



A bio-economic approach for predicting monthly irrigation water demands

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Abstract

Hydrologic models typically predict irrigation water demands via bio-physical processes in-line with FAO-56
10 standards. However, irrigation demands also depend heavily on the economic behaviour of farmers, particularly their responses to water and crop prices. This study develops a novel method for predicting monthly irrigation water demand that integrates bio-physical processes with an economic profit maximization framework. This method yields a set of simple parametric equations for predicting annual crop areas and monthly water use as a function of both weather and prices. We apply this method to the Australian Murray-
15 Darling Basin (MDB) with a dataset covering 13 regions and 12 irrigation activities between 2004-05 and 2021-22. Model parameters are obtained using structural estimation, with a joint system of physical and behavioural equations solved by non-linear least squares. Validation results show strong performance for water use particularly in the southern basin (annual in-sample R^2 0.94, cross-validated R^2 0.90). Performance is weaker in the northern basin partly on account of data quality issues (annual in-sample R^2 0.84, cross-
20 validated R^2 0.71). The model is applied to measure the effects on water demand of long-term adjustment in the irrigation sector, including the emergence of almond and cotton crops in the southern basin. The results show that new almond plantings have contributed to a 40 per cent increase in peak summer demands in the lower Murray since 2014. In future, this bio-economic approach could provide a foundation for integrated hydro-economic models capable of analysing complex water policy issues, including environmental water
25 management, water market design and climate change adaptation.

Keywords

Water, Economics, Irrigation, Murray-Darling Basin



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Highlights

- This study presents a new model of irrigation water demand which integrates bio-physical processes and economic behaviour
- In addition to irrigation water use, the model also predicts irrigation crop areas, production and profit
- 35 • The model parameters are calibrated to the Murray-Darling Basin via a structural estimation approach involving a joint system of non-linear equations
- The model is validated against historical data for the period 2004-05 to 2021-22. Performance is better in the southern basin than the northern basin
- The model is applied to quantify long-term changes in irrigation demands in the basin, including
- 40 • growth in almond water use in the lower southern basin
- In future, the model could be used to support integrated hydrologic-economic simulation models



1 Introduction

Models play a central role in the management of water in large, regulated river systems. Within the Australian Murray–Darling Basin (MDB) hydrologic models are central to all aspects of water management, from river operations to water accounting and compliance, to long-term planning and policy decisions.

Predicting water demand is a key aspect of these models. Short-term forecasts of water demand are routinely used to support river operations; however, the same demand models are also used to examine long-term policy issues. For example, in the MDB, models are used to assess the effects of recovering water for the environment under the Basin Plan (Kirby et al., 2014; MDBA 2020). Assessments of long-term climate change (CSIRO 2020; Fowler et al., 2022; Kirby et al., 2014) also depend heavily on water demand models and their responses to rainfall, temperature and water availability.

In hydrologic models, irrigation demands are typically simulated via bio-physical crop and soil water balances in line with FAO-56 standards (see Allen, 1998; MDBA 2018). While these approaches have proven effective, particularly for short-term predictions, they ignore economic factors such as water and crop prices which in practice have a large bearing on irrigation activity and water demand (Brennan, 2006; Scheierling et al., 2006; Wheeler et al., 2008). In contrast, economic models represent irrigation farmers as profit maximizing businesses with water demands that respond to prices of inputs (i.e., water) and outputs (i.e., crops). Economic models employ either mathematical programming or statistical methods to link water demand with prices (Adamson et al., 2007; Hall et al., 1994; Grafton & Jiang, 2011; Hughes et al., 2023). However, economic models are more abstract, with higher spatial and temporal resolution (i.e., annual), less bio-physical detail and often less emphasis on predictive power and validation. As a result, economic models play a lesser role in river management, often being applied within research literature to examine the economics of water markets, or other water policy issues (see Kirby et al., 2014; Qureshi et al., 2013; Hughes et al., 2023).

In this paper, a bio-economic approach for predicting monthly irrigation water demand is developed and applied to the MDB. This approach combines bio-physical systems from hydrologic models with an economic profit maximization framework. While representing the key bio-physical and economic processes the model is designed to have a simple parametric form which is empirically tractable.

This study extends the annual economic demand model of Hughes et al. (2023) in several directions: introducing a monthly time-step, coverage of northern MDB regions, along with more biophysical and economic structure. In related work, Ahmed et al. (2024) present an approach to downscale annual outputs from the Hughes et al. (2023) model to a monthly time-step. In contrast with Ahmed et al. (2024), this study



develops a fully dynamic monthly model with irrigator crop planting and water use decisions dependent on prices and weather conditions in each month.

While the approach involves additional economic and bio-physical structure it remains heavily data-driven with parameters estimated from historical data for the period 2004–05 to 2021–22. A comprehensive dataset was constructed to support this work, with various improvements and extensions made to the data of ABARES (2021). A structural estimation approach is developed with parameters jointly estimated via a non-linear system of physical (hydrologic) and behavioural (economic) equations. This structural approach widens the potential scope for applications beyond short-term water demand prediction and forecasting to counter factual simulation modelling.

These demand models are intended to support development of integrated hydro-economic simulation models which represent both physical river systems and the economics of irrigation and water markets (Brouwer & Hofkes, 2008). As a first step, Hughes et al. (2025 unpublished) use these water demands to develop an economic model of the MDB linked to a monthly hydrologic model (John et al., 2025 unpublished). In this sense, the work has similarities with previous hydro-economic models of the MDB including those of Kirby et al. (2013) and Qureshi et al. (2013); as well as the CALVIN model of California (Draper et al., 2003; Howitt et al., 2010). While conceptually similar, our demand system differs from Qureshi et al. (2013) and Howitt et al. (2010) in at least two respects. First, the parameters are obtained by statistical estimation rather than calibration (i.e., Positive Mathematical Programming, Howitt et al., 2012). Second, our approach yields reduced-form demand equations (predicting water use as a function of weather conditions and water and crop prices) which can be used as a stand-alone system or embedded in simulation models.

In future, fully integrated (aka “holistic”) hydro-economic models have the potential to address many contemporary water policy questions, including those related to the design of water markets, environmental management and climate change adaptation (Brouwer & Hofkes, 2008). While the value of integrated models is often recognized (see Quinlivan, 2022) they have seen limited use in the MDB to date outside of the research literature. This study helps address at least one long-standing constraint to integrated modelling: different time-steps between hydrologic (e.g., daily/monthly) and economic (e.g., annual) models.

In this paper, we describe our bio-economic water demand system in detail, including the data, bio-physical and economic assumptions and the estimation methods. We then present validation results measuring in-sample and cross-validated out-of-sample performance. Finally, we apply the demand system to isolate key trends in water demand in the MDB in recent years, particularly increased demand in the lower Murray due to new almond plantations.



2 The Murray-Darling Basin

The Murray–Darling Basin (MDB) drains an area of over 1,000,000 km² across South-Eastern Australia.

105 Irrigated agriculture accounts for more than 90% of total water use in the region. The southern MDB (sMDB)—which includes the Murray River and its connected tributaries (Figure 1)—accounts for around three quarters of agricultural water use in the MDB (and around half of national agricultural water use).

The sMDB is subject to a more temperate climate, with winter dominant inflows and relatively large on-river storages which support inter-year carryover reserves. In contrast, the northern MDB (nMDB) has a
110 sub-tropical climate with highly variable and summer dominant inflows and smaller on-river storage capacity. As a result, a greater percentage of water use in these regions is obtained from flood-plain harvesting and on-farm dams (as opposed to extraction from regulated rivers).

Irrigation within the sMDB is diverse with a mix of perennials (e.g., fruit and nut trees and grapevines), irrigated pasture and annual cropping (e.g., rice and cotton). Perennial crops are concentrated in
115 the lower Murray, dairying / irrigated pasture in northern Victoria (e.g., Goulburn–Broken) and rice and cotton in NSW (Murray and Murrumbidgee, see Figure 1). The nMDB is dominated by cotton, which accounts for over 80 per cent of water use in the region.

The last decade has seen much structural change, with declines in some activities—such as dairy in northern Victoria—and expansion in others, particularly almonds in the lower Murray regions and cotton in
120 the Murrumbidgee (see Zeleke & Luckett, 2025). These trends have been driven by commodity prices and technology, including new cotton varieties suited to the southern Australian climate.



Figure 1: The Murray–Darling Basin



125 3 Data

As documented in previous studies (Hughes et al., 2023; Qureshi et al., 2013) the collation of economic and hydrologic data for MDB presents many challenges. The dataset constructed for this study draws on a wide range of sources, with the key variables listed in Table 1. The data covers the period 2004–05 to 2021–22, for 15 catchment regions (Table 2) and 12 irrigation activities (Table 3).

130 For this study, three related but distinct measures of water use are defined: diversions D , allocation use U and water applied W (see Figure A1, Appendix A). Diversions are physical volumes of surface water extracted from river systems; water applied refers to volumes applied by irrigation farmers to crops, while allocation use reflects usage by water right holders of annual water allocations. Allocation use includes held environmental water which is effectively “used” (i.e., released from storage) but not diverted, while irrigation
135 water applied excludes conveyance losses, but may also include groundwater applied to crops.

Agricultural data, including areas irrigated, yield, and water use, are derived from ABARES (2021) and ABS (2022) are available annually from 2005–06 to 2020–21 for all regions. Further adjustments were applied to better align irrigation data with diversion and water accounting records, particularly on a spatial basis. This process drew on additional data sources including aerial photography data (SunRISE Mapping and
140 Research, 2022), irrigation operator data (Murray Irrigation Limited) and industry reports (Cotton Australia, Australian Almond Board). As in Hughes et al., (2023) and Qureshi et al., (2013), Geographic Information System (GIS) methods were used to translate ABS (2022) regional data to catchment regions.

Monthly allocation usage data were obtained from state agencies for Victorian and NSW regions (but were not available for SA and QLD). Monthly reference evapotranspiration and rainfall are derived from the
145 Australian SILO daily dataset (Jeffrey et al., 2001) for representative locations within each irrigation region. Crop coefficients are sourced from MDBA (2018) and from Ahmed et al. (2024).

