

Response to Referee 1

1. Summary of the manuscript

Title: Basin-Scale Geometric Focusing: A Probabilistic-Geometric Framework for Global Tsunami Hazard Assessment and the 2025 Kamchatka Peninsula Tsunami

Authors: A. Abdolali, M.-A. Y.-H. Lam, U. Kadri, M. Malej, M. Filimonov, F. Shi

The manuscript proposes a hybrid probabilistic-geometric framework to identify coastal areas exposed to transoceanic tsunamis at global scale, combining Gutenberg–Richter-type seismic weighting with ray-tracing simulations. The central idea is to use ray density and coastal ray-termination points as a rapid proxy for bathymetry-induced focusing or shadowing. The approach is validated using the 2025 Kamchatka event through phase-corrected FUNWAVE-TVD simulations and comparison with DART and tide-gauge records. The work shows a reasonable spatial correlation between the geometric proxy and the maximum amplitudes obtained from the hydrodynamic model. The article is interesting and potentially useful as a rapid screening tool, although it still requires a more rigorous delimitation of its scope, of the probabilistic component, and of what can properly be referred to as “hazard assessment”.

Response: We are very grateful to the reviewer for his/her constructive critiques and comments. In the following, we state the referee’s comments (in black) followed by the response and actions taken (in blue).

2. Structure of the manuscript by sections

- 2.1:** The introduction presents the problem of transoceanic tsunami propagation and the role of bathymetry in energy focusing and defocusing. It also justifies the use of ray tracing as an alternative to full numerical models and positions the contribution of the study.
- 2.2:** Section 2: Probabilistic Tsunami Hazard Assessment. This section builds the probabilistic component from a historical catalogue, a global Gutenberg–Richter law, and a spatial KDE over a curve representing fault zones. The resulting output is an annual rate, λ , used as a weight for the rays.
- 2.3:** Section 3: Ray Tracing Methodology. This section formulates the ray equations, describes the mesh and numerical implementation, and presents several global experiments designed to separate the effects of fault geometry, bathymetry, and probabilistic weighting.
- 2.4:** Section 4: M8.8 Kamchatka Peninsula event. This section introduces two source scenarios (NOAA and USGS), describes the FUNWAVE simulation, applies a phase correction, and compares results with observations to evaluate the predictive capability of the geometric approach.
- 2.5:** Section 5: Conclusions. The conclusions summarize the findings and present the method as an efficient tool for prioritizing exposed areas. The closing section is generally clear, although in some places it overextends the actual scope of the method.

Response: We thank the reviewer for the accurate summary and the section-by-section breakdown.

3. General comments

- 3.1:** The article addresses a relevant problem with an attractive methodological combination: using ray tracing to capture the geometric organization of tsunami energy and adding seismic weighting to prioritize more likely sources. The validation with a real event and with a higher-fidelity hydrodynamic model adds credibility, and the comparison with real observations clearly improves the manuscript.

Response: We agree with the reviewer and thank them for recognizing the value of the methodology and the observational validation. In addition to the Kamchatka event, based on reviewer feedback, we have now added the 2011 Tohoku event as a secondary validation case to demonstrate the model’s robustness. This addition is reflected in the Abstract (Page 1, Line 6), the Introduction (Page 3, Line 78), and a newly added Section 5 (Page 20, Lines 390–411) in the revised manuscript.

3.2: As currently presented, the method appears closer to an “exposure screening” tool, or a preliminary prioritization tool for transoceanic corridors, than to a complete Probabilistic Tsunami Hazard Assessment framework. Uncertainties are not propagated, aleatory and epistemic variability are not explicitly treated, and the final coastal indicator is a normalized geometric proxy rather than a directly interpretable physical intensity measure. I recommend either lowering the expectations in the title and abstract and moderating the conclusions (what would be enough), or substantially strengthening the probabilistic component.

Response: We completely agree with this fundamental distinction. We have removed claims of a full “Probabilistic Tsunami Hazard Assessment” (which requires physical intensity exceedance probabilities) and reframed the manuscript around “Exposure Assessment” and “Focusing Potential.” We updated the Title (Page 1, Lines 1–3), Abstract (Page 1, Lines 3 and 11), and manuscript body (e.g., Page 2, Lines 32 and 44–46) to ensure the geometric proxy is accurately described as a measure of relative coastal exposure and transoceanic focusing corridors.

3.3: The probabilistic formulation requires stronger justification. The use of a single global b -value, a completeness threshold selected by visual inspection, and a KDE over a curve constructed from epicentres substantially simplifies the problem. This simplification may be acceptable for an exploratory study, but it should be better justified, preferably through sensitivity analyses that help clarify the limitations of the approach.

Response: We completely agree with the referee that relying on visual inspection to determine the catalog completeness is insufficient. In the revised manuscript, we have updated our methodology to formally assess the Magnitude of Completeness (M_c).

Completeness Threshold & b -value: We utilized the Maximum Curvature (MAXC) method (Woessner and Wiemer, 2005), which identified the peak of the non-cumulative frequency–magnitude distribution near $M_w = 7.05$. However, because raw MAXC estimates often fall within the catalog’s detection ‘roll-over’ region and can underestimate the true magnitude of complete reporting, we adopted a conservative operational threshold of $M_c = 7.4$. We now state explicitly in the text that this threshold represents a modeling choice rather than an exact, immutable catalog property. To directly address the reviewer’s request for sensitivity analysis, we evaluated the stability of the estimated b -value over nearby completeness thresholds (e.g., $M_c = 7.3$ to 7.5), confirming that the value remains stable and the principal basin-scale exposure patterns are not materially altered. Using the $M_c = 7.4$ dataset ($N = 212$ events), the b -value was calculated via the Aki-Utsu maximum likelihood estimator along with its formal standard error ($\sigma_b = b/\sqrt{N}$), yielding $b = 1.05 \pm 0.07$. This updated mathematical formulation and its associated uncertainties have been fully integrated into Section 2.1 (Page 5, Lines 96–105) and Figure 1d.

1D KDE Formulation: Regarding the KDE, we agree that justification was needed regarding the projection of 2D epicentral scatter onto a 1D curve. While resolving 3D geometry is critical for local nearshore modeling, this dimensional reduction is physically and computationally justified for basin-scale, transoceanic exposure screening. Far-field hazard is governed primarily by macro-scale, along-trench seismic productivity, making a 1D longitudinal projection an established, robust methodology for large-scale analysis (Scourse et al. 2018). Furthermore, the 500 km smoothing bandwidth (σ) is physically grounded in subduction zone scaling laws. Because great megathrust earthquakes ($M_w \geq 8.5$) are restricted by down-dip width saturation (~ 200 km), rupture propagation is forced longitudinally along the trench (Allen and Hayes, 2017). Empirical scaling formulas confirm characteristic rupture lengths of 440–588 km for M_w 8.8–9.0 events. Our 500 km bandwidth physically scales the probability density to match these realistic rupture lengths, explicitly avoiding the severe epicentral over-fitting that occurs with narrower bandwidths (which artificially treat massive ruptures as point-sources). This is quantitatively validated by our parametric sensitivity sweep: spatial variance stabilizes at the 500 km threshold, while correlation analysis ($r > 0.95$) demonstrates that macro-scale hazard clustering is statistically stable and independent of the chosen bandwidth. These physical justifications and sensitivity analyses have been added to the revised manuscript in Section 2.2 (Pages 5–6, Lines 119–137) and Appendix A (Figure A1).

3.4: The validation is promising but still limited for supporting global claims. The use of a single case study (the 2025 Kamchatka event) and one main spatial metric (Spearman’s $\rho = 0.66$) is a useful first step, but it is not sufficient to demonstrate general robustness. The manuscript would benefit from a more quantitative discussion of sensitivity to the selected wave period, the 500 m termination criterion, the effective source length and width, and the stability of the results with respect to ray-tracing parameters.

Response: We agree with the reviewer that validation required strengthening. To support global claims, we have added the 2011 Tohoku tsunami as a secondary validation case, comparing ray densities to FUNWAVE maximum wave envelopes and evaluating them statistically (Section 5, Pages 20–21, Lines 390–411).

Wave-period sensitivity and ray-tracing stability: We agree that the selected period requires explicit sensitivity assessment. Long-period seismic tsunamis are generally weakly dispersive during deep-ocean

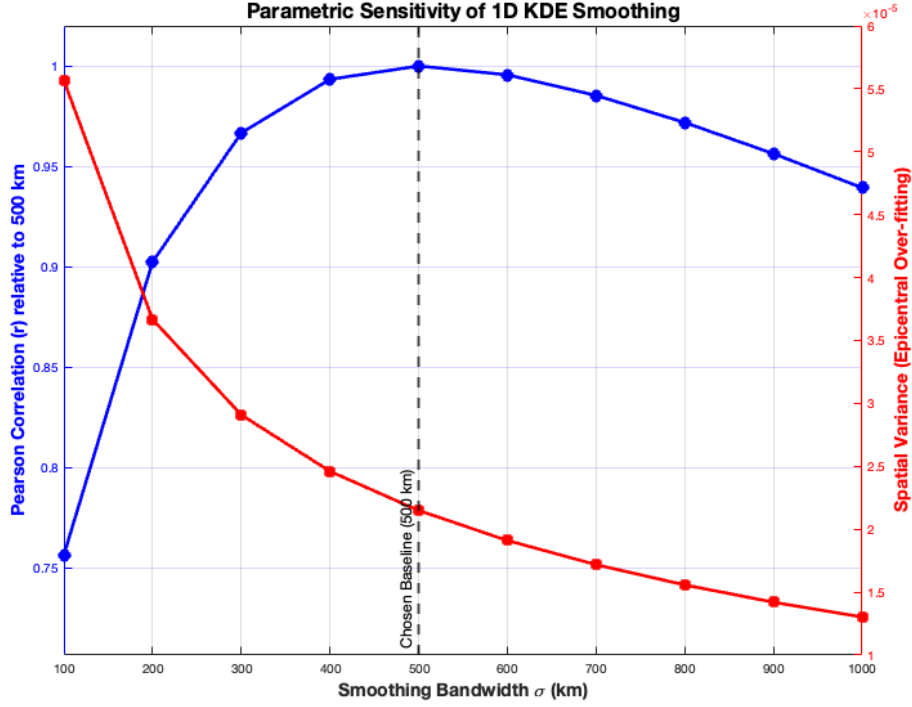


Figure 1: Parametric sensitivity of the 1D Kernel Density Estimation (KDE) smoothing bandwidth (σ). The red curve (right axis) shows spatial variance; values stabilize at the 500 km baseline, consistent with characteristic rupture lengths of great (M_w 8.8–9.0) megathrust events. The blue curve (left axis) shows the Pearson correlation coefficient ($r > 0.95$) relative to the 500 km baseline, demonstrating geographic stability across the bandwidth spectrum.

propagation, so bathymetric refraction is expected to dominate the principal basin-scale pathways. Nevertheless, trajectories may retain some period dependence through the finite-depth dispersion relation. We therefore evaluated representative periods within the adopted range and found that the principal focusing corridors remain stable, although local ray densities and coastal termination patterns vary. For the Kamchatka and Tohoku validation cases, the characteristic periods were estimated independently from the source dimensions and observational spectra (Section 3.3, Page 9, Lines 204–208).

