the value a=1 has been added to the captions of Figures 2, 3, and 4.

Line 222. ".. it allows for performing a return period analysis strictly valid within $\Delta\tau$, ..". If $\Delta\tau$=1 year, does the analysis refer to a 1-year window? This should be clarified.

We added the clarification that $\Delta\tau$ is equal to 1 year, similarly to the Annual Maxima return period analysis (see lines 234).

Line 231-232. The change in return period cannot be infinite in the case of decreasing return period; since it is bounded below to $\Delta\tau$. I suggest rephrasing the sentence to clarify that the

"change" refers to the admissible variation in the parameter values.

Thanks for having highlighted this point. Indeed, we referred to the change in the process parameters. Based on the suggestion, we modified the text to clarify this point in detail.
The changed text now reads (lines 257-258):

Such a change in the mean frequency or magnitude increases with the timeframe length, ultimately reaching an infinite value at the asymptote $t = \frac{T_0(x)-\Delta\tau}{a}$, in the case of decreasing return period (see Figure 5).

Lines 283-284. I suggest adding a short paragraph describing how the method could be applied to a real case study: i) fitting a time-varying GEV model to the data; ii) evaluating the parameters to check if the weak non-stationarity condition holds depending on a - for the return period of interest T and the relevant time horizon t; iii) computing the quantile of interest under weak non-stationarity using Eq. (5), to be used for design purpose. The procedure could be also framed within a cost-benefit analysis for system design.

Based on the suggestion, we added some text in the conclusions to clarify how the proposed framework can be applied. Specifically, we again highlighted that the GEV parameters and the trends affecting them should be known a priori (e.g., by GEV estimation analysis, or by modelling scenarios), and that the proposed approach is meant to determine the maximum future timeframe, t, for which the proposed approximation may work within a reasonable estimation error, driven by the parameter a.
The text reads (lines 320-322):

As a result, once the GEV parameters are known from fitting models or forecasted scenarios, the maximum timeframe for the validity of the weakly non-stationarity analysis can be retrieved in the case of decreasing return period for specific values of the parameter a, which in turn drives the estimation error (e.g., Figure 7).

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