Monthly water market price data were also obtained from ABARES based on underlying state water trade register data (processed using the method of Sanders et al. 2019). The availability and quality of water price data is generally lower in the northern MDB, given less mature and active markets in these regions.
150 Finally, diversion data are mostly annual, with monthly data available for the Murray regions and the Goulburn



Table 1: Variable descriptions and data sources

Variable	Units	Description	Data sources
W_{ijt}	ML	Irrigation water use in region i for crop j in period t	ABARES/ABS ¹
L_{ijt}	Ha	Area of crop j irrigated in region i in period t	ABARES/ABS ¹
Y_{ijt}	t	Quantity of production for crop j in region i in period t	ABARES/ABS ¹
P_{jy}^y	\$ / t	Output price for crop j in year y	ABARES/ABS ¹
P_{it}^w	\$ / ML	Market price for water allocations in region i period t	MDB states ²
A_{it}	ML	Water allocations available for use in region i period t	MDB states ²
U_{iy}	ML	Usage of water allocation in region i in year y	MDB states ²
U_{iy}^{env}	ML	Usage of environmental water in region i in year y	MDBA ³
D_{it}	ML	Diversions in region i in period t	MDBA ³
ET_{it}^0	mm	Reference evapotranspiration for region i in month m	SILO ⁴
R_{it}	mm	Rainfall in region i in month m (excluding estimated run-off)	SILO ⁴

¹ABARES (2021), ABS (2022), SunRISE Mapping and Research (2022), Cotton Australia, (<https://www.cottondata.com.au/>), Australian Almond Board (<https://australionalmonds.com.au/>), Murray Irrigation

155 Limited Annual Reports (<https://www.murrayirrigation.com.au/>)

²NSW Department of Climate Change, Energy, the Environment and Water, Water NSW, Victorian Water Register (<https://waterregister.vic.gov.au/>), SA Department of Environment and Water, all processed by ABARES

³MDBA annual take reports (https://www.mdba.gov.au/sites/default/files/publications/annual-water-take-report-2022-23_1.pdf), monthly data MDBA personal communications

160 ⁴SILO climate data (<https://www.longpaddock.qld.gov.au/silo/>) (Jeffrey et al., 2001)



Table 2: Model regions ($i \in I$)

Regions, $i \in I$	State	Trading Zone
Goulburn–Broken–Loddon–Campaspe	Vic.	Northern Vic.
Vic. Murray (above)	Vic.	Murray above
Vic. Murray (below)	Vic.	Murray below
NSW Murray (above)	NSW	Murray above
NSW Murray (below)	NSW	Murray below
SA Murray	SA	Murray below
Murrumbidgee	NSW	Murrumbidgee
Lachlan	NSW	Lachlan
Macquarie–Castlereagh	NSW	Macquarie
Namoi	NSW	Namoi
Gwydir	NSW	Gwydir
Barwon–Darling	NSW	Barwon–Darling
NSW Border Rivers	NSW	Border Rivers
QLD Border Rivers	QLD	Border Rivers
Condamine–Balonne	QLD	Condamine

165 **Table 3: Model activities ($j \in J$)**

Irrigation activities, $j \in J$	Crops,	Season/s, $s \in S$
Other Horticulture	Tree crops (excl. almonds)	Perennial
Grapes	Grapes (wine and table)	Perennial
Almonds	Almonds	Perennial
Rice	Rice	Summer
Cotton	Cotton	Summer
Pasture (Perennial)	Pasture	Perennial
Pasture (Winter)	Pasture	Winter
Hay (Perennial)	Hay	Perennial
Hay (Winter)	Hay	Winter
Vegetables	Vegetables	Perennial
Other (Summer)	Other field crops	Summer
Other (Winter)	Other field crops	Winter



4 The model

4.1 Sets

The model is defined over three main sets: regions I (Table 2), irrigation activities J (Table 3) and time T .

170 While the model has a monthly time-step, some variables are annual. In the below equations, the time index varies between the generic t and the specific year y and month m as required. While 15 regions are defined (Table 2, Figure 1) the two Queensland regions are omitted from much of the analysis due to data limitations.

Nine irrigation crop types are defined following those available in the irrigation data (Table 3). Crops are defined to be either *perennial*, *summer* or *winter* in type. Perennial crops have a fixed planted area, and
175 require irrigation all year round, while winter and summer subject to annual planting decisions and seasonal irrigation. Here the *Other* crops include both annual and summer types and *Pasture* and *Hay* includes both perennial and winter types, leading to a total of 12 distinct irrigation activities.

4.2 Crop irrigation requirements

Following hydrologic models, short-term crop water requirements are determined by bio-physical drivers
180 (Figure 2). Specifically, water requirements \bar{w}_{ijm} (for crop j in region i in month m) are a function of potential crop evapotranspiration ET_{im}^0 less effective rainfall ER_{im} :

$$\bar{w}_{ijm} \propto \max(k_{jm}^c ET_{it}^0 - ER_{it}, 0)$$

Here k_{ijm}^c are pre-defined ‘crop coefficients’ and ET_{it}^0 is the reference ET for region i in time period t (year y , month m) and both ET_{im}^0 , and ER_{im} are functions of weather data. Following hydrological model
185 conventions, crop water requirements are defined in mm units (later converted to ML / ha, via the 1 /100 factor in Eq. 2).



The approach adopted in this model differs from typically FAO-56 systems in at least two respects: the time-step is monthly rather than daily, and there is no explicit soil moisture balance. To improve empirical performance of this simplified approach, we calibrate both effective rainfall and crop water requirements via the parametric equations:

$$ER_{im} = \beta_f^{ER0} \min(\max(R_{it} - \beta_f^{ER1} \cdot ET_{it}^0 + \beta_f^{ER2}, 0), R_{it}) \quad (1)$$

$$\bar{w}_{ijt} = \frac{1}{100} \cdot \frac{1}{\beta_{ij}^{w0}} \max(k_{jt}^c ET_{ijt}^0 + k_{jt}^{SM} - ER_{ijt}, 0) \quad (2)$$

$$w_{ijm}^{nbr} = \beta^{w3} \cdot \bar{w}_{ijm}$$

where β are parameters to be estimated.

In the absence of a soil moisture balance we introduce a monthly soil moisture recharge / depletion target k_{jt}^{SM} which is used to account for pondage in the case of rice crops (but is set to zero for all other crops). Lastly, for almond crops we also account for the lower water requirements of non-bearing almond trees, where w^{nbr} is the per hectare water requirement for non-bearing tree areas

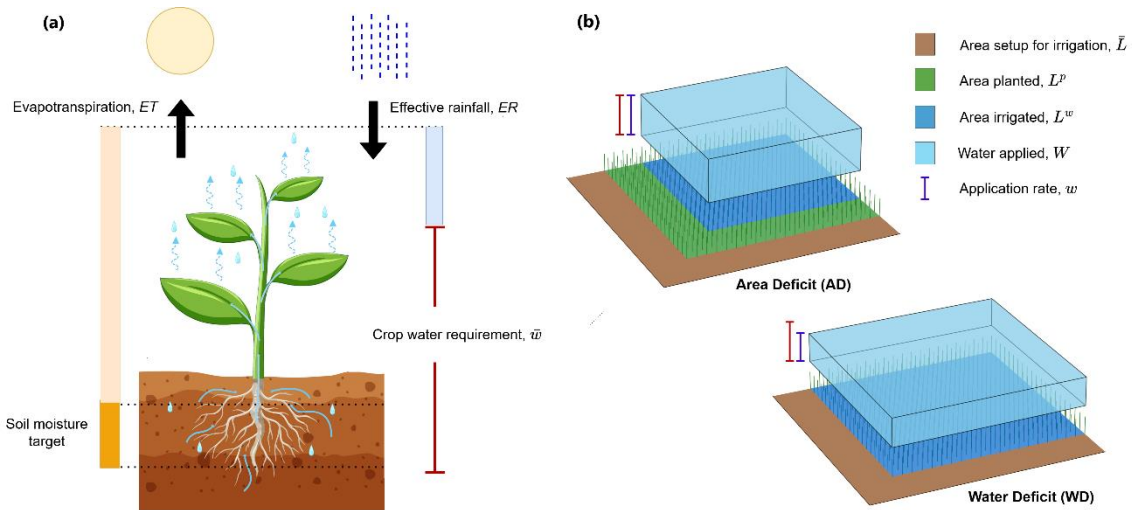


Figure 2: Water demand model crop water requirements and land constraints. (a) Crop water balance bio-physical processes. (b) Two forms of deficit irrigation: Area Deficit (AD) and Water Deficit (WD)



4.3 Crop areas

As shown in Figure 2, an upper limit of area planted, \bar{L}_{ijt} , is defined to reflect the maximum area that could potentially be used, while, L_{ijt}^p , reflects the estimated area actually planted. For perennial crops (i.e., fruit and nut trees, wine grapes, vegetables) area planted is held fixed (at observed levels $\bar{L}_{ijt} = L_{ijt}^p$). For annual crops, areas planted can vary each year in response to water prices and other factors, with \bar{L}_{ijt} calibrated from the data with an allowance for a linear time-trend (to account for historical shifts in irrigation development)¹:

$$\bar{L}_{ijt} = \beta_{ij}^{l0} \cdot (1 + \beta_{ij}^{l1} \cdot t) \quad (3)$$

The model allows for two different forms of deficit irrigation: Area Deficit (AD) and Water Deficit (WD) as shown in Figure 2. Under AD, water application rates w_{ijt} are fixed at \bar{w}_{ijt} but area irrigated $L_{ijy,m}^w$ can be less than area planted. Under WD the planted area is subject to a variable application rate $w_{ijt} \leq \bar{w}_{ijm}$. The two approaches are the result of different assumptions on crop yield responses, as outlined in the next section.

Water use for crop j in region i in period t is then defined as area watered L_{ijm}^w times application rate w_{ijt} , with an allowance for non-bearing areas L_{ijt}^{nbr} (in the case of almond crops):

$$W_{ijm} = L_{ijm}^w \cdot w_{ijm} + L_{ijt}^{nbr} \cdot w_{ijm}^{nbr} \quad (4)$$

$$L_{ijm}^w \leq (L_{ijt} - L_{ijt}^{nbr}), \quad L_{ijm}^w \leq L_{ijt}^p, \quad w_{ijm} \leq \bar{w}_{ijm}$$

4.4 Crop production

As is standard in economic models, we assume crop production is subject to diminishing marginal productivity. The model adopts two alternative functional forms or “technologies” for crop production, labelled Area Deficit (AD) and Water Deficit (WD) (consistent with Figure 2 above):

¹ Three of the crops are specified with multiple season types: pasture and hay (winter and perennial) and other (summer and winter). However, crop data is only available for annual areas (with no summer, winter, perennial splits). For these three crops, limits on area planted in each season type are a fixed share of an estimated annual crop limit (in the model estimation, the sum of perennial, winter and summer estimates are compared with the annual crop activity data).



$$f_{ij}^y(L_{ijt}^p) = \beta_{ij}^{y0} \left(L_{ijt}^p - \frac{L_{ijt}^{p2}}{2 \cdot \beta_j^{y1} \bar{L}_{ijt}} \right) \quad (5 \text{ AD})$$

$$f_{ij}^y(L_{ijt}^p) = \beta_{ij}^{y0} \cdot L_{ijt}^p \quad (5 \text{ WD})$$

$$y_{ijt} = f_{ij}^y(L_{ijt}^p) - \sum_m \dot{k}_{ijt}^s (f_{ij}^y(L_{ijt}^p) - f_{ij}^y(L_{ijt}^w)) \quad (6 \text{ AD})$$

$$y_{ijt} = \left(1 - \sum_m \dot{k}_{ijt}^s \left(\frac{w_{ijt}}{\bar{w}_{ijt}} - 1 \right)^2 \right) f_{ij}^y(L_{ijt}^p) \quad (6 \text{ WD})$$

$$Y_{ijt} = \max[y_{ijt}, 0]$$

$$k_{ijt}^s = \beta_j^{s0} \left(1 - \frac{\min[ER_{it}, k_{jm}^c ET_{it}^0]}{k_{jm}^c ET_{it}^0} \right)$$

$$\dot{k}_{mj} = \left(\frac{k_{jm}^c}{\sum_m k_{jm}^c} \right), \quad \dot{k}_{ijt}^s = \dot{k}_{mj} \cdot k_{ijt}^s$$

Here f_{ij}^y is a production function linking crop area planted to potential crop production (production in the absence of any water stress) and k_{ijt}^s is an FAO-56 type water stress coefficient. The AD and WD forms may be better suited to crops, with the choice of functional form to be guided by empirical testing and / or domain knowledge. For example, decreasing land productivity is likely to occur where there exists heterogeneity across individual farms within a region (for a given crop type) due for example to variations in land quality or crop variety. As such, AD may be more appropriate for broadly defined crop categories (i.e., other crops, vegetables, other horticulture) or larger regions. Assumptions for the crops in the MDB model are shown in Table 3.

Note that, the above crop yield response approach simplifies FAO-56 by assuming that water stress in each month has an independent additive effect on yields (ensuring that short-run crop yield responses and water demands are independent of past and future months). An allowance is also made for yield penalties: where water stress is severe enough to impact future yields or impose other costs beyond the loss of the current year's production. In these cases, $\sum_m \dot{k}_{ijt}^s$ can be greater than 1, and y_{ijt} can be negative. These penalties can



245 be viewed as estimates of future lost production (e.g., as a result of tree damage or death) and related costs (such as re-planting of trees) due to current season water stress.