500m cutoff: We agree that this criterion needed quantitative justification. We expanded the text to clarify that the scope of this framework is strictly basin-scale transoceanic focusing. As tsunami waves enter water shallower than 500 m, they require a proportionally higher resolution mesh to resolve accurately. Implementing such high resolution along global coastlines would impose heavy computational costs, directly defeating the primary purpose of this study as a rapid, efficient exposure screening tool. Furthermore, continuing to propagate rays into these shallow zones using our existing coarse deep-water mesh would yield unphysical and inaccurate results. Therefore, the 500 m cutoff acts as a necessary and strict “hand-off” boundary, safely isolating basin-scale focusing from local nearshore amplification that requires specialized high-resolution hydrodynamic modeling (Section 3.3, Page 9, Lines 211–221).

Source dimensions: We agree and have clarified in the text how the effective source dimensions (L_{eff} and W_{eff}) are inherently captured within the 1D kinematic ray release. First, the effective source length dictates the physical extent of the ray-release line; by seeding rays along this exact fault length, the spatial footprint of the rupture is naturally preserved. Second, the effective dimensions govern the initial tsunami wavelength. As detailed in our methodology, L_{eff} and W_{eff} are used to calculate the characteristic length (L_c), which then dictates the exact characteristic wave period assigned to the rays (e.g., 53 minutes for Kamchatka). Thus, releasing rays perpendicularly along the fault strike natively encodes both the 2D source dimensions and the physical directivity (Section 4, Page 13, Lines 288–297).

- 3.5:** The manuscript would benefit from a more rigorous and consistent use of the terms hazard, exposure, and risk. In several places, the ray-based output is presented as “hazard”, whereas the actual model output is a normalized geometric proxy of coastal ray convergence. Unless exceedance probabilities of physical intensity measures are computed at coastal locations, the terminology should be adjusted toward “relative exposure”, “focusing potential”, or “screening-level hazard indicator”.

Response: We completely agree. We have conducted a thorough review of the manuscript and systematically replaced "hazard" with "relative exposure," "focusing potential," and "exposure screening" when referring to the ray-tracing outputs (e.g., Title, Page 1, Line 2; Abstract, Page 1, Line 3; Section 2 title, Page 3, Line 81).

- 3.6:** The writing is generally understandable, but in some places it is repetitive. The introduction repeats the computational-efficiency argument for ray tracing several times. The paragraphs around the discussion of numerical models, Satake (1988), and Tehranirad et al. (2015) could be shortened and merged into a more concise rationale.

Response: We agree. We have consolidated the three separate paragraphs discussing the computational efficiency of ray tracing in the introduction into a single, concise rationale summarizing the transition from Satake (1988) to the Boussinesq benchmarking by Tehranirad et al. (2015) in the revised manuscript (Introduction, Page 2, Lines 37–44).

- 3.7:** The discussion of Figure 3 is informative but somewhat over-narrative. It could be shortened and made more quantitative, for example, by reporting changes in normalized coastal focusing for selected regions rather than relying mainly on descriptive expressions.

Response: We agree with this critique. We have revised the discussion of Figure 3, replacing qualitative adjectives with quantitative metrics. The text now explicitly reports the percentage changes in the normalized exposure index for key regions (e.g., the artificial overestimation along Northeast Australia in the uniform scenario vs. the bathymetry-corrected reduction) in Section 3.3 (Page 10, Lines 248–261).

4. Main comment

My main concern is that the manuscript currently makes claims that are broader than what is demonstrated by the analysis. In its present form, the work reasonably validates a geometric proxy for transoceanic coastal exposure, but not a complete global probabilistic hazard framework. This distinction is important because it affects the title, the abstract, and the interpretation of the results. The authors should either substantially reinforce the probabilistic treatment and uncertainty quantification, or reformulate the scope of the manuscript as a global screening/prioritization tool for transoceanic tsunami exposure.

Response: We agree with the reviewer's concern. We have lowered the tone of the manuscript, reformulating the scope to strictly present transoceanic tsunami exposure, as recommended, particularly throughout the Abstract (Page 1) and Conclusions (Pages 21–22, Lines 407–422).

5. Specific comments

- 5.1:** Lines 1–3 (title): There is a missing space in "AProbabilistic-Geometric". In addition, the title is somewhat too ambitious. I suggest replacing "Global Tsunami Hazard Assessment" with "Global Tsunami Exposure Screening" or an equivalent formulation, unless the probabilistic component is substantially strengthened.

Response: We agree. Thank you for catching the typo. We have fixed the spacing and changed the title to "Basin-Scale Geometric Focusing: A Probabilistic-Geometric Framework for Global Tsunami Exposure Assessment and the 2025 Kamchatka Peninsula Tsunami" to accurately reflect the scope of the work (Page 1, Lines 1–3).

- 5.2:** Lines 5–10 (abstract): The abstract works well overall, but it mixes "hazard", "exposure", and "coastal focusing patterns" as if they were equivalent. The authors should clarify from the outset what variable the ray model predicts and what it does not predict.

Response: We agree. We have revised the abstract to cleanly separate these concepts. We explicitly state from the outset that the ray model maps relative geometric convergence (focusing potential and exposure) rather than absolute wave height probabilities (Abstract, Page 1, Lines 1–11).

- 5.3:** Lines 22–33: The exclusion of non-seismic tsunamis and nearshore processes is well motivated, for which decisions is the method suitable? And not?.

Response: We agree with the need to explicitly define the method's scope and utility, and we have expanded the introduction to clearly delineate these boundaries. Specifically, we clarified that non-seismic tsunamis (e.g., submarine landslides and volcanic eruptions) are highly dispersive and typically have less influence on cross-basin propagation; therefore, this framework is restricted to seismically generated tsunamis. Consequently, this method is highly suitable for rapid, basin-scale exposure analysis, probabilistically identifying the most exposed coastal areas, and prioritizing distant coastlines for high-resolution modeling using rapid geometric computations. Conversely, it is explicitly not suitable for assessing non-seismic events due to their dispersive nature, nor is it appropriate for evaluating near-source impacts,

site-specific inundation mapping, or local evacuation planning, which require specialized near-field hydrodynamic tools (Introduction, Page 2, Lines 23–36).

- 5.4: Lines 35–55: This part repeats the justification for ray tracing several times. It could be shortened considerably without losing content. The central argument is already clear from the references to Satake (1988) and Tehranirad et al. (2015).

Response: We agree. We have consolidated the manuscript to avoid repetitive arguments (Introduction, Page 2, Lines 37–44).

- 5.5: Lines 61–75: The introduction of the probabilistic component is well placed, but a sentence is needed to clarify that λ is not a direct probability of coastal amplitude exceedance.

Response: We agree. We have added a clarifying sentence to the introduction of the probabilistic component, explicitly stating that λ serves as a source weighting factor to calculate relative ray density, and does not represent a direct probability of coastal amplitude exceedance (Page 3, Lines 63–65).

- 5.6: Lines 94–106: In Figure 1 and its caption, magnitude is described as being on the “Richter scale”. For large events, this should be revised and the appropriate magnitude scale should be used, most likely Mw. In addition, “AMagnitude of Completeness” requires typographical correction.

Response: We agree. We have replaced all mentions of “Richter scale” with “Moment Magnitude (M_w)” and fixed the typo in Section 2.1 (Page 5, Lines 96–105) and the Figure 1 caption (Page 4, Line 7).

- 5.7: Lines 109–126: The construction of the curve Γ and the KDE is a delicate parts of the manuscript. how Γ is generated?,

Response: We agree it needed clarification. We clarified in the text that the curve Γ was constructed by tracing major tsunamigenic fault systems and subduction interfaces globally, discretizing this path into 25 km intervals. To account for the spatial scatter of historical earthquakes, epicenters falling within a 400 km search radius were orthogonally projected onto the nearest segment of this curve prior to applying the 1D Gaussian smoothing (Section 2.2, Page 5, Lines 113–117).

- 5.8: Lines 184–191: The normalized ray termination count is used as a proxy for amplitude. This is a crucial assumption and deserves a more nuanced discussion. Is it a proxy for maximum amplitude, incident energy, or relative probability of impact?

Response: We agree this is a crucial assumption. The normalized ray-termination count is interpreted as a relative geometric focusing or exposure index. It indicates where ray trajectories preferentially converge along the coastal boundary, but it is not a probability of impact, an absolute wave-amplitude estimate, or a direct measure of incident energy flux. We have revised the manuscript to make this distinction explicit (Section 3.3, Page 10, Lines 220–224).

- 5.9: Lines 230–240: The summary of the Kamchatka event includes information on impacts and damage that should be supported by specific references if retained.

Response: We agree. Citations have been added for the run-up heights (e.g., Severo-Kurilsk, Hawaii) and impacts, referencing the NOAA/NCEI Global Historical Tsunami Database and USGS event summaries (Section 4, Page 12, Lines 269–273).

- 5.10: Lines 284–303: The phase correction appears useful, but the current presentation mixes several elements (compressibility, elasticity, shortest path, and DART observations) in a dense block. I suggest separating more clearly the method, the data, and the result of the correction.

Response: We agree and have streamlined this block. We now briefly summarize that standard Boussinesq models assume an incompressible ocean over a rigid bed, causing theoretical phase speed overestimations. We reference Abdolali et al. (2019, 2026) for the formulation incorporating water compressibility, Earth elasticity, and background density, separating the pure kinematic time-shift application from the hydrodynamic amplitude outputs (Section 4.1.1, Page 16, Lines 328–336).