4.5 Water use

Monthly diversions in each region are modelled as a function of total water applied to crops, as detailed below. This function implicitly represents the net effects of other sources of water used for irrigation (such as
250 groundwater), non-irrigation diversions such as domestic water along with conveyance losses.

$$D_{iym} = \beta_i^{d0} \sum_j W_{ijym} + \beta_{im}^{d1} + \beta_i^{d2} \cdot \frac{R_{iym}}{\bar{R}_{im}} \quad (7)$$

where R_{iym}/\bar{R}_{im} is monthly rainfall relative to the long-run monthly mean.

Allocation use explicitly accounts for total irrigation water applied W_{iy} and environmental water use U_{iy}^e , and implicitly the net effects of any ‘unregulated’ surface water extraction (such as flood plain
255 harvesting):

$$U_{iy} = \beta_i^{u0} W_{iy} + \beta_i^{u1} U_{iy}^{env} + \beta_i^{u2} + \beta_i^{u3} \frac{R_{iym}}{\bar{R}_{im}} \quad (8)$$

In Appendix A.3, we present a simple statistical model for predicting annual environmental water use as a function of held environmental water entitlements, allocations and rainfall.

4.6 Irrigation costs and profits

260 Annual profits from water use π_{ijy} in region i in year y for activity j are defined as:

$$\pi_{ijy} = P_{jy}^y \cdot y_{ijy} - \sum_m (P_{iym}^w + \beta_i^{c0}) W_{ijym} - \beta_j^{c2} L_{ijy}^p \left(1 + \beta^{c1} \cdot \frac{L_{ijy}^p}{2L_{ijy}} \right) \quad (9 AD)$$

Here regional profits equal revenue from crop production (net of any yield penalties), less water costs and less planting costs. Water costs include both the market price of water P_{it}^w and any usage / delivery costs β_i^{c0} . Planting costs are assumed quadratic with respect to area planted (such that per hectare costs increase linearly).

265 Note this profit function (and associated water demand functions) are short-run in nature excluding any capital investment costs.



5 Estimation

5.1 Model reduced-form

The above model can be framed as an economic optimization problem where irrigators make crop planting and water use decisions to maximize expected profits given prevailing and expected prices, climate conditions and other physical constraints:

$$\max_{L_{ijt}^p, W_{ijt}} \pi_{ijy}$$

subject to Eq. 1-6 and 9.

From the first order conditions of this problem, a set of reduced-form input-demand functions can be derived (see Appendix A.4). Short-run (monthly) water demand given crop area planted are shown below for both forms:

$$W_{ijt} = \bar{w}_{ijt} \bar{L}_{ijt} \beta_j^{y1} \left(1 - \frac{w_{ijt}(P_{it}^w + \beta_t^{c0})}{P_{jt}^y \cdot \beta_{ij}^{y0} \cdot \dot{k}_{ijt}^s} \right) \quad (10 \text{ AD})$$

$$W_{ijt} = \bar{w}_{ijt} \cdot L_{ijt}^p \left(1 - \frac{\bar{w}_{ijt}(P_{ijt}^w + \beta_j^{c0})}{2 \cdot P_{jt}^y \cdot \beta_{ij}^{y0} \cdot \dot{k}_{ijt}^s} \right) \quad (10 \text{ WD})$$

$$0 \leq W_{ijm} \leq L_{ijm}^p \bar{w}_{ijm}$$

Crop planting decisions are slightly more complex given decisions are made at planting time, with uncertainty over the prices and weather conditions that will prevail over the rest of the cropping season. However, with some minor simplifying assumptions, parametric functions can be derived linking area planted with crop and water prices (as at the time of planting, (equations 11 AD, 11 WD in Appendix A.4). In particular, we assume that expected water prices over the crop season are equal to prices at planting time, and that expected water requirements and water stress can be approximated by linear functions of planting time prices (see Appendix A.4 for full details).

5.2 Numerical methods

The estimation is set up as a simultaneous equation non-linear least squares problem, with the parameters chosen to minimize the total weighted squared error of the model predictions relative to the historical data for



290 W_{ijt} , L_{ijt} , Y_{ijt} , U_{it} and D_{it} (see Appendix A.5). This approach to estimation offers flexibility to accommodate differences in data availability and quality across regions and years. The system approach also makes full use of the available data, allowing the limited monthly data (for variables such as diversions) to influence monthly irrigator crop planting and water use parameters where possible.

The estimation is also flexible enough to handle regions where water price data P_{it}^w are missing or of
295 low quality due to thin markets (including the Macquarie, Gwydir, Namoi and NSW Border Rivers). In these regions an additional equation is added to the system, imputing a shadow water price as a function of water supply data. These shadow prices are then used in-place of observed data in the water use and crop area functions (see Appendix A.5)

Given a set of starting values and bound constraints, non-linear optimisation methods, specifically
300 the L-BFGS algorithm (Liu & Nocedal, 1989) are used to find the error minimising parameters. Here bound constraints also provide an opportunity to impose various feasibility and economic conditions (i.e., diminishing marginal productivity). The estimation is implemented in Julia via the JuMP modelling language (Lubin et al., 2023) using the NLOPT (L-BFGS) solver (Johnson & Schueller, 2021). For further details see Appendix A.5.



305 6 Results

6.1 Validation

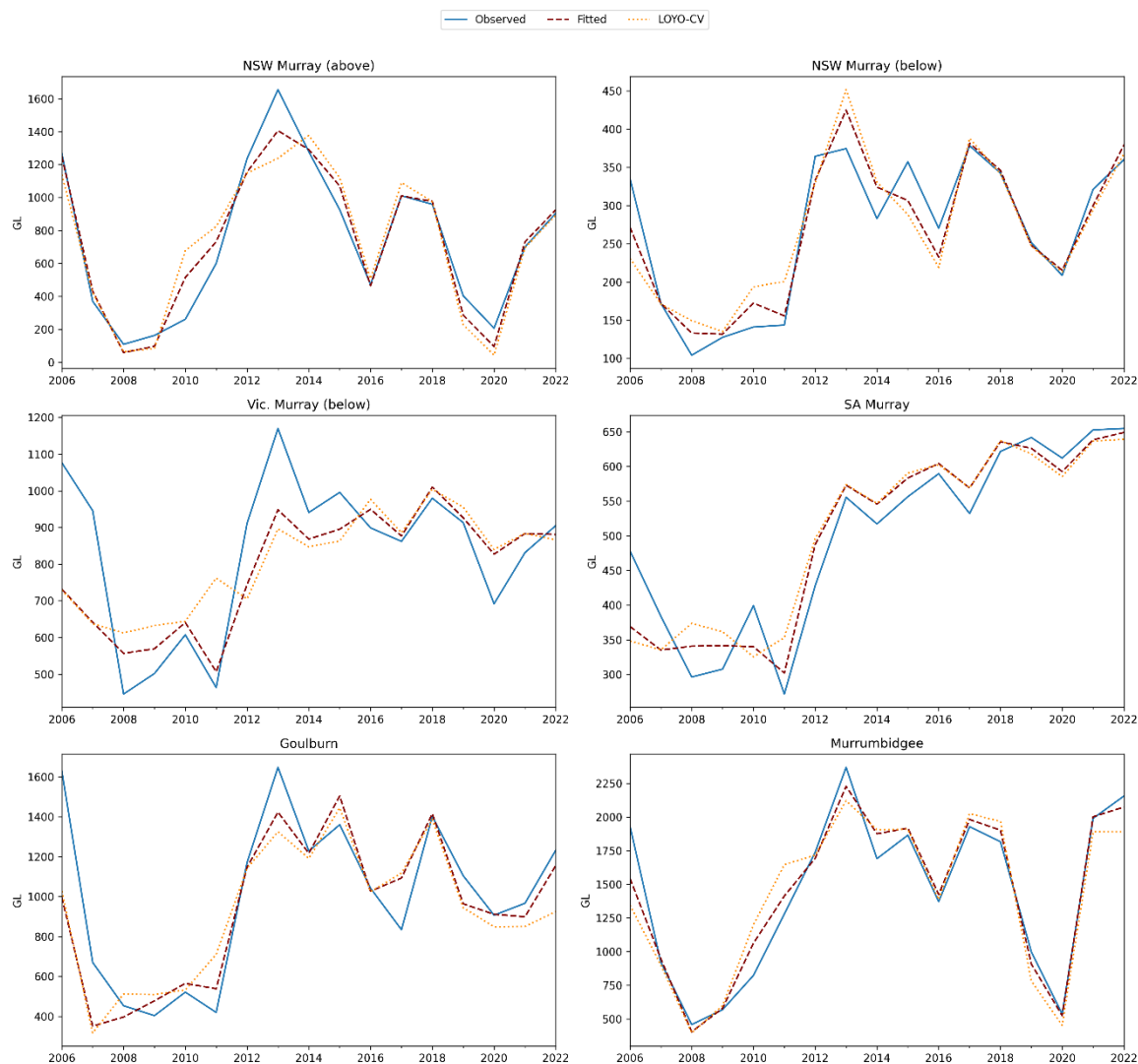
Validation results are shown in Figure 3 to 5 (with further detail in Appendix B). Actual data are compared against both in-sample fitted values, and cross-validated (Leave-One-Year-Out, LOYO) predictions.

310 The Cross-validated (LOYO) Symmetric Median Absolute Percentage Error (SMAPE) for MDB annual diversions is 12.8 per cent ($R^2 = 0.94$), with better performance in southern MDB regions (SMAPE 10.0 per cent, $R^2 = 0.92$) than northern regions (SMAPE 20.5 per cent, $R^2 = 0.67$) (see Appendix B, Table B1 and B3). While direct comparisons with other models are difficult (due to differences in spatial and temporal extents and reported metrics) the performance for annual diversions in the Murray regions (Table B2) appears similar to that reported for the Murray *Source* hydrological model (see MDBA 2018, Table 1).

315 In the northern MDB performance is constrained somewhat by larger volumes of unregulated water use (i.e., flood-plain harvesting and on-farm dams) and the absence of accurate water market price data. Within the model unregulated water use volumes are represented implicitly via statistical relationships with rainfall (within the use and diversion equations, Eq. 7 and Eq. 8, and the shadow price functions, see Appendix A.5). These statistical approaches are relatively effective in-sample, but performance is lower out-of-sample.
320 Further, note that the observed diversion data in these regions are themselves subject to measurement error, as some forms of unregulated take are not metered and have to be approximated (MDBA 2024).

In regions where reliable monthly diversion data are available (the Murray) the model replicates the seasonal (i.e., summer dominant) pattern of water demands reasonably well (Figure 6) with better performance in the larger regions (Vic. Murray below and NSW Murray above). While direct comparisons are again
325 difficult, the monthly performance for the Murray regions (particularly R^2 , see Table B5) appears similar to that reported for the Murray *Source* model (see MDBA 2018).

At a crop level, performance is stronger for the largest crops including rice, cotton and pasture and generally weaker for other crops and hay (Figures B1-B3, Appendix B) with the model replicating historical variation in area planted (ha) and production (t) as well as water applied. As shown in Figure B3, annual
330 variation in production for horticulture, grapes and vegetables is not well explained by the model. Much of this variation is unrelated to water use, reflecting external factors such as changes in observed crop varieties within each category (e.g., the mix of wine and table grapes) due both to actual changes and measurement (sampling) error.



335 **Figure 3: Observed, fitted and LOYO-CV annual allocation use U_{ty} southern MDB regions**

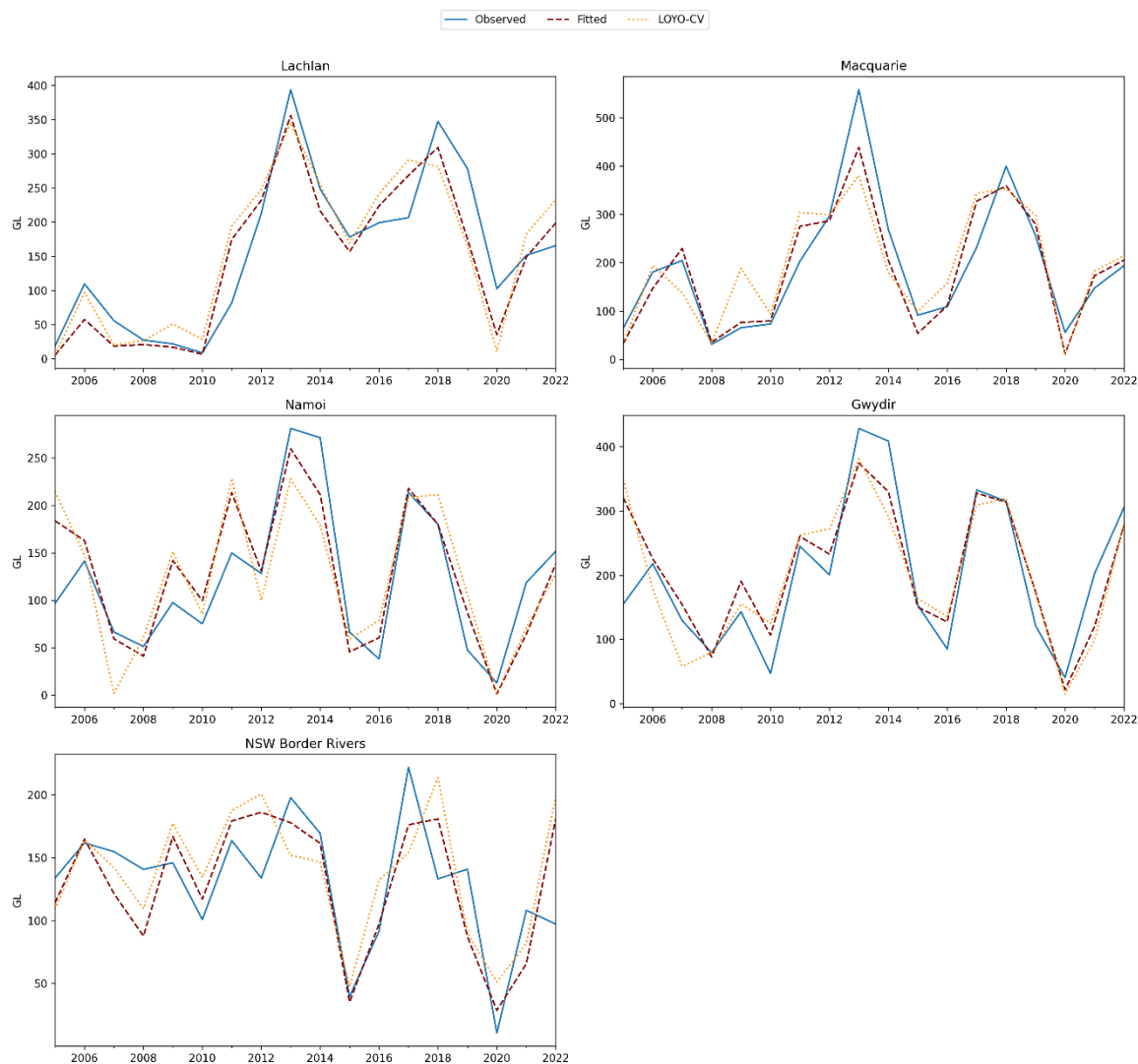


Figure 4: Observed, fitted and LOYO-CV allocation use U_{ity} by region, northern MDB regions

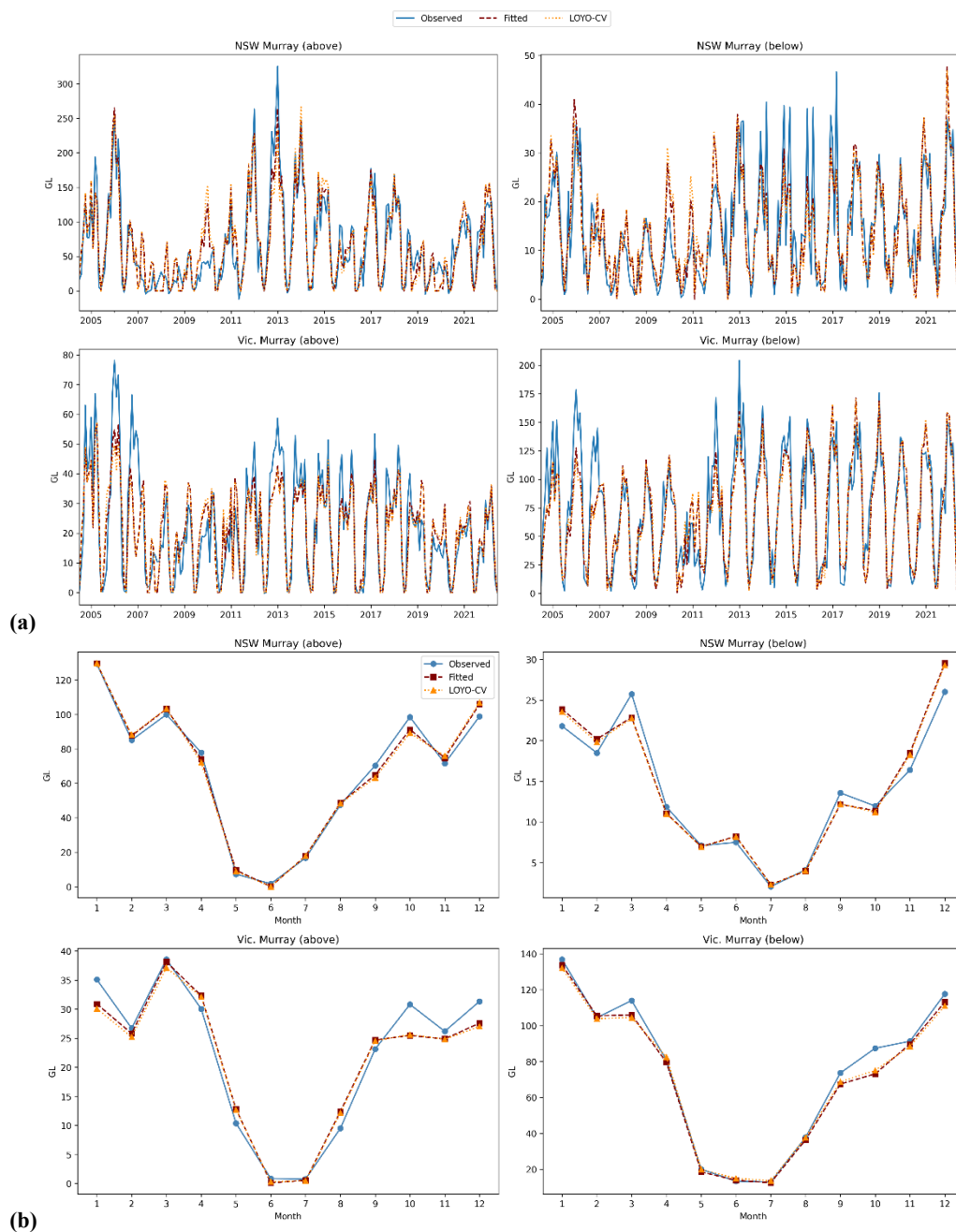


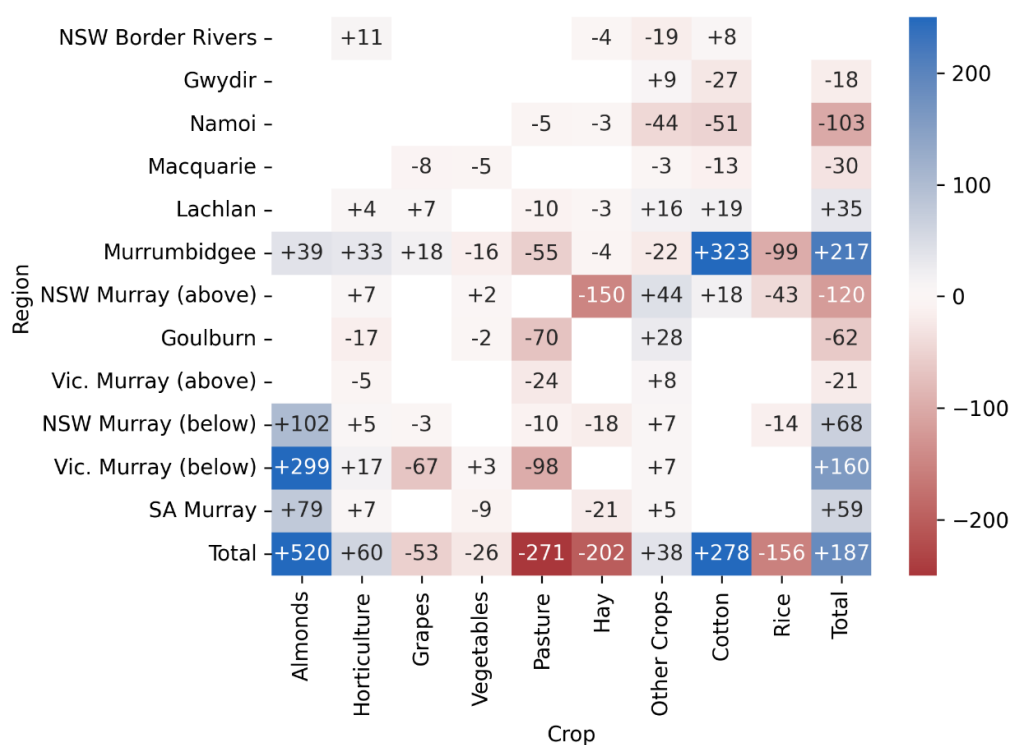
Figure 5: (a) Observed, fitted and LOYO-CV monthly diversions D_{it} (GL) in Murray regions 2005-06 to 2021-22. (b) Mean diversions by month (GL) for Murray regions (mean of 2005-06 to 2021-22)



340 6.2 Water demand trends

The estimated model can be applied to measure long-term historical changes in irrigation water demands by region and crop type. The model equations identify both changes in irrigation development (i.e., area set-up for annual crops, area planted to perennial crops, maturity of trees) and technology (i.e., improvements in yields and water use efficiency). As such, the model can be applied to separate long-term structural change from the effects of annual climate and price variability.

Here we apply the model equations to estimate water demands under both 2006-07 and 2021-22 development and technology, in each case under a repeat of historical climate and price variability (2004-05 to 2021-22). Figure 6 presents the long-term changes in average water applied by crop and region.



350 **Figure 6: Effect of long-term change in irrigation development (2006-07 to 2021-22) on median water applied W_{ij} (GL) by region and crop**



The results highlight the large increase in water demand from new almond plantings in the lower Murray regions, along with the recent expansion of cotton in the southern MDB, offset to some extent by declines in pasture, hay, rice and wine grapes. In contrast with the southern MDB, water demands in the northern regions have remained relatively stable over time.

The growth in water demand in lower Murray regions is of interest to water managers, as it has raised concerns over the ability of the river system (and therefore the water market) to support large downstream flows. Figure 7b provides an estimate of monthly water demands (diversions) in the lower Murray regions (NSW, Vic. and SA Murray) under 2022 development levels (and a repeat of 2004-05 to 2021-22 prices and climate). Figure 7a shows the long-term trend in lower Murray diversions for the peak month of January, which increased around 40 per cent since 2014.