- 5.11: Lines 333–340: Spearman’s $\rho = 0.66$. How many coastal points are used? What is the statistical significance? Does it change if selected regions are excluded or if the coastal proximity criterion is modified?

Response: We agree this needed statistical rigor. To address the coastal proximity criterion and strictly isolate far-field propagation physics from localized coseismic displacement, we explicitly excluded all coastal points within a 1,000 km radius of the epicenter. Applying this region exclusion filter left 161 valid far-field coastal observation points for the analysis. For these 161 points, the spatially corrected Spearman correlation is $\rho = 0.65$. To determine the statistical significance while properly accounting for

the spatial autocorrelation of coastal wave heights, we implemented a rigorous spatial block-bootstrap resampling scheme (partitioning the coastline into contiguous 500 km segments). This methodology yielded an unbiased 95% confidence interval of [0.35, 0.83]. Because the confidence interval is strictly positive and bounded well above zero, the monotonic relationship is highly statistically significant ($p < 0.01$). The details of this proximity filtering and the statistical significance are now fully documented in the revised Section 4.2 (Page 19, Lines 366–376). Following the reviewer suggestion, we added Tohoku 2011 event and performed similar analysis for it in section 5.

6. Typos and local style improvements

6.1: Review spacing and typographical errors throughout the manuscript, including “large-scale ray-tracing”, “farfield”, “AMagnitude”, and “wether”.

Response: Corrected.

6.2: Standardize the use of “far-field” and “near-field” throughout the text.

Response: Corrected.

7. Final assessment

I recommend Major revisions. The manuscript presents an interesting and potentially useful probabilistic-geometric framework for the rapid identification of transoceanic tsunami focusing corridors. The combination of recurrence-based source weighting, ray tracing, FUNWAVE simulations, and observational comparison is valuable. However, the current framing is broader than what the analysis demonstrates. In particular, the manuscript should more clearly distinguish between probabilistic tsunami hazard assessment, exposure screening, and relative geometric focusing. The probabilistic component remains simplified, the uncertainty and sensitivity analysis is limited, and the validation relies mainly on one event and one principal spatial correlation metric. These issues affect the interpretation of the results and the claims made in the title, abstract, and conclusions. I consider major revision appropriate. The manuscript has clear potential if the scope is reframed and the methodological assumptions are better justified.

Response:

We appreciate the referee’s constructive and detailed comments. We have thoroughly revised the manuscript in response to these suggestions.

Response to Referee 2

1. Summary and Recommendation

Title: Basin-Scale Geometric Focusing: A Probabilistic-Geometric Framework for Global Tsunami Hazard Assessment and the 2025 Kamchatka Peninsula Tsunami

Authors: A. Abdolali, M.-A. Y.-H. Lam, U. Kadri, M. Malej, M. Filimonov, F. Shi

This manuscript presents a hybrid probabilistic-geometric framework that couples a simplified Probabilistic Tsunami Hazard Assessment (PTHA) with global wave ray tracing to map far-field coastal tsunami exposure and validates the approach against the 2025 M8.8 Kamchatka event using FUNWAVE-TVD simulations, DART records, and tide-gauge observations.

The paper is clearly written, timely, and addresses a genuine operational need: a computationally cheap screening tool to prioritise where expensive high-fidelity inundation modelling should be applied. The experimental design that isolates fault geometry, bathymetric refraction, and recurrence weighting (Fig. 3) is clean and pedagogically effective, and the multi-source validation (numerical model + two independent observational networks) is in my opinion a good practice. I also found valuable the explicit phase-lag correction for ocean compressibility, sea-floor elasticity and background density, which is applied transparently as a purely kinematic time-shift.

We are very grateful to the reviewer for their constructive critiques and comments. In the following, we state the referee's comments (in black) followed by the response and actions taken (in blue).

2. General Comments

2.1: That said, I have several concerns that, in my view, require major revision before the paper could be considered for publication. They can be divided into three groups: (i) the physical completeness of the ray-based amplitude proxy and the treatment of caustics; (ii) a mismatch between the probabilistic claims and what the method actually delivers; and (iii) the strength of the validation and its supporting statistics relative to the “global” and “rigorous” framing; plus some additional methodological and presentation issues (comments 6-8). None of these is fatal, and I think all are addressable within the scope of the study.

Response: We agree with this clear summary of concerns. In response: i) We have clarified that we use a kinematic ray density approach precisely to avoid dynamic ray-tube caustic singularities in the deep ocean (Page 9–10, Lines 223–225); ii) We have fundamentally lowered the tone from “Probabilistic Hazard Assessment” to “Recurrence-Weighted Exposure Screening” throughout the manuscript; iii) We have added the 2011 Tohoku event as a secondary validation and implemented rigorous block-bootstrap statistics (Page 20, Lines 390–411).

2.2: The amplitude proxy is physically incomplete (ray-tube spreading / energy flux). The hazard proxy is the normalised count of ray terminations along the coast. A count is not an amplitude: two coastal segments receiving the same number of rays can experience very different wave heights depending on the geometric convergence of the ray tube. Classical geometrical optics provides a natural amplitude estimate via conservation of energy flux within the ray tube (the spreading Jacobian), optionally combined with Green's law toward shallower water. By using a count rather than a ray-tube-based energy-flux estimate, the manuscript leaves the most quantitatively valuable part of the method untapped and weakens the link between the proxy and the FUNWAVE maximum-amplitude envelope it is compared against. I would recommend either (a) implementing an energy-flux / ray-tube amplitude estimate, or (b) explicitly justifying the count proxy and discussing the conditions under which count density and amplitude diverge.

Response: We partially agree with the reviewer's theoretical observation regarding classical geometrical optics, but we specifically utilize a kinematic design for our screening scope. We have clarified in the revised manuscript that our methodology utilizes **kinematic ray tracing combined with a ray-density proxy**, rather than a classical dynamic ray-tube amplitude solver. While solving the transport equation for energy flux (the spreading Jacobian) provides direct amplitude estimates, it requires additional treatment near caustics where the classical geometrical-optics approximation formally breaks down. Our framework

instead tracks kinematic ray trajectories and computes their spatial density, providing a stable geometric proxy for relative focusing suitable for basin-scale exposure screening. By instead tracking the kinematic trajectories of the wave packets and computing their spatial density (binned coastal counts), we establish a robust geometric proxy for relative energy focusing that bypasses the mathematical singularities inherent in classical amplitude calculations. This aligns with our reframing of the tool for exposure screening rather than absolute hazard assessment. This is explicitly justified in Section 3.3 (Page 9–10, Lines 222–227).

2.3: Caustics in the deep ocean are not addressed. Rays are terminated at 500 m depth to avoid nearshore caustics, but caustics (ray crossings) also form in the open ocean - precisely at the focusing zones that are the object of the study, where geometrical optics formally diverges (unbounded amplitude). The manuscript should state how deep-water caustics are detected and handled (e.g. amplitude capping or a Maslov/Gaussian-beam correction) and discuss the implications for the reliability of the identified hotspots. I may be missing something here, but I could not find any mention to this issue in the manuscript.

Response: We agree that this needed explanation. Because of our adoption of a kinematic ray-density framework that does not solve the transport equation for ray amplitudes, classical caustic singularities do not arise within the model. Instead, regions where rays converge simply appear as regions of increased ray density, which are interpreted as relative focusing rather than singular amplitude predictions. Because we do not solve the classical transport equation (which mathematically diverges to infinity when ray tubes collapse), ray crossings do not produce mathematical singularities in our model. Instead, deep-water caustics naturally and stably emerge simply as zones of high ray concentration. We have added a paragraph to Section 3.3 (Pages 9–10, Lines 223–225) to explicitly define this “kinematic ray tracing” design choice and explain how the density approach inherently stabilizes the identification of transoceanic focusing hotspots.

2.4: The “probabilistic” component reduces to a recurrence rate and yields no hazard curves. The authors themselves show (L137–143) that with a single global b -value and a uniform magnitude range, the energy-weighted field reduces to a constant multiple of the recurrence rate ($\Lambda = C \cdot \lambda$). There is no logic tree, no epistemic uncertainty, no zone-dependent maximum magnitude, and no slip distribution; the final product is a normalised 0–1 ray-count field rather than amplitudes at given return periods. The claim of “a probabilistic interpretation of exposure consistent with modern hazard assessment standards” therefore overstates the method relative to the state of the art (e.g. Davies et al., 2018, which is cited). I recommend either producing actual hazard quantities (amplitude vs. return period) or reframing the contribution honestly as a recurrence-weighted exposure screening and moderating the PTHA language accordingly.

Response: We fully agree with this assessment and the reference to state-of-the-art PTHA standards (e.g., Davies et al., 2018). We have thoroughly reframed the manuscript as a “recurrence-weighted exposure screening tool,” removing all claims of providing absolute probability exceedance or hazard curves. We noted this reduction in Section 2 (Page 6, Lines 149–155).

2.5: Single-event validation does not support a “global / rigorously validated” claim. The validation rests on one source region (Kuril-Kamchatka). A global framework should be tested across basins and source geometries. I strongly recommend adding at least one validation case in a different setting (e.g. Tohoku 2011, Maule 2010, or Sumatra-Andaman 2004) before claiming global applicability, and softening “rigorously validated” if only one case is retained.

Response: We agree with the reviewer. We have added the 2011 Tohoku event as a full secondary validation case, running both the ray-tracing and FUNWAVE simulations. The results confirm the model’s stability across different basin geometries, which is detailed in Section 5 (Pages 20–21, Lines 390–411).

2.6: Statistical rigour of Spearman $\rho = 0.66$. $\rho = 0.66$ is a moderate monotonic association ($\sim 44\%$ of rank variance). No confidence interval or significance test is reported, and co-located coastal pairs are strongly spatially autocorrelated, which inflates the effective sample size and apparent significance. I would ask the authors to report a confidence interval, to use a resampling scheme that accounts for spatial autocorrelation (e.g. block bootstrap or an effective- N correction), and to show the sensitivity to the coastal sampling/binning choices.

Response: We completely agree with the reviewer that standard significance tests ignore the spatial autocorrelation of coastal wave heights, which artificially inflates degrees of freedom and narrows confidence intervals. To provide unbiased confidence bounds and properly account for spatial autocorrelation, we implemented a rigorous spatial block-bootstrap resampling scheme.