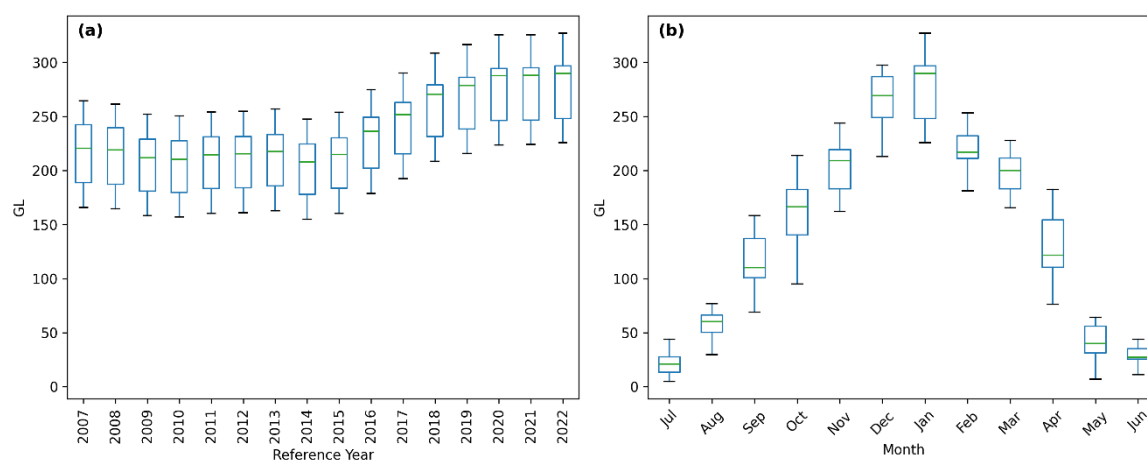


Figure 7 (a) Effect of annual change in irrigation development (2006-07 to 2021-22) on lower-Murray January diversions (GL) (total for Vic. Murray below, NSW Murray below and SA Murray) (b) Monthly lower-Murray diversions (GL) (total for Vic. Murray below, NSW Murray below and SA Murray) under 2021-22 irrigation development



7 Discussion and conclusions

This study introduces a new approach to modelling irrigation water demand in a regulated river system, one that integrates bio-physical processes with the economic behaviour of irrigation farmers. The model yields a set of parametric equations which can be estimated as a system against suitable set historical data. While computationally intensive, this approach is highly flexible and can be tailored to suit the available data.

Validation results show the model can replicate historical variation in water demands in the Murray Darling Basin with good accuracy. Within the southern MDB, where data quality is highest, annual water diversions are predicted with Symmetric Median Absolute Percentage Error (SMAPE) of 12.7 per cent, and an R^2 of 0.90 (based on a Leave-One-Year-Out cross-validation). While monthly validation data are currently limited, the model can represent monthly diversions in the key Murray River regions well.

To demonstrate the model, we present an analysis estimating long-term changes in water demands by region and crop in the MDB between 2006-07 and 2021-22. These results highlight the significant structural changes within the southern MDB over the period, including the emergence of almonds and cotton and declines in pasture, hay, rice and grapes. These changes have also altered the spatial pattern of water use, increasing demand in the lower Murray regions.

While these demand models can be used directly for short-term forecasting or historical analysis (as above) they have been developed primarily for simulation modelling, with the bio-physical and economic structure intended to support counterfactual scenario analysis such as alternative climate or policy scenarios. In recent work, Hughes et al. (2025) use this demand system as the basis for a new monthly economic model of irrigation production and water markets in the MDB. This economic model has already been applied to simulate climate change outcomes in the southern MDB drawing on water availability (i.e., allocation) scenarios from a simplified hydrological model (John et al., 2025). A longer-term the goal is to develop fully integrated models with two-way feedback between economic and hydrological processes (“holistic” models in the terminology of Brouwer & Hofkes, 2008). With this goal in-mind. there remains scope to further refine and extend the water demand system to improve skill and generalization.

As would be expected, model performance is weaker in the northern basin. This is partly due to the larger volumes of un-regulated water use (i.e., flood-plain harvesting and on-farm dams) and related data quality issues. Additional data on these un-regulated water sources and / or better-quality water price data would help model predictions in these regions. In addition, although the model can produce reasonable monthly use patterns in the Murray regions, further refinement would be required to support integration with hydrological models drawing on a larger sample of monthly data. Diversion and allocation use predictions



400 could also be improved if the model explicitly represented non-irrigation demand components (such as conveyance and town water).

One key question is the model's ability to generalize, particularly to climate conditions outside the range of historical data. It is important to note this issue applies equally, if not more so, to traditional biophysical demand models, which rely on statistical calibration (particularly crop area / water availability curves, see MDBA 2018) and standard crop coefficients (derived from historical observation). The economic structure of this model—where crop areas respond to profit drivers—should improve generalization compared with models where crop areas are held fixed or are based on fixed statistical relationships. Further, the estimation is designed to be updated annually, such that demand models can at least capture recent technological changes.

410 Regardless, for some applications it may be advisable to supplement these demand models with external information. For example, temperature increases under climate change are likely to alter the locations at which crops can be feasibly grown (e.g., improving the viability of horticultural crops in cool regions and reducing it in warm regions). To account for this, water demand models might need to be combined with some form of temperature-based crop suitability analysis.

415



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Author contributions

Neal Hughes was responsible for conceptualisation, methodology, project administration, software development, validation, data curation and writing of the original draft. April Zhao contributed to software development, data curation and methodology and review of the draft. Rhys Downham contributed to data

510 curation and review of the draft.

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515

Data availability

A data repository is available on Mendeley Data (DOI: 10.17632/cf5sk9n5bn.1)



Appendix A: Additional model detail

For this study, three related but distinct measures of water use are defined: diversions D , allocation use U and water applied W (Figure A1). Diversions are physical volumes of surface water extracted from river systems; water applied refers to volumes applied by irrigation farmers to crops, while allocation use reflects usage by water right holders of annual water allocations. Allocation use includes held environmental water which is effectively “used” (i.e., released from storage) but not diverted, while irrigation water applied excludes conveyance losses, but may also include groundwater applied to crops.

A.1: Allocation use, water applied and diversions

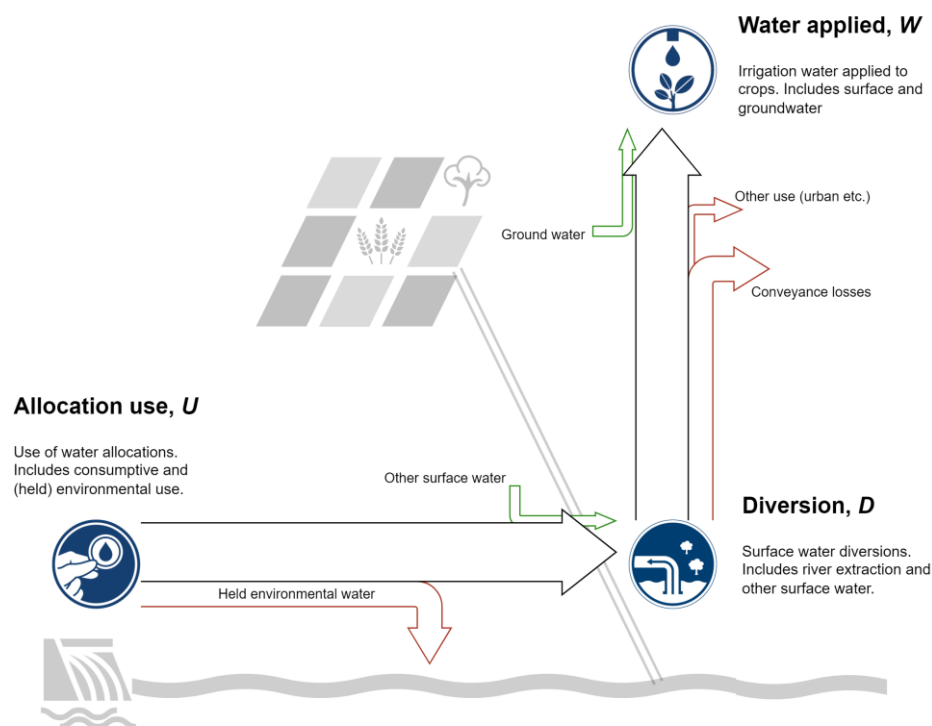


Figure A1: Three measures of water use: allocations, diversions and water applied



530 A.2 Other water allocation supply

While this study is concerned with water demand, some components of the demand model depend on historical estimates of water availability, particularly the predicted shadow prices \hat{P}_{it}^w in regions with inadequate price data. Although historical water accounting data (including monthly allocations) are readily available for most regions in the MDB, estimation of monthly water availability is not straight forward, requiring the imposition of accounting rules, simplifying assumptions and some statistical calibration.

Adapting the approach set out in Hughes et al., (2023) water allocations available for use at in year t month m can be defined as:

$$A_{iyt} = \sum_h C_{iyh} + \sum_h a_{iyh} \cdot E_{ih} - U_{iyt} + T_{iyt} - F_{iyt} + A_{iyt}^{other}$$

where C_{iyh} are allocations carried over from the previous (financial year), a_{iyh} are allocation percentages and E_{ih} water entitlements, U_{iyt} are monthly water use volumes, T_{iyt} monthly net inter-region trade volumes (net imports), and F_{iyt} user forfeits. Our data includes the two main entitlement classes h in each region (e.g., high and low reliability in Victoria, and General and High Security in NSW) which are combined to estimate total allocations.

Historical data is available on most of these components (see Hughes et al., 2023), although trade and forfeit data are only available annually, with monthly values imputed pro-rata. The unobserved A_{iyt}^{other} term represents other sources of allocations (beyond the two main entitlement classes) including those against conveyance (bulk) water entitlements, town and stock and domestic entitlements, and supplementary / non-regulated entitlements (water extracted directly from rivers or via flood-plain harvesting during periods of high flow). These latter sources of water allocation represent a larger share of supply in the northern MDB regions. Given historical data on all other components A_{iyt}^{other} can be computed as a residual by exploiting changes in opening carryover between years (see Hughes et al., 2023).

A.3 Environmental water demands

The water demand system focuses on consumptive (i.e., irrigation) use. While historical data on environmental water use is available, for any out-of-sample applications these would also need to be modelled. As part of this study simple reduced-form equations were also estimated to predict environmental water use as a function of held environmental water rights (entitlements and allocations) and rainfall:

$$U_{iy}^{env} = \beta^{env0} \cdot A_{iy}^{env} + \beta^{e1} A_{iy}^{env} \cdot R_{iy} + \beta^{e2} E_{iy}^{env} + \beta^{env3} E_{iy}^{env} \cdot R_{i,y-1}$$



$$A_{iy}^{env} = \sum_h \max_m \{a_{iym}\} \cdot \delta_{ihy} \cdot E_{ih}$$

$$E_{iy}^{env} = \sum_h \delta_{ihy} \cdot E_{ih}$$

560 where δ_{ihy} is the share of entitlement class h held by the environment in region i year y .