Spatial Autocorrelation & Confidence Interval: To preserve the local spatial dependence structure, the analyzed coastline was partitioned into contiguous 500 km segments along the coastal trajectory. Resampling these contiguous blocks with replacement over 10,000 iterations yielded a spatially corrected

Spearman correlation of $\rho = 0.65$, with an unbiased 95% confidence interval of $[0.35, 0.83]$. Because the confidence interval is strictly positive and bounded well above zero, the monotonic relationship is statistically significant ($p < 0.01$).

Sensitivity to Binning Choices: To address the sensitivity to our coastal binning parameter, we evaluated alternative block sizes. As mathematically expected, smaller blocks (300 km) narrowed the 95% confidence interval to $[0.44, 0.78]$, while larger blocks (800 km) widened it to $[0.32, 0.83]$. Crucially, the lower bound remained strictly positive across all physically reasonable binning choices, confirming robust statistical significance regardless of the specific block size. This comprehensive statistical methodology is now fully detailed in the revised Section 4.2 (Page 19, Lines 368–376). Following the reviewer suggestion, we added Tohoku 2011 event and performed similar analysis for it in section 5.

2.7: Dispersion and the handling of multiple wave periods. Each ray is assigned a period between 15 min and 3 h, but the manuscript does not explain how the different periods are aggregated into the final field, nor how the non-dispersive assumption is reconciled with the use of a dispersive Boussinesq model (FUNWAVE) as the reference. Over transoceanic distances frequency dispersion is not negligible. Please clarify the period-aggregation procedure and discuss the dispersion assumption quantitatively.

Response: We agree and appreciate the reviewer’s careful reading. We have updated Sections 3.3 and 4.2 to explicitly address the dispersion assumption and clarify the period-aggregation strategy.

Period Aggregation and Dispersivity: While real wavefields are spectral, long-period seismic tsunamis are generally weakly dispersive during deep-ocean propagation, meaning bathymetric refraction is expected to dominate the principal basin-scale pathways. Nevertheless, wave trajectories may retain some period dependence through the finite-depth dispersion relation. By evaluating representative periods across the 15-minute to 3-hour spectrum, we found that while local coastal termination patterns exhibit minor frequency-dependent variations, the principal transoceanic focusing corridors remain highly stable. For the global analysis, we aggregated this discrete period spectrum by merging the independent ray fields, which effectively bounds these minor frequency-dependent variations to construct a robust, composite exposure footprint (Section 3.3, Page 9, Lines 204–208).

This physical reality directly informs our twofold rationale for utilizing the dispersive Boussinesq model (FUNWAVE) as our reference:

- (a) **High-Fidelity Benchmark:** Over transoceanic distances, cumulative dispersion does alter the final waveform shape, amplitude, and exact arrival time. FUNWAVE serves to precisely capture these detailed hydrodynamics for the Kamchatka event.
- (b) **Validation of the Kinematic Proxy:** By comparing the models, we demonstrate that a computationally inexpensive geometrical tracer method accurately isolates the primary transoceanic propagation paths (energy corridors). FUNWAVE validates that frequency dispersion does not fundamentally alter these macro-scale geometric trajectories, thereby justifying the ray-tracing framework as a robust rapid screening tool.

2.8: Code and data availability. For a methods paper whose central contribution is the ray-tracing framework, “available from the corresponding author upon request” does not meet FAIR requirements. Only the pre-existing FUNWAVE-TVD code is released. Depositing the ray-tracing code and the derived datasets in a citable repository with a DOI (e.g. Zenodo) is necessary to meet FAIR standards.

Response: We agree. The complete unstructured ray-tracing codebase (MATLAB), input meshes, source files, and resulting density datasets have been deposited in a Zenodo repository. The Data Availability statement has been updated (Page 25, Lines 438–440).

2.9: Writing economy. While reading the introduction I had the feeling of being told three times why ray tracing is cheaper than full numerical modelling; the point is made at L35-39, again around L40-44 (Satake) and once more at L44-55 (Tehrani et al.). Once is enough, and the introduction would gain pace if these passages were merged. Something similar happens with Section 3.3: the description of Figure 3 reads like a guided tour (“a striking artifact”, “a very different picture emerges”, “dismantled almost entirely”). Since all panels share the same normalised 0–1 coastal scale, the authors have the numbers at hand: stating, for two or three regions (NE Australia, the Pacific Northwest, Chile), by how much the coastal metric actually changes between panels b and d would replace adjectives with evidence and make the case far more convincing.

Response: We agree. We have merged the repetitive intro paragraphs into a single concise rationale (Introduction, Page 2, Lines 37–44). For Figure 3, we removed descriptive adjectives and explicitly reported the numerical changes in normalized coastal focusing for the specific regions discussed (Section 3.3, Page 10, Lines 248–261).

3. Specific Comments

- 3.1:** L3, L73, L184: “over 9,000”, “more than 9,000”, and “In total, 9,000 rays were released” are inconsistent. Please use a single exact figure throughout the paper.

Response: We agree. ‘In total, 9,000 rays were released’ is correct. We made it consistent across the manuscript (e.g., Abstract, Page 1, Line 4; Section 3.3, Page 9, Line 221).

- 3.2:** L50-54 and 157-160: The argument that seismic tsunamis are “ideal for the geometric optics approach” because they are non-dispersive is reasonable, but it sits in tension with using a dispersive Boussinesq model as ground truth. Please reconcile.

Response: We partially agree; while the waves are weakly dispersive, finite depth dynamics still apply. This is addressed and reconciled. (See response to comment 2.7; Section 3.3, Page 9, Lines 204–208).

- 3.3:** L137-143 (magnitude-energy paragraph): The demonstration that $\Lambda = C \cdot \lambda$ is honest and welcome, but it undercuts the “probabilistic” framing; this should be reflected in the abstract and conclusions (see General comment 2.4).

Response: We agree. The text has been updated to reflect the new “exposure screening” scope, clearly stating that $\Lambda = C \cdot \lambda$ supports a recurrence-weighted proxy rather than true probabilistic energy amplitudes (Section 2, Page 6, Lines 149–155).

- 3.4:** L103-108: $b = 1.05$ from 212 events with $M_c = 7.4$ determined “by visual inspection”. Please justify M_c more quantitatively (e.g. maximum-curvature or goodness-of-fit method) and report the uncertainty on b .

Response: We agree. We re-calculated M_c using the Maximum Curvature (MAXC) method, mathematically verifying $M_c = 7.4$. The uncertainty for the b -value (via Aki-Utsu) is now reported as $b = 1.05 \pm 0.07$ (Section 2.1, Page 5, Lines 96–105).

- 3.5:** L117-125: KDE with $\sigma = 500$ km is justified by rupture length, and 400-600 km is said to give similar patterns. It would be good to see this sensitive shown quantitatively, even as supplementary material.

Response: We agree. We have now included a quantitative bandwidth sensitivity analysis (Section 2.2, Page 6, Lines 133–135; Figure A1, Page 26), demonstrating that the large-scale hazard patterns remain stable for bandwidths between 400 and 600 km while avoiding overfitting at smaller bandwidths.

- 3.6:** Figure 1 caption (around L94-98): magnitude is described as being on the “Richter scale”. For the large events that dominate this catalogue, moment magnitude (M_w) is the appropriate scale and should be used instead.

Response: We agree. Corrected globally to M_w (e.g., Section 2.1, Page 5, Line 100; Figure 1 Caption, Page 4).

- 3.7:** L168-172 and L270-271: “1 arc-second resolution” for GEBCO 2025 contradicts the cited reference (L408), which states 15 arc-second intervals. The same error is repeated at L270-271 (“downscaled from a resolution of 1 arc-second to 4 arc-minutes”). Please correct this factual inconsistency in both places and confirm the actual resolution used.

Response: We agree. Thank you for catching this. It has been corrected to the accurate 15 arc-second interval grid throughout the manuscript (Section 3.2, Page 9, Line 196; Section 4.1, Page 14, Line 308).

- 3.8:** L168-173: Ray-tracing mesh resolution is 10-28 km. The seamounts and fracture zones responsible for focusing may be under-resolved, and barycentric interpolation with precomputed gradients smooths exactly those features. Please add a ray-tracing mesh-resolution sensitivity test (a FUNWAVE resolution test is already provided, L278-280; an equivalent for the rays is needed).

Response: We agree and appreciate the reviewer’s careful attention to the mesh resolution. While standard Cartesian meshes require very fine, uniform resolution to capture bathymetric peaks without smoothing them out, our unstructured mesh bypasses this limitation by explicitly forcing nodal generation directly on high-gradient features during the mesh creation process.

Figure A2 details the spatial distribution of the mesh resolution. As shown in the figure, the adaptive variable-resolution mesh explicitly tracks bathymetric gradients. The finest resolutions (down to 10.77 km) are concentrated strictly along the exact critical refractive structures the reviewer highlighted—such as seamount chains, mid-ocean ridges, and subduction trenches (indicated by the red/orange zones). Conversely, the coarser resolutions (up to 28.35 km) are permitted only in the relatively flat abyssal plains where refraction gradients are naturally minimal.