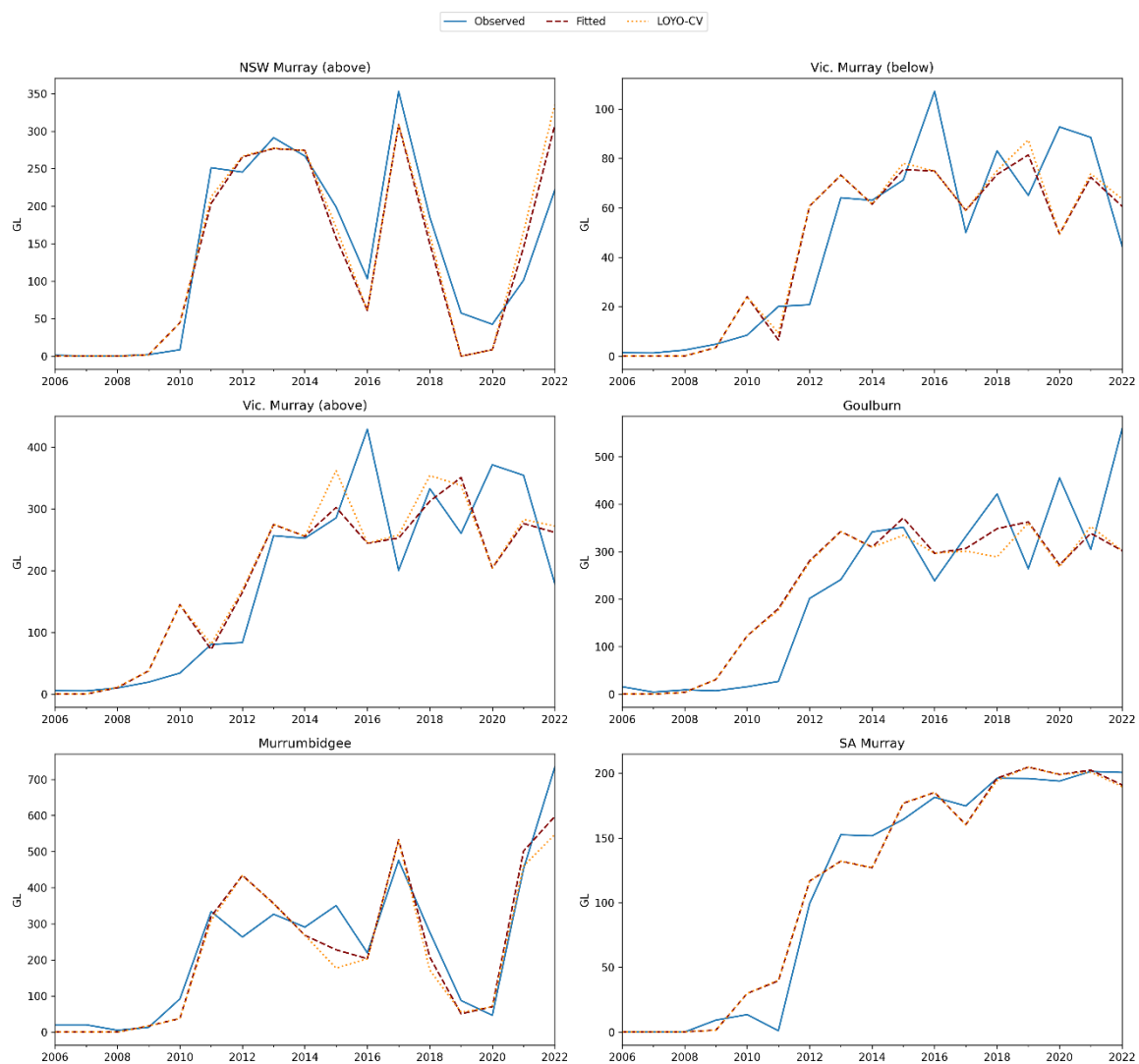
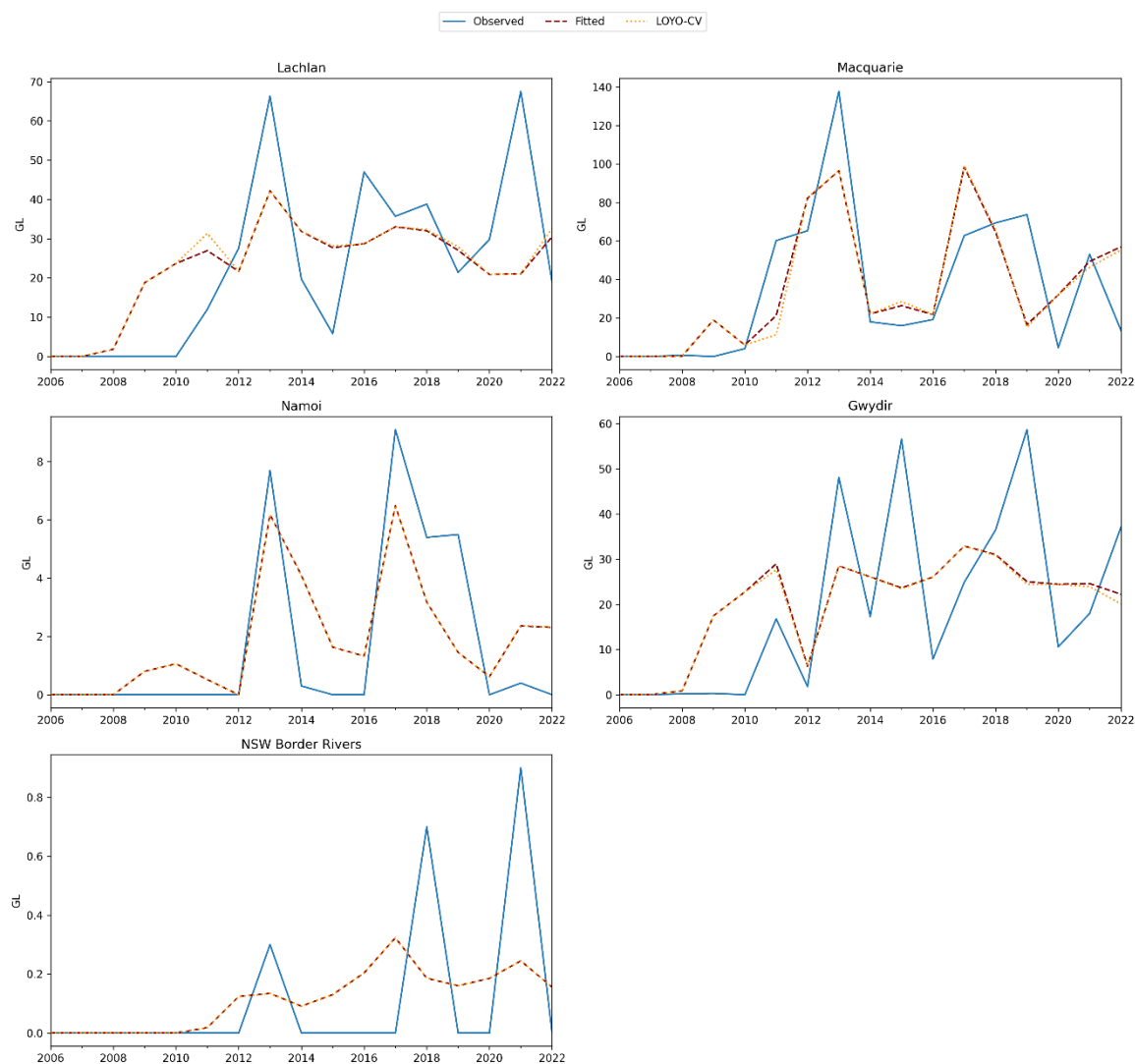


Figure A2: Observed and fitted environmental water use by region, 2004-05 to 2021-22



565 **Figure A3: Observed and fitted environmental water use by northern MDB region, 2004-05 to 2021-22**



A.4 Model reduced-form

The irrigation problem involves selecting crop areas and water use to maximise expected profit:

$$\max_{L_{ijt}^p, W_{ijt}} \pi_{ijt}$$

570 subject to equations 1 to 6.

The first order conditions can then be used to derive a set of demand functions, linking crop areas and water use with water and crop prices:

$$\frac{\partial \pi_{it}}{\partial W_{ijt}} = 0, \quad \frac{\partial \pi_{it}}{\partial L_{ijt}^p} = 0$$

575

$$0 \leq W_{ijm} \leq L_{ijm}^p \bar{w}_{ijm}$$

$$0 \leq L_{ijt}^p \leq \bar{L}_{ijt}$$

The water use functions for both the AD and WD forms are straightforward to derive:

$$W_{ijt} = \bar{w}_{ijt} \bar{L}_{ijt} \beta_j^{y1} \left(1 - \frac{w_{ijt}(P_{it}^w + \beta_i^{c0})}{P_{jt}^y \cdot \beta_{ij}^{y0} \cdot \dot{k}_{ijt}^s} \right) \quad (8 \text{ AD})$$

$$W_{ijt} = \bar{w}_{ijt} \cdot L_{ijt}^p \left(1 - \frac{\bar{w}_{ijt}(P_{ijt}^w + \beta_j^{c0})}{2 \cdot P_{jt}^y \cdot \beta_{ij}^{y0} \cdot \dot{k}_{ijt}^s} \right) \quad (8 \text{ WD})$$

580

With $m0$ denoting the planting month and h the harvest month, the first-order condition for L_{ijt}^p under the AD from can be written as:

$$\begin{aligned} & \mathbf{E}_{m=m0} \left[\frac{\beta_{ij}^{c1} \cdot \beta_{ij}^{c2}}{\bar{L}_{ijt}} + \sum_{m=m0}^h (1 - \mathbf{I}_{ijt}) w_{ijt} (\beta_{ij}^{c0} + P_{it}^w) \right] \\ &= \mathbf{E}_{m=m0} \left[\left(1 - \sum_{m=m0}^h \dot{k}_{ijt}^s \mathbf{I}_{ijt} \right) P_{jt}^y \beta_{ij}^{y0} \left(1 - L_{ijt}^p \cdot \frac{1}{\beta_j^{y1} \bar{L}_{ijt}} \right) \right] \end{aligned}$$

585

$$\mathbf{I}_{ijt} = \begin{cases} 0, & \text{if } L_{ijt}^w \geq L_{ijt}^p \\ 1, & \text{if } L_{ijt}^w < L_{ijt}^p \end{cases}$$



Here the first-order condition requires the expected marginal costs of planting a crop (including water and non-water costs) to equal the expected marginal revenues. Expectations are required because there is uncertainty at planting time over future prices P_{it}^w, P_{jt}^y , water requirements w_{ijt} and the occurrence of deficit irrigation and water stress. We simplify further by assuming expected prices over the cropping season are equal to prices in the planting month. We can then derive a function for L_{ijt}^p as

$$L_{ijt}^p = \beta_j^{y1} \bar{L}_{ijt} \left(1 - \frac{\beta_{ij}^{c2} + (\beta_{ij}^{c0} + P_{i,m0}^w) \cdot w_{ijt}^*}{P_{jt}^y \beta_{ij}^{y0} \cdot (1 - k_{ij,m0}^*)} \right) \cdot \left(1 + \frac{\beta_{ij}^{c1} \cdot \beta_{ij}^{c2} \cdot \beta_j^{y1}}{P_{jt}^y \beta_{ij}^{y0} \cdot (1 - k_{ij,m0}^*)} \right)^{-1} \quad (9)$$

$$w_{ij,m0}^* = \mathbf{E}_{m=m0} \left[\sum_{m=m0}^h (1 - \mathbf{I}_{ijt}) \bar{w}_{ijt} \right]$$

$$k_{ij,m0}^* = \mathbf{E}_{m=m0} \left[\sum_{m=m0}^h \dot{k}_{ijt}^s \mathbf{I}_{ijt} \right]$$

where $w_{ij,m0}^*$ are the expected (at the time of planting $m0$) annual water requirements and $k_{ij,m0}^*$ expected annual yield penalties.