We have expanded Section 3.2 (Pages 8–9, Lines 184–190) to explicitly detail this feature-tracking mesh

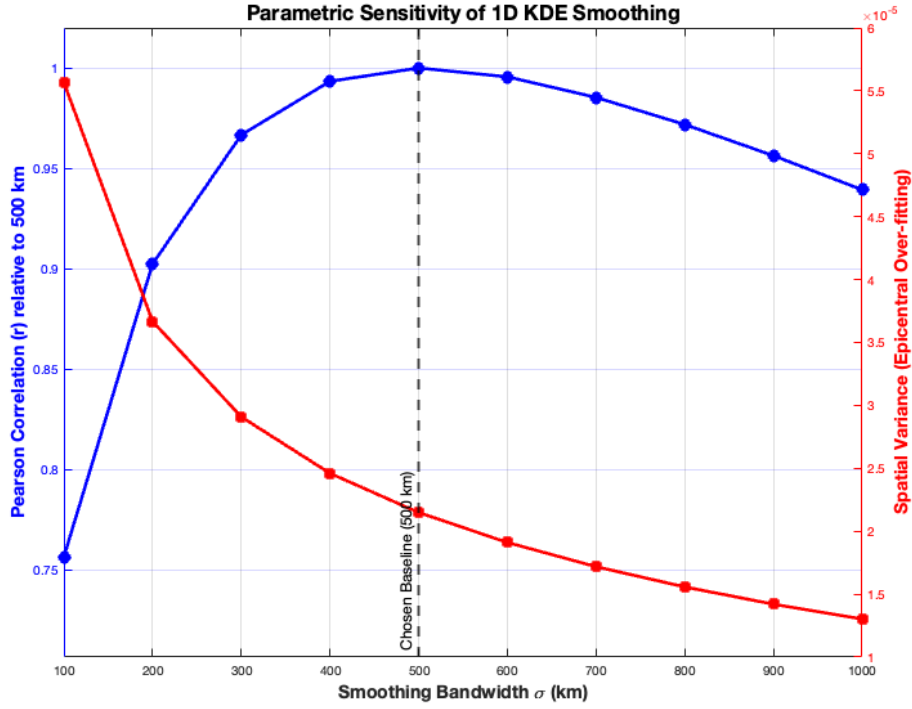


Figure 1: Parametric sensitivity of the 1D Kernel Density Estimation (KDE) smoothing bandwidth (σ). The red curve (right axis) shows spatial variance; values stabilize at the 500 km baseline, consistent with characteristic rupture lengths of great (M_w 8.8–9.0) megathrust events. The blue curve (left axis) shows the Pearson correlation coefficient ($r > 0.95$) relative to the 500 km baseline, demonstrating geographic stability across the bandwidth spectrum.

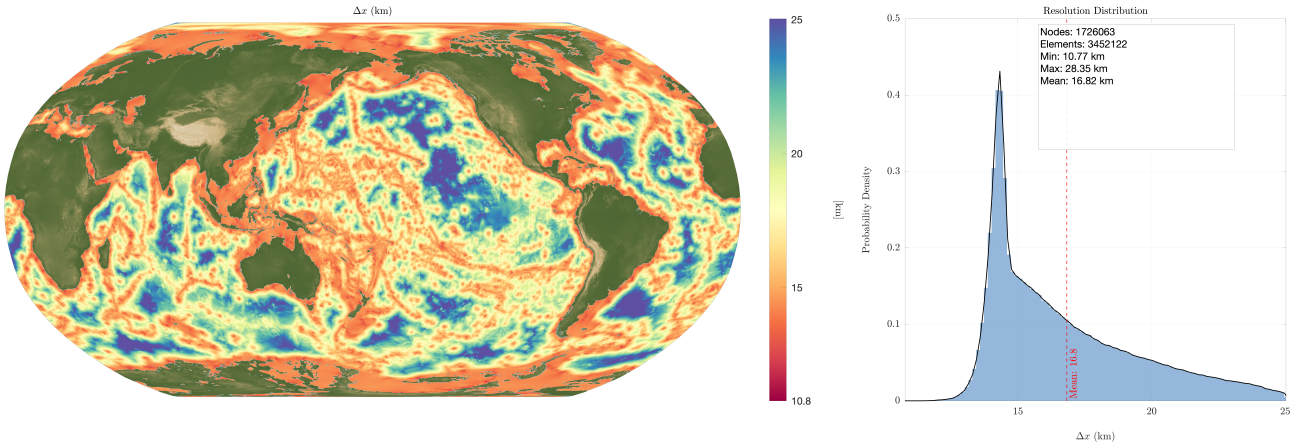


Figure 2: Spatial distribution and statistical properties of the variable-resolution unstructured ray-tracing mesh. (Left) Global map of the element sizes (Δx in km) demonstrating the feature-tracking nodal generation. Finer resolutions (red/yellow, ~ 10.8 – 15 km) are explicitly forced along high-gradient bathymetric features such as seamount chains, mid-ocean ridges, and subduction trenches to preserve critical refractive boundaries. Coarser resolutions (blue, up to 28.35 km) are restricted to relatively flat abyssal plains. (Right) Probability density distribution of the element sizes, detailing the specific mesh geometry of 1,726,063 nodes and 3,452,122 elements with a mean resolution of 16.82 km.

generation process and included the resolution map and distribution statistics (Appendix A, Page 27) to directly address this concern.

- 3.9:** L179-184: Rays are emitted perpendicular to the fault and oceanward only. Real radiation directivity depends on slip distribution and dip. Please discuss the impact of this simplification on the radiation pattern.

Response: We agree. We added text clarifying that while real directivity relies on 3D rupture complexities, the dominant macro-scale energy flux of large subduction megathrusts radiates strictly orthogonal to the fault strike. Weighting the initial ray density by the vertical seafloor deformation envelope accurately initializes these primary radiation lobes for far-field propagation (Section 3.3, Page 9, Lines 208–213).

- 3.10:** L181-184: The 500 m termination depth is a key parameter; please justify the choice and test sensitivity (see General comment 2.3 on caustics).

Response: We agree. We expanded the text to clarify that the scope of this framework is strictly basin-scale transoceanic focusing, and to justify the 500 m termination depth from both a physical and computational standpoint. Physically, this depth truncates the simulation just before the complex nearshore zone. In water shallower than 500 m, non-linear shoaling and extreme refraction dominate, where the standard geometrical optics approximation fails. Moreover, computationally, tsunami waves entering these shallower waters require a proportionally higher-resolution mesh to resolve accurately. Implementing such high resolution along global coastlines would impose heavy computational costs, directly defeating the primary purpose of this study as a rapid, efficient exposure screening tool. Furthermore, continuing to propagate rays into these shallow zones using our existing coarse deep-water mesh would yield unphysical and inaccurate results. Therefore, the 500 m cutoff acts as a necessary and strict "hand-off" boundary. It safely isolates basin-scale transoceanic focusing from local nearshore amplification, which must be handled by specialized, high-resolution hydrodynamic modeling (Section 3.3, Page 9, Lines 215–221).

- 3.11:** L230-240: the Kamchatka event summary reports specific run-up heights, casualties, and damage. These should be supported by explicit references if retained.

Response: We agree. Citations to the NOAA/NCEI Database and USGS summaries have been added (Section 4, Page 12, Lines 269–273).

- 3.12:** L278-280: The sensitivity simulations are described as being conducted on "higher resolution 8 and 16 arc-minute grids", but 8 and 16 arc-minutes are coarser than the 4 arc-minute production grid, not finer. Either this is a typo ("coarser") or the sensitivity test only explores lower resolutions, in which case the stated conclusion that 4 arc-minutes is "adequate" is not supported by a convergence test toward finer grids. Please clarify.

Response: We agree. Thanks for pointing out the typo. It should be 'lower' (Section 4.1, Page 15, Line 317).

- 3.13:** L334-340: Please specify exactly how co-located FUNWAVE-ray pairs are extracted (segment length, number of points, near-field exclusion criterion) and report the statistics described in General comment 2.6. In particular, please detail the near-field exclusion criterion applied; near the Kamchatka source region, the ray-tracing approximation breaks down due to initial source dynamics and strong diffraction, meaning near-field coastal segments must be strictly excluded to avoid biasing the global far-field statistical correlation.

Response: We agree. We updated the text to explicitly detail that data was extracted by joining outputs onto a uniform coastal grid. Crucially, to prevent finite-fault dynamics and strong local diffraction from artificially biasing the global correlation, all coastal segments within a 1,000 km radius of the epicenters were strictly excluded from the statistical pairs prior to running the block bootstrap (Section 4.2, Page 19, Lines 366–371).

- 3.14:** L356-384 (Conclusions): Language such as "rigorously validated", "robust", and "defensible" should be moderated given $\rho = 0.66$ and the single-event validation.

Response: We agree. The tone has been lowered throughout the conclusions (Pages 21–22, Lines 407–422). The addition of the Tohoku case also removes the "single-event" limitation.

4. Technical Corrections

- 4.1:** L137: "wether" → "whether".

Response: Corrected.

4.2: Ensure all figure colour scales state explicitly that the coastline metric is a normalised ray-count proxy (0–1), not wave amplitude, to avoid reader misinterpretation.

Response: The description is added to the figure caption and text.

Basin-Scale Geometric Focusing: A Probabilistic-Geometric Framework for Global Tsunami ~~Hazard~~ Exposure Assessment and the 2025 Kamchatka Peninsula Tsunami

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Abstract. We present a hybrid probabilistic-geometric framework that integrates probabilistic earthquake statistics with large-scale ray-tracing simulations to efficiently map global coastal tsunami exposure. Utilizing a catalog of historical tsunami-genic events and the Gutenberg-Richter relation, we derive probabilistic weights for ~~over~~ 9,000 rays released across potential fault zones. The simulated ray pathways reveal persisting bathymetry-driven energy convergence patterns that govern far-field coastal focusing and shadowing. The geometric framework's predictive power is demonstrated using the 2025 M8.8 Kamchatka Peninsula earthquake and the 2011 M9.0 Tohoku-oki event. Validation against ~~the 2025 M8.8 Kamchatka earthquake~~ these events utilizes phase-corrected FUNWAVE-TVD simulations and in-situ DART observations. The resulting ray-based coastal focusing patterns display ~~a substantial~~ strong qualitative and quantitative spatial agreement (e.g., Spearman's $\rho = -0.66$ $\rho = 0.66$ and $\rho = 0.63$, respectively) with the transoceanic maximum wave amplitudes from the high-fidelity FUNWAVE-TVD model. This agreement confirms the hybrid probabilistic-geometric approach as a scalable and computationally efficient tool for rapidly identifying ~~coastal hotspots of transoceanic tsunami impact~~ transoceanic focusing potential and prioritizing coastal regions for high-resolution exposure assessment, mapping relative geometric convergence.

1 Introduction

Transoceanic tsunamis, typically generated by large-magnitude megathrust earthquakes in subduction zones, represent one of the most far-reaching and destructive natural hazards to coastal populations worldwide. As these long-wavelength waves traverse entire ocean basins, their energy does not disperse uniformly. Instead, the complex three-dimensional structure of ocean bathymetry, comprising mid-ocean ridges, fracture zones, seamount chains, continental shelves, and deep-ocean trenches, acts as an intricate system of refractive lenses. This bathymetry-driven refraction can channel, redirect, and sometimes trap tsunami rays, producing regions of pronounced geometric focusing. The resulting distribution of tsunami energy at landfall is highly

20 heterogeneous: some coastlines, even thousands of kilometers from the source, experience severe and often unexpected wave amplification, while other areas may remain largely unaffected, forming classical shadow zones. This basin-scale sensitivity to bathymetric structure highlights the need for reliable tools capable of resolving the principal ray pathways that govern far-field tsunami exposure, especially given that these spatially variable pathways originate from the heterogeneous source distribution shown in Figure 1a.