Following a similar approach, we can derive the crop area planted function for the WD form as:

$$L_{ijt}^p = \bar{L}_{ijt} \cdot \frac{1}{\beta_{ij}^{c1} \beta_{ij}^{c2}} \cdot (P_{jt}^y \beta_{ij}^{y0} \cdot (1 - k_{ij,m0}^*) - \beta_{ij}^{c2} - (\beta_{ij}^{c0} + P_{ij}^w) \cdot w_{ij,m0}^*)$$

$$k_{ij,m0}^* = \mathbf{E}_{m=m0} \left[\sum_m \dot{k}_{ijt}^s \left(\frac{w_{ijt}}{\bar{w}_{ijt}} - 1 \right)^2 \right]$$

$$w_{ij,m0}^* = \mathbf{E}_{m=m0} \left[\sum_m w_{ijt} \right]$$

To proceed further we assume that the expectations terms in both equations can be approximated by a linear function of the price at the time of planting:

$$k_{ij,m0}^* = \beta_{ij}^{ek0} + \beta_{ij}^{ek1} P_{ijm0}^w$$

$$w_{ij,m0}^* = \beta_{ij}^{ew0} + \beta_{ij}^{ew1} P_{ijm0}^w$$



605 This leaves us with the following crop area functions for the AD and WD forms:

$$L_{ijt}^p = \beta_j^{y1} \bar{L}_{ijt} \left(1 - \frac{\beta_{ij}^{c2} + (\beta_{ij}^{c0} + P_{i,m0}^w) \cdot (\beta_{ij}^{ek0} + \beta_{ij}^{ek1} P_{ijm0}^w)}{P_{jt}^y \beta_{ij}^{y0} \cdot (1 - \beta_{ij}^{ek0} - \beta_{ij}^{ek1} P_{ijm0}^w)} \right) \times$$

$$\left(1 + \frac{\beta_{ij}^{c1} \cdot \beta_{ij}^{c2} \cdot \beta_j^{y1}}{P_{jt}^y \beta_{ij}^{y0} \cdot (1 - \beta_{ij}^{ek0} - \beta_{ij}^{ek1} P_{ijm0}^w)} \right)^{-1} \quad (9 \text{ AD})$$

$$L_{ijt}^p = \bar{L}_{ijt} \cdot \frac{1}{\beta_{ij}^{c1} \beta_{ij}^{c2}} \cdot (P_{jt}^y \beta_{ij}^{y0} \cdot [1 - \beta_{ij}^{ek0} - \beta_{ij}^{ek1} P_{ijm0}^w] - \beta_{ij}^{c2} - (\beta_{ij}^{c0} + P_{ij}^w) \cdot [\beta_{ij}^{ew0} + \beta_{ij}^{ew1} \cdot P_{ijm0}^w]) \quad (9 \text{ WD})$$

where $\beta_{ij}^{ek0}, \beta_{ij}^{ek1}, \beta_{ij}^{ew0}, \beta_{ij}^{ew1}$ are additional parameters to be estimated (as outlined in Appendix A.5 below).

610



A.5 Estimation

The parameter estimation problem is set-up as a non-linear minimization problem, where the parameters are chosen to minimize the weighted sum of squared prediction errors:

$$\min_{\beta} \epsilon^2$$

$$\epsilon^2 = \sum_{ijt} \left(\delta_{ijt}^L (L_{ijt} - \hat{L}_{ijt}) \right)^2 + \sum_{ijt} \left(\delta_{ijt}^W (W_{ijt} - \hat{W}_{ijt}) \right)^2 + \sum_{ijt} \left(\delta_{ijt}^Y (Y_{ijt} - \hat{Y}_{ijt}) \right)^2 +$$

$$\sum_{it} \left(\delta_{it}^U (U_{it} - \hat{U}_{it}) \right)^2 + \sum_{it} \left(\delta_{it}^{U^e} (U_{it}^e - \hat{U}_{it}^e) \right)^2 + \sum_{it} \left(\delta_{it}^D (D_{it} - \hat{D}_{it}) \right)^2 + \sum_{it} \left(\delta_{it}^P (P_{it}^W - \hat{P}_{it}^W) \right)^2$$

subject to Equations 1 to 11

Table A1: Parameters to be estimated

Parameter	Description / function
$\beta_i^{er0}, \beta_i^{er1}, \beta_i^{er2}$	Effective rainfall
$\beta_{ij}^{w0}, \beta_j^{w2}, \beta^{w3}$	Crop water requirements
$\beta_j^{s0}, \beta_j^{s1}$	Crop water stress
$\beta_{ij}^{y0}, \beta_j^{y1}, \beta_j^{y2}$	Crop yield response
$\beta_{ij}^{l0}, \beta_{ij}^{l1}$	Crop area constraint
$\beta_j^{c1}, \beta_j^{c2}$	Crop planting costs
β_i^{c0}	Water delivery charge
$\beta_i^{d0}, \beta_{im}^{d1}, \beta_i^{d2}$	Diversions
$\beta_i^{u0}, \beta_i^{u1}, \beta_i^{u2}, \beta_i^{u3}$	Water allocation use
$\beta_i^{env0}, \beta_i^{env1}, \beta_i^{env2}, \beta_i^{env3}$	Environmental water use

The estimation weights δ^L, δ^Y are scaled by mean application rates and yields respectively, such that all variables are in comparable units (i.e., ML). Weights are also increased linearly over-time to put greater emphasis on recent years of data.

In some northern basin regions (Macquarie, Namoi, Gwydir, Border Rivers) water market price data is deemed to be of limited quality and is replaced with an imputed “shadow” price \hat{P}_{it}^W estimated as a non-linear function of water availability (Equation 10). These shadow water prices are then used in-place of observed data (in Equations 8, 9, 8b, 9b). In all other regions, actual observed water market prices are used.



$$P_{it}^W = \beta_i^{P0} + \beta_i^{P1} \dot{A}_{it} + \beta_i^{P2} \log(\dot{R}_{it}^{12}) + \beta_i^{P3} y + \beta_i^{P4} \dot{A}_{it}^{\left(\frac{1}{\beta_i^{P5}-1}\right)} \quad (12)$$

$$P_{it}^W \geq 0$$

$$\dot{A}_{it} = \frac{A_{it}}{\sum_h E_{ih}}$$

where \dot{A}_{it} is the estimated monthly allocation water supply volume relative to entitlement volume, and \dot{R}_{it}^{12} the 12-month moving average of regional rainfall relative to the long run annual mean.

To determine the expected crop water requirements and water stress parameters $\beta_{ij}^{ek0}, \beta_{ij}^{ek1}, \beta_{ij}^{ew0}, \beta_{ij}^{ew1}$ an iterative bootstrapping approach is employed. First the main model parameters are estimated as above given an initial guess for the expectation terms ($k_{ijt}^* = 0, w_{ijt}^* = \bar{w}_{ijt}$). Next $\beta_{ij}^{ek0}, \beta_{ij}^{ek1}, \beta_{ij}^{ew0}, \beta_{ij}^{ew1}$ are estimated by solving:

$$\min_{\beta_{ij}^{ek0}, \beta_{ij}^{ek1}, \beta_{ij}^{ew0}, \beta_{ij}^{ew1}} \sum_{ijt} \left(\delta_{ijt}^k (k_{ijt}^* - \hat{k}_{ijt}^*) \right)^2 + \sum_{ijt} \left(\delta_{ijt}^w (w_{ijt}^* - \hat{w}_{ijt}^*) \right)^2$$

This process then proceeds iteratively until convergence.



Appendix B: Additional validation results

640 **Table B1: Symmetric Median Absolute Percentage Error (SMAPE) for annual allocation use U , diversion D and water applied W by region. Fitted and cross-validated (Leave-One-Year-Out, LOYO)**

Region	U (Fitted)	U (LOYO)	D (Fitted)	D (LOYO)	W (Fitted)	W (LOYO)
NSW Murray (above)	14.4	13.3	4.6	8.2	11.2	14.4
Vic. Murray (above)	15.5	20.3	10.0	14.1	22.1	22.9
Vic. Murray (below)	8.9	14.3	7.5	8.0	6.1	6.6
NSW Murray (below)	8.0	10.0	9.4	11.0	11.1	12.6
Murrumbidgee	3.9	11.1	5.7	6.7	9.0	10.4
Goulburn	10.1	12.8	12.2	11.2	11.7	15.0
SA Murray	5.4	5.9	5.4	5.8	10.6	11.6
Southern MDB	8.1	12.7	8.4	10.0	11.2	12.3
Lachlan	23.8	21.0	29.0	33.5	14.5	17.0
Macquarie	15.6	24.6	13.6	25.7	16.9	21.4
Namoi	24.9	24.9	11.2	10.2	16.9	25.8
Gwydir	14.8	20.0	17.7	12.8	20.1	26.3
NSW Border Rivers	22.9	27.7	19.3	23.3	29.2	42.0
Northern MDB	18.2	24.9	17.6	20.5	17.8	25.8
MDB	12.1	17.1	10.9	12.8	13.4	16.6

Table B2: Symmetric Median Absolute Percentage Error (SMAPE) for water applied, area planted and production by crop. In-sample and cross-validated (Leave-One-Year-Out)

Region	W (Fitted)	W (LOYO)	L (Fitted)	L (LOYO)	Y (Fitted)	Y (LOYO)
Almonds	3.6	4.1			6.0	6.3
Cotton	14.7	27.8	11.2	22.5	15.0	22.6
Grapes	4.9	5.9			14.8	16.6
Hay	16.5	21.7	17.4	18.5	33.7	28.9
Horticulture	4.6	5.2			23.0	26.1
Other Crops	14.7	20.2	10.3	10.3	21.2	21.7
Pasture	13.4	15.7	11.2	22.2		
Rice	14.6	15.7	18.0	17.6	23.3	29.5
Vegetables	3.4	5.5			23.3	23.9
All crops	6.9	9.2	3.3	1.9		

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Table B3: R^2 for annual allocation use, diversion and water applied by region. In-sample and cross-validated (Leave-One-Year-Out)

Region	U (Fitted)	U (LOYO)	D (Fitted)	D (LOYO)	W (Fitted)	W (LOYO)
NSW Murray (above)	0.94	0.84	0.90	0.82	0.93	0.85
Vic. Murray (above)	0.41	0.32	0.67	0.60	0.51	0.43
Vic. Murray (below)	0.52	0.28	0.67	0.59	0.84	0.75
NSW Murray (below)	0.89	0.76	0.81	0.70	0.73	0.66
Murrumbidgee	0.95	0.85	0.93	0.86	0.83	0.74
Goulburn	0.73	0.65	0.49	0.38	0.75	0.66
SA Murray	0.89	0.81	0.51	0.25	0.06	-0.06
Southern MDB	0.94	0.90	0.95	0.92	0.95	0.92
Lachlan	0.81	0.71	0.74	0.60	0.23	0.25
Macquarie	0.86	0.68	0.79	0.58	0.70	0.44
Namoi	0.83	0.65	0.83	0.58	0.68	0.28
Gwydir	0.87	0.78	0.90	0.79	0.90	0.77
NSW Border Rivers	0.45	0.12	0.50	0.15	0.72	0.30
Northern MDB	0.84	0.71	0.83	0.67	0.80	0.61
MDB	0.96	0.93	0.96	0.94	0.95	0.92

650 **Table B4: R^2 for water applied, area planted and production by crop. In-sample and cross-validated (Leave-One-Year-Out)**

Region	W (Fitted)	W (LOYO)	L (Fitted)	L (LOYO)	Y (Fitted)	Y (LOYO)
Almonds	0.96	0.94			0.95	0.94
Cotton	0.87	0.75	0.85	0.28	0.86	0.67
Grapes	0.73	0.51			-0.58	-1.00
Hay	0.45	0.30	0.30	0.06	-0.12	-0.45
Horticulture	0.31	0.04			-1.16	-1.55
Other Crops	0.14	-0.19	-0.02	-0.34	-0.04	-0.31
Pasture	0.81	0.74	0.46	0.07		
Rice	0.96	0.89	0.94	0.88	0.85	0.80
Vegetables	0.84	0.72			-5.43	-5.19
All crops	0.93	0.89				