25 While tsunamis can be triggered by non-seismic sources such as submarine landslides or volcanic eruptions, the vast majority of basin-wide events are generated by submarine earthquakes, which account for approximately 80% of all recorded damaging tsunamis (Davies et al., 2018; Behrens et al., 2021). Non-seismic tsunamis, although potentially devastating in the ~~near field~~near-field, typically involve more localized sources that produce shorter-wavelength waves. These waves are more susceptible to frequency dispersion, causing their energy to decay more rapidly with distance and limiting their far-field significance

30 (Løvholt et al., 2008; Torsvik et al., 2021). For this reason, the present study focuses exclusively on the far-field propagation of seismically generated tsunamis. ~~Accordingly, the framework is not intended to replace detailed near-field inundation or run-up modeling, but to identify which distant coastlines merit such analysis. We explicitly exclude the complex, nonlinear, and site-specific nearshore processes of shoaling, breaking, inundation, and run-up, which constitute a separate modeling domain. This far-field focus is critical because the identification of transoceanic energy corridors serves as a necessary first step for prioritizing resources and allocating~~ To specifically isolate the influence of ocean basin geometry, rays are intentionally released in the oceanward direction. Shoreward-directed waves are purposely excluded, as their propagation is predominantly governed by local near-source complexities rather than basin-scale topography. This directional constraint not only isolates the transoceanic focusing mechanics but also allows for the use of a coarser computational mesh optimized for deep water. Therefore, this framework is designed to guide decision-making for rapid, basin-scale transoceanic exposure assessment and

35 to prioritize distant coastlines for computationally intensive nearshore modeling (run-up and inundation) ~~to the most vulnerable coastal communities. Conversely, the method is explicitly not suitable for assessing non-seismic events due to their dispersive nature, leaving site-specific inundation mapping and local evacuation planning to specialized near-field hydrodynamic tools.~~

Given the strong influence of bathymetric refraction on transoceanic tsunami propagation, ~~there are fundamentally two approaches for examining these~~ evaluating basin-scale convergence ~~patterns. The first is to employ~~ traditionally relies on

45 full numerical models, which ~~resolve the detailed physics of tsunami generation and evolution, but~~ become computationally prohibitive ~~when applied to global domains or to the~~ for large scenario ensembles ~~required for hazard analysis. The second option is to utilize more cost-effective.~~ Alternatively, geometrical-optics methods ~~, so-called ray tracing, which captures the dominant effects of~~ (ray-tracing) capture dominant bathymetric refraction at a fraction of the computational cost. ~~In one of the earliest demonstrations of this concept, Satake (1988) adapted seismic ray-tracing techniques to tsunami propagation, showing~~

50 Foundational studies (Satake, 1988) demonstrated that tsunami rays ~~can accurately reconstruct the principal propagation pathways and identify the bathymetric features responsible for focusing and defocusing. These foundational results established ray tracing as a powerful and efficient tool for exploratory analysis, scenario screening, and rapid basin-scale hazard evaluation.~~

A recent study by Tehranirad et al. (2015) further reinforces the utility of ray tracing as a robust proxy for tsunami energy assessment. ~~Focusing on a large-scale volcanic flank collapse at Cumbre Vieja Volcano (La Palma) in the Atlantic, they~~

55 ~~compared accurately reconstruct principal pathways. More recently, Tehranirad et al. (2015) benchmarked ray-tracing predictions with against~~ high-resolution Boussinesq simulations using dispersive models (FUNWAVE-TVD and found a), finding strong spatial correlation between ray convergence ~~patterns~~ and maximum surface elevations. ~~This effectively benchmarked the geometric-optics approach against a fully nonlinear dispersive model, demonstrating that continental-shelf ray concentration reliably identifies zones of elevated tsunami impact. Importantly, that study dealt with a landslide-generated tsunami, where~~ shorter wavelengths and significantly higher kd values, where k is the wavenumber and d is the undisturbed water depth, produce stronger dispersion conditions typically cited as limitations for ray-based methods. Yet the agreement remained high, implying that for ~~Since~~ seismically generated tsunamis, which are ideal for the geometric optics approach because they are ~~are~~ dominated by the non-dispersive shallow-water approximation, where the wave celerity $C_p = \omega/k = \sqrt{gd}$ does not depend on wave length, and exhibit lower dispersion over transoceanic distances, ray tracing should be even more accurate. Thus, the validation by Tehranirad et al. (2015) directly addresses one of the central criticisms of ray theory and strengthens its relevance for the ray-tracing method serves as a robust proxy for rapid scenario screening and global-scale seismic tsunami hazard assessment, including the Kamechatka case analyzed herein exposure assessment.

However, identifying the pathways of geometric focusing represents only one component of the broader hazard exposure assessment problem. A realistic global hazard exposure assessment must also reflect the inhomogeneous recurrence rates of tsunamigenic earthquakes across subduction systems. Early ray-based studies were inherently deterministic, typically initiated from a single, hypothetical earthquake source. Such simplified treatments do not capture the full range of seismic variability across global subduction systems and therefore cannot produce a comprehensive or realistic depiction of global tsunami hazard exposure. Modern hazard assessments must confront two key challenges: (1) the probabilistic nature of earthquake occurrence across distributed and heterogeneous source zones, and (2) the computational scalability required to evaluate tens of thousands of plausible rupture scenarios. This latter requirement motivates the use of Probabilistic Tsunami Hazard Assessment (PTHA), a framework that quantifies risk by estimating the exceedance probabilities (or return periods) of tsunami intensities at specific locations. PTHA couples statistical models of earthquake recurrence, often based on the Gutenberg-Richter relation, with numerical or analytical models of tsunami generation and propagation. Comprehensive reviews such as Grezio et al. (2017) and global assessments like Davies et al. (2018) outline how PTHA can be applied at regional to global scales global scales by integrating multiple source zones to construct hazard curves and probability maps.

To address this methodological gap, we introduce a hybrid probabilistic geometric modeling framework that couples classical ray tracing ray tracing with earthquake probability statistics to produce a fast, scalable, and scientifically robust global hazard exposure mapping tool. Our approach integrates Gutenberg-Richter-based frequency-magnitude relationships with large-ensemble ray tracing ray tracing simulations, enabling efficient evaluation of far-field tsunami exposure while retaining the essential bathymetric controls. In our implementation, more than 9,000 rays are released from candidate fault segments worldwide, each ray assigned a probabilistic weight (representing the annual rate, λ) derived from local and regional seismicity rates. It is important to clarify that in this framework, λ serves strictly as a source weighting factor to modulate the relative density of the ray field; it does not represent a direct probability of coastal amplitude exceedance. This approach allows us

to rapidly quantify coastal energy convergence and identify global tsunami hotspots most susceptible to long-range focusing, while providing a probabilistic interpretation of exposure consistent with modern hazard assessment standards.

This paper is organized as follows. Section 2 presents the Probabilistic Tsunami ~~Hazard-Assessment~~ (PTHA)-Exposure Assessment methodology. Historical seismological data are analyzed to map the spatial distribution of tsunamigenic sources and to estimate their return periods for earthquakes exceeding magnitude 7.4. Section 3 introduces the wave ~~ray-tracing~~ ray-tracing method, based on the principles of geometrical optics. This approach simulates tsunami propagation paths from the ~~PTHA-identified~~ identified source regions, with the resulting ray patterns revealing how ocean basin geometry and bathymetric refraction generate zones of energy focusing and shadowing. These patterns provide a first-order assessment of regional tsunami ~~hazard~~ exposure. Section 4 presents a validation case study of the M8.8 Kamchatka Peninsula earthquake. It begins with a high-fidelity Boussinesq-type simulation using the FUNWAVE-TVD model, hereafter referred to as FUNWAVE, to compute tsunami wave amplitudes. Next, the ray-tracing model is applied to predict tsunami energy convergence zones (focal points) for the same event. The resulting numerical simulations are compared against in-situ observations to assess both the accuracy of the ray-tracing predictions of focal zones and the wave heights simulated by FUNWAVE. Section 5 extends this verification framework by applying the combined ray-tracing and hydrodynamic models to the 2011 M9.0 Tohoku-oki tsunami as a secondary validation case. Finally, Section 6 summarizes the findings and provides concluding remarks.

2 Probabilistic Tsunami ~~Hazard~~ Exposure Assessment

To create a spatially variable weighting scheme for tsunami sources, a simplified ~~Probabilistic Tsunami Hazard Assessment~~ (PTHA)-recurrence-weighted exposure screening was conducted (Grezio et al., 2017). The process is based on deriving earthquake recurrence parameters from a historical catalog NGDC/WDS (2024) of tsunamigenic events and applying these parameters to a set of discrete source points along known and potential earthquake zones. The final output is the mean annual rate of occurrence for earthquakes, which serves as a quantitative weight for the subsequent ~~ray-tracing~~ ray-tracing model.

The historical catalog comprises approximately 1,600 global events. Figure 1a illustrates the spatial distribution of these events, where color indicates the magnitude of tsunamigenic earthquakes recorded from 500 BC to 2025. Panel b shows the occurrence rate, while panel c presents the decadal histogram color-coded by magnitude. The establishment of the World-Wide Standardized Seismograph Network (WWSSN) in 1965 is marked by a vertical red line.

2.1 Gutenberg–Richter Recurrence Relationship

The frequency-magnitude distribution of earthquakes is modeled by the Gutenberg–Richter (G-R) law:

$$\log_{10}(N) = a - bM \quad (1)$$

where N is the cumulative number of earthquakes with a magnitude greater than or equal to M . The parameter b describes the relative proportion of small to large earthquakes, while the a -value represents the overall seismic productivity of a region.

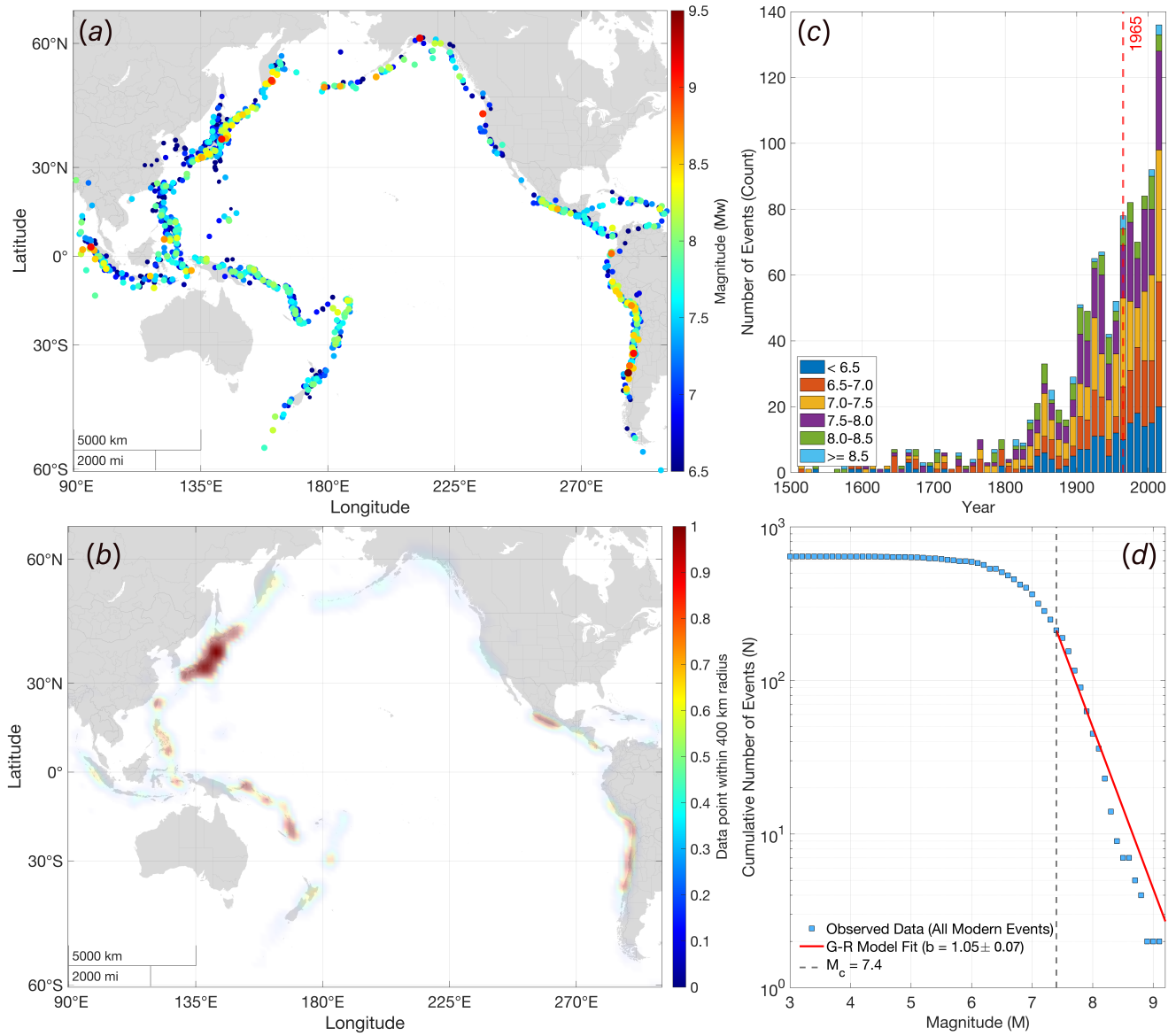


Figure 1. (a) Historical tsunamigenic earthquakes based on ~1600 events from ~500 BCE to 2025. The colormap indicates event magnitude on the Richter scale (M_w); (b) Spatial density plot (heatmap) showing earthquake clustering, calculated with a 400 km radius of influence; (c) Histogram of historical events per decade since 1500, with bars color-coded by event magnitude. The vertical red line at 1965 marks the start of modern event records used in this study; (d) b-value estimate, showing the Gutenberg–Richter model fit (red line) to historical data since 1965 for events of magnitude ≥ 7.4 . Basemap created with Natural Earth data.

To determine a stable b -value, the catalog was filtered to include only the modern instrumental period (1965 onward), corresponding to the establishment of the World-Wide Standardized Seismograph Network (WWSSN). The Magnitude of Completeness (M_c) was initially assessed using the Maximum Curvature (MAXC) method (?), which defines the completeness threshold by identifying the maximum value of the first derivative of the frequency-magnitude curve. For this dataset, the raw MAXC peak occurs at $M_w = 7.05$. However, because raw MAXC estimates frequently fall within the catalog's detection roll-over region and can therefore underestimate true completeness, we adopted a conservative operational threshold of $M_c = 7.4$.

We emphasize that this threshold represents a modeling choice to ensure absolute catalog completeness rather than an exact, immutable catalog property. To ensure robustness, we evaluated the stability of the estimated b -value over nearby completeness thresholds (e.g., $M_c = 7.3$ to 7.5), confirming that the principal basin-scale recurrence patterns are not materially altered. Based on the resulting complete subset of 212 events, a b -value of 1.05 ± 0.07 was calculated using the Aki–Utsu maximum likelihood estimator (Figure 1d), where the standard error is given by $\sigma_b = b/\sqrt{N}$ (Aki, 1965; Utsu, 1965).

A Magnitude of Completeness (M_c) of 7.4 was ~~determined by visual inspection of the~~ mathematically determined using the Maximum Curvature (MAXC) method, identifying the point of maximum first derivative in the frequency-magnitude ~~distribution (consistent with completeness levels found in global subduction-zone catalogs)~~ curve. Based on the resulting subset of 212 events, a b -value of ~~1.05~~ 1.05 ± 0.07 was calculated using the Aki–Utsu maximum likelihood estimator (Figure 1d) (Aki, 1965; Utsu, 1965). This value is consistent with the global average ($b = 1$), indicating a robust fit (Frohlich and Davis, 1993).

2.2 Spatial Seismicity Rate Estimation

For the sake of applicability, a polygonal curve $\Gamma(Lat, Lon)$ was first defined to pass through the historical epicenters, representing both known and potential earthquake zones. The curve Γ was constructed by interpolating through the historical epicenters and aligning with major subduction interfaces and fault boundaries identified from global tectonic maps.

The continuous curve Γ was constructed by tracing the major tsunamigenic fault systems and subduction interfaces globally, discretizing this path into 10 km intervals. To account for the spatial ~~variability in the number of historical occurrences, as shown in~~ scatter of historical earthquakes, epicenters located within a [400] km search radius were orthogonally projected onto the nearest segment of this curve before applying the 1D Gaussian smoothing.

To account for spatial variability in historical seismic occurrence (Figure 1b), a spatially varying $a(\Gamma)$ value was derived using one-dimensional Kernel Density Estimation (KDE) along the along-trench coordinate Γ . This approach mitigates data sparsity by smoothing the influence of ~~each historical event over its surrounding area~~ historical events over their surrounding segments. A Gaussian kernel $f(\Gamma)$ was applied at the projected 1D location (Γ_i) of each ~~filtered historical~~ event:

$$f(\Gamma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\Gamma-\Gamma_i)^2}{2\sigma^2}} \quad (2)$$

~~where a~~ While near-field inundation modeling requires resolving complex 3D fault geometries, basin-scale transoceanic exposure screening is predominantly governed by macro-scale, along-trench variations in seismic productivity (Scourse et al., 2018). Therefore, this dimensional reduction to a 1D longitudinal projection is an established methodology for macro-scale hazard

analysis (Scourse et al., 2018). A smoothing bandwidth of $\sigma = 500$ km was selected to ~~mitigate the spatial sparsity of the~~
~~historical catalog (values about that value, i.e., between 400~~ satisfy established empirical scaling laws for subduction interface
earthquakes. Because great megathrust events ($M_w > 8.5$) are restricted by down-dip width saturation at approximately 200
155 km, their ruptures are forced to propagate longitudinally (Allen and Hayes, 2017). Established length-scaling relations indicate
~~that the characteristic along-strike rupture length for M_w 8.8–600 km, yield similar large-scale patterns).~~ Physically, this
~~length-scale corresponds to the characteristic rupture dimensions of great subduction zone earthquakes ($M8+$), ensuring~~
~~that the probabilistic weight is distributed across the entire potentially rupturing fault segment rather than being artificially~~
~~concentrated at historical epicenters. The continuous curve representing the relative seismicity rate along the potential fault~~
160 ~~zones is the sum of all kernels~~ 9.0 earthquakes spans roughly 440–588 km (Blaser et al., 2010; Allen and Hayes, 2017). The
500 km KDE bandwidth therefore physically scales the probability density to match realistic rupture extents, eliminating the
severe epicentral over-fitting that occurs with narrower bandwidths. A quantitative statistical evaluation (Figure A1) confirmed
that spatial variance stabilizes at this 500 km threshold, while the macro-scale spatial clustering remains stable and independent
of the selected bandwidth ($r > 0.95$). This curve was subsequently normalized to match the total observed annual rate of events
165 above M_c from the modern catalog, so that local variations do not inflate regional seismic productivity, producing an absolute
annual rate density for each source point.

The final weight assigned to each source point i is its total annual rate of significant tsunamigenic earthquakes ($M \geq 7.4$),
calculated by applying the spatially determined $a(\Gamma)$ from the KDE to the Gutenberg-Richter relationship and integrating
over the relevant magnitude bins. This yields a single representative annual rate, λ , which serves as the input likelihood for the
170 tsunami ray-tracing models. The corresponding ~~hazard-exposure~~ map, showing the average return period $T = 1/\lambda$, is presented
in Figure 2. ~~The resulting hazard-~~

The resulting exposure map reveals significant spatial heterogeneity in earthquake recurrence. The Kuril-Kamchatka and
Japan Trenches in the northwest Pacific, alongside the Peru-Chile Trench in the southeast, emerge as the most seismically
active source zones, exhibiting return periods between 100 and 1,000 years (red to orange segments). This creates a dominant
175 ~~'hazard-exposure~~ dipole' across the Pacific Basin. Additionally, the Aleutian Trench and the Sumatra-Andaman subduction
zone show high recurrence rates, highlighting them as critical sources for transoceanic tsunami generation. In contrast, mid-
ocean ridges and transform boundaries generally exhibit longer return periods (blue segments), reflecting their lower potential
for generating large tsunamigenic displacements.