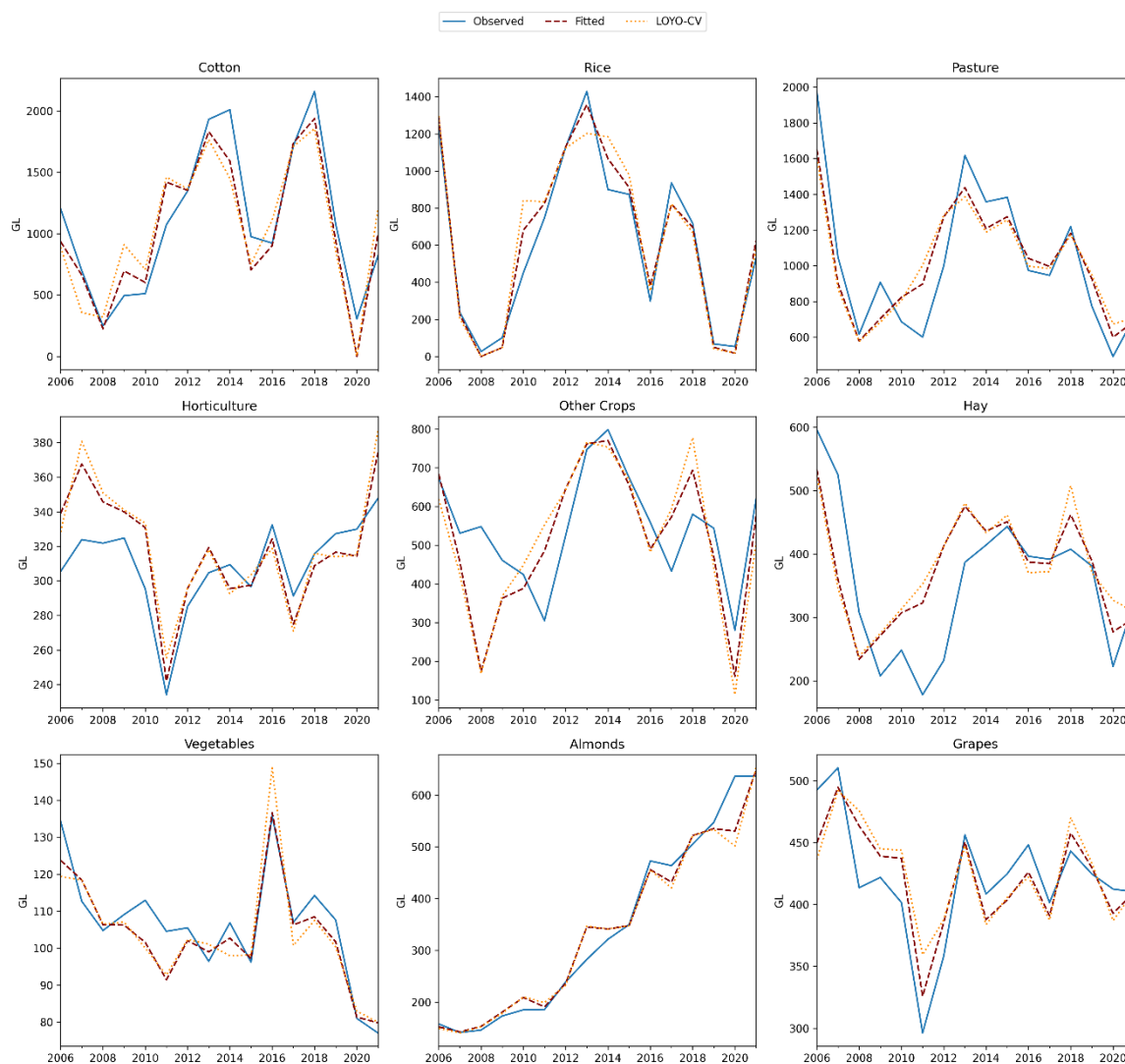


Figure B1: Observed, fitted and hindcast annual water applied W_{ty} by crop

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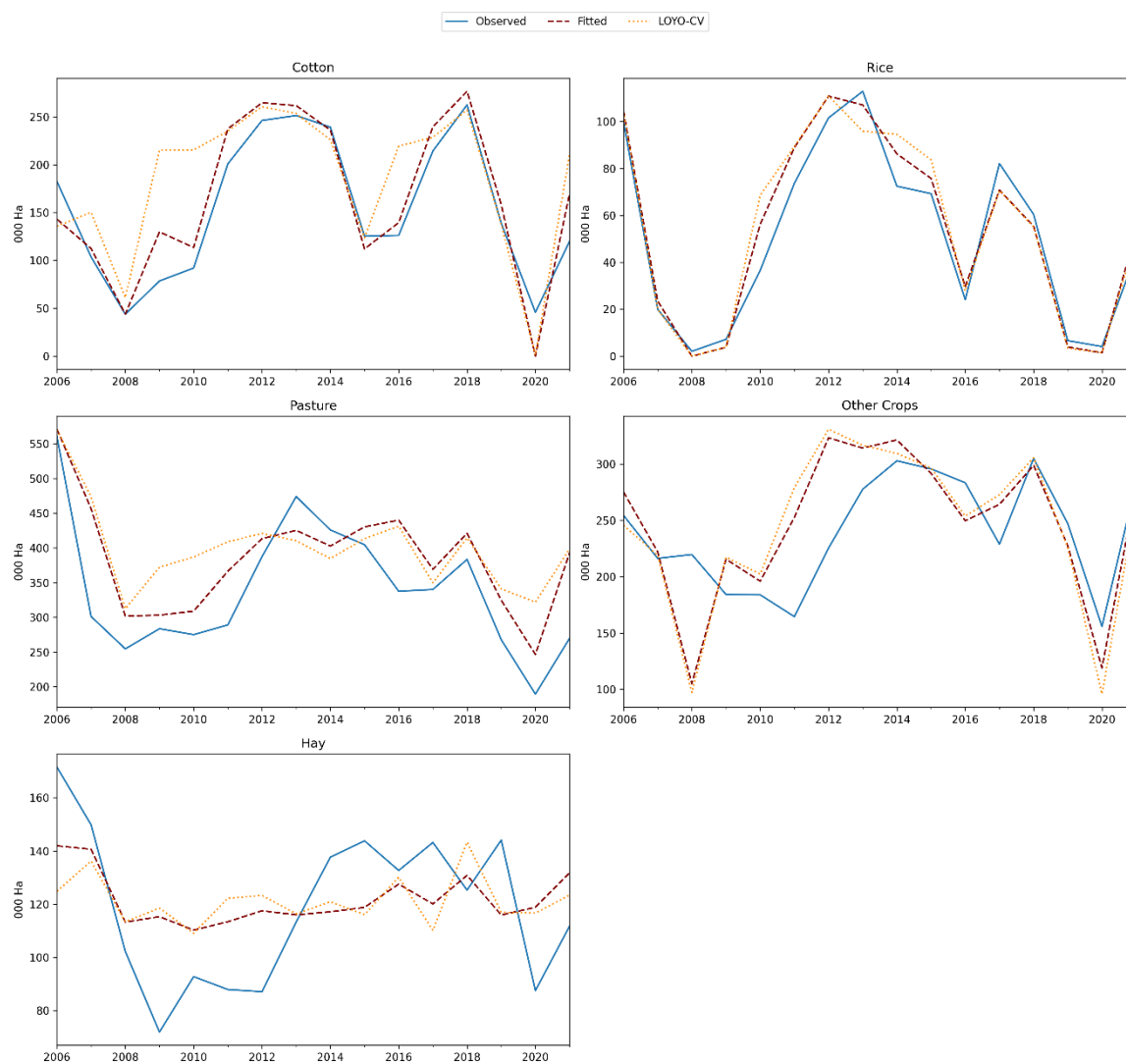


Figure B2: Observed, fitted and hindcast annual area planted by crop

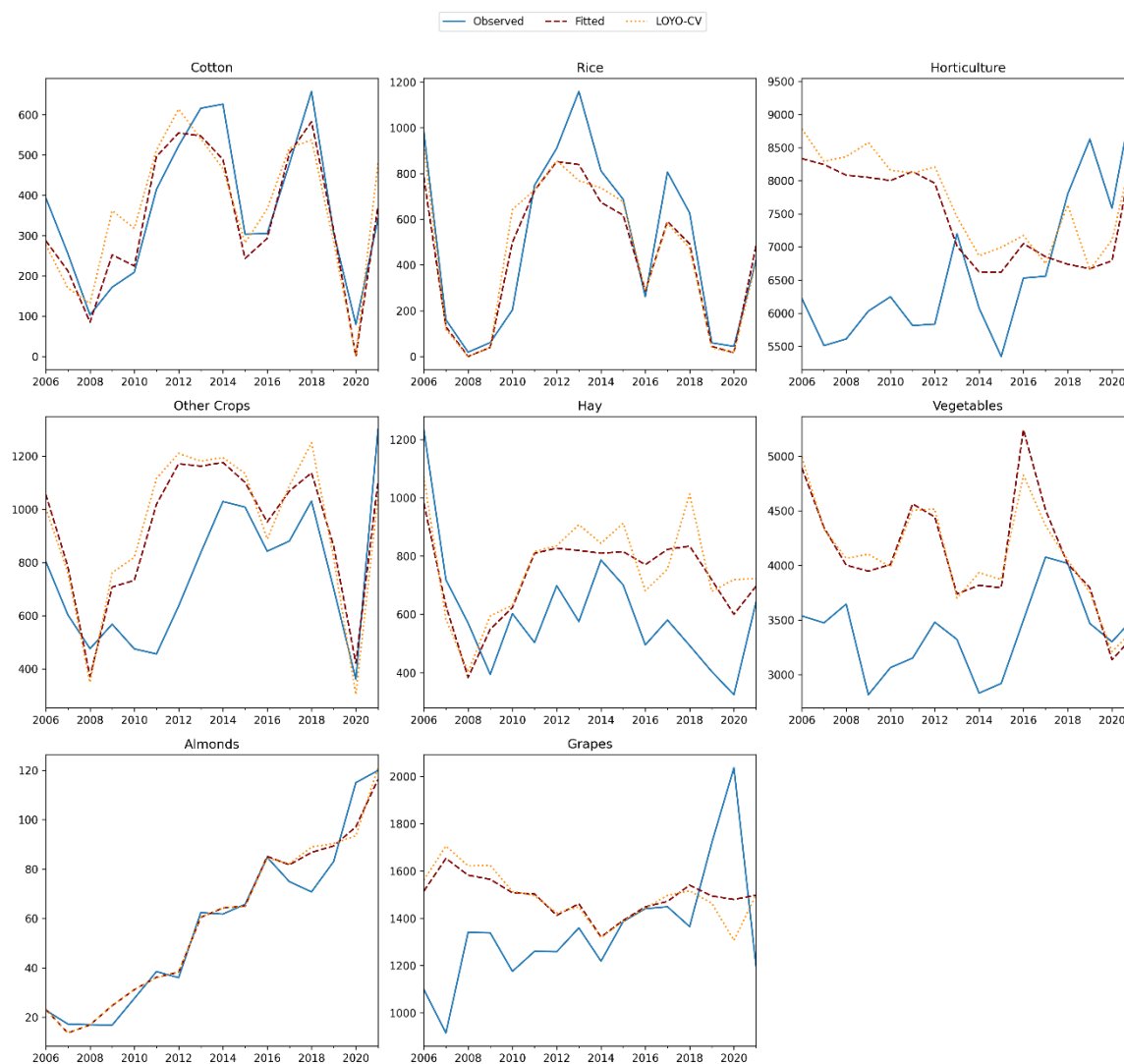


Figure B3: Observed, fitted and hindcast production (quantity) by crop



Table B5: R^2 and SMAPE for monthly diversions. In-sample and cross-validated (Leave-One-Year-Out)

Region	R^2		SMAPE	
	D (Fitted)	D (LOYO)	D (Fitted)	D (LOYO)
NSW Murray (above)	0.85	0.79	37.0	42.5
NSW Murray (below)	0.70	0.66	33.5	36.0
Vic. Murray (above)	0.76	0.71	31.2	33.6
Vic. Murray (below)	0.83	0.80	19.9	20.9
Total Murray	0.88	0.85	28.8	32.3