In this study we adopt a recurrence-based weighting, though we also examined ~~wether-whether~~ magnitude-energy formula-
180 tion would alter the resulting coastal exposure patterns. Using the same magnitude bins between $M = 6.5$ and 9.5, we computed
an annualized energy metric Λ_j for each segment j by multiplying each bin's rate by $E(M) \propto 10^{1.5M}$ and summing across
bins. However, since the PTHA implementation employs a single regional b -value and the same magnitude range for all source
segments, the Gutenberg-Richter distribution has an identical shape everywhere. Under these assumptions Λ_j reduces to a
constant multiple of the recurrence rate ($\Lambda_j = C\lambda_j$), and thus energy-weighted ray-density field would be indistinguishable
185 from the recurrence-weighted field up to a uniform rescaling. For this reason we present the recurrence-based maps, explicitly.

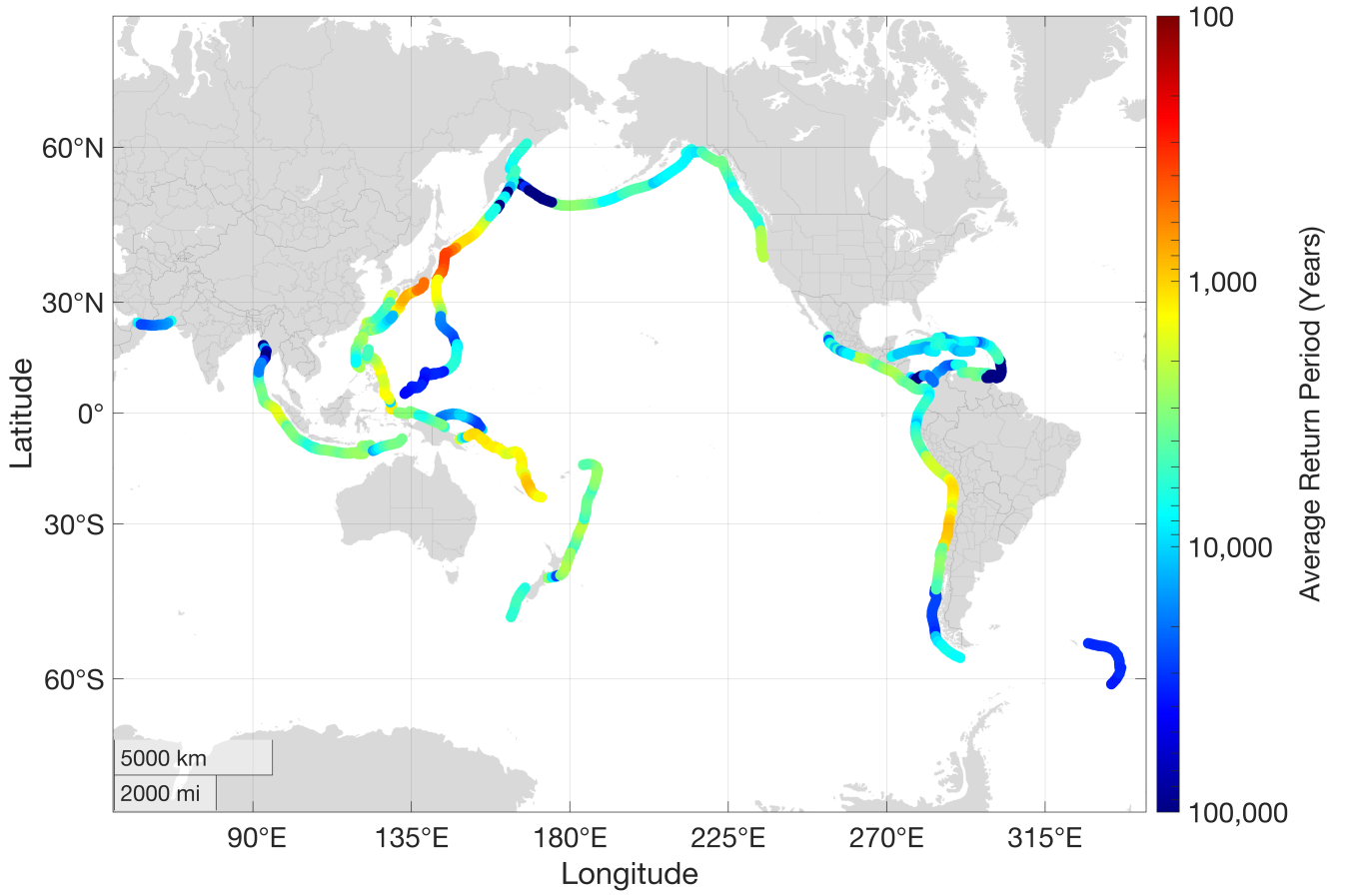


Figure 2. Return period of tsunamigenic earthquakes ($M > 7.4$) along the potential faults, based on historical data analysis, with a spatial resolution of 25 km. Basemap created with Natural Earth data.

3 ~~Ray-Tracing~~ Ray-tracing Methodology

To simulate global tsunami propagation and identify regions of energy focusing or shadow zones, we use a numerical wave ray-tracing model based on the geometrical optics approximation. This approach tracks the trajectory of wave energy packets (rays) by solving a set of coupled ordinary differential equations (ODEs) over time. The theoretical framework follows standard
190 formulations for surface gravity waves propagating over variable bathymetry and currents (Halsne et al., 2023).

3.1 Governing Equations

The model employs a kinematic ray-tracing approach. Rather than solving the dynamic transport equation for wave amplitude, which is prone to caustic singularities over complex deep-ocean bathymetry, we strictly solve the kinematic equations to track

the trajectories of discrete wave energy packets. The state of a wave ray is defined by its position vector, $\mathbf{x} = (x, y)$, and wave
 195 number vector, $\mathbf{k} = (k_x, k_y)$. Their evolution in time t is governed by:

$$\frac{d\mathbf{x}}{dt} = \mathbf{c}_g + \mathbf{U} \quad (3)$$

$$\frac{d\mathbf{k}}{dt} = -\nabla_h \omega - (\mathbf{k} \cdot \nabla_h) \mathbf{U} \quad (4)$$

Here, $\mathbf{U} = (U, V)$ is the ambient current velocity vector, $\mathbf{c}_g$ is the intrinsic group velocity vector, and ω is the intrinsic
 angular frequency of the wave. For far-field tsunami applications, ambient currents are negligible compared to the wave speed,
 200 so we set $\mathbf{U} = 0$ in Equations 3–4.

The angular frequency ω is related to the wave number magnitude $k = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$ and local water depth d through the
 linear dispersion relation:

$$\omega^2 = gk \tanh(kd) \quad (5)$$

Wave refraction is governed by the term $-\nabla_h \omega$ in Equation 4 (where $\mathbf{U} = 0$). For the characteristic period spectrum
 205 evaluated in this study, the waves strictly satisfy the shallow-water limit even in the deepest oceanic trenches. Because the
phase speed remains overwhelmingly depth-dependent, the bathymetric refraction gradients that dictate the ray trajectories are
fundamentally unaffected by frequency dispersion. Since ω depends on both k and d , its horizontal gradient is primarily driven
 by bathymetric variations:

$$\nabla_h \omega = \frac{\partial \omega}{\partial d} \nabla_h d \quad (6)$$

210 and because no diffraction term is included here, rays cannot reproduce energy leakage around obstacles. This shows that
 changes in the wave number vector, and thus the propagation direction, are directly proportional to the local sea-floor gradient,
 $\nabla_h d$, which is the main mechanism for tsunami refraction across ocean basins.

3.2 Numerical Implementation

The system of ODEs (Equations 3 and 4) is solved numerically to trace each ray's trajectory using a fourth-order Runge-
 215 Kutta scheme. Rays propagate over a global, unstructured triangular mesh ~~with approximately 1.3 million nodes and 2.6~~
~~million comprising 1,726,063 nodes and 3,452,122~~ elements, with spatial resolution varying from ~~10 to 28 km~~. ~~The 10.77 to~~
~~28.35 km (Figure A2). The variable-resolution unstructured~~ mesh is generated based on the shallow-water ~~group-velocity~~ wave,
 ensuring that tsunami wavelengths are adequately resolved. During the mesh generation process, bathymetric gradients and all
relevant geometric features that affect the wave refraction under study are explicitly taken into account. This feature-tracking

220 preserves the steep bathymetric gradients necessary for accurate refraction, forcing finer nodal placement (down to 10.77 km) along critical structures like seamount peaks and trench fracture lines, while allowing coarser resolutions (up to 28.35 km) in relatively flat abyssal plains, without requiring a uniform, ultra-high resolution domain.

Bathymetric data are sourced from 2025 General Bathymetric Chart of the Oceans dataset (GEBCO, 2025) with ~~4~~15 arc-second resolution. Using a time step of 180 s and Courant-Friedrichs-Lewy $CFL < 1$, the depth-dependent resolution is set as
225 the product of the time step and local group velocity, ensuring that no ray travels more than one element per time step.

At each time step, environmental parameters are evaluated dynamically, as rays can lie anywhere within a mesh element. Water depth at a ray's latitude-longitude location is computed using barycentric interpolation from the nodal values of the containing element. The bathymetric gradient, $\nabla_h d$, which drives wave refraction, is precomputed for all elements to improve computational efficiency.

230 3.3 Numerical Simulations

Rays are initialized along potential tsunami-generating fault lines, shown in Figure 2, at 25 km intervals. ~~Each ray is assigned a characteristic tsunami period ranging from~~ To ensure comprehensive coverage of potential refractive pathways, rays are initialized across a discrete spectrum of characteristic tsunami periods spanning the typical empirical range of 15 -minutes to 3 -hours , with the hours at each source location. By aggregating this discrete period spectrum globally, the kinematic model
235 effectively bounds the frequency-dependent variations in transoceanic energy focusing to construct the final exposure footprint. While real near-field radiation directivity depends on detailed 3D rupture kinematics—such as variable slip distribution, rake, and dip, the dominant macro-scale energy flux of large subduction megathrust earthquakes radiates predominantly orthogonal to the fault strike. Therefore, the initial propagation direction for all rays is set perpendicular to the ~~fault segment and oriented oceanward.~~ local fault segment. This simplification natively encodes the primary radiation lobes required for transoceanic
240 far-field propagation without necessitating complex 3D volumetric initializations. Furthermore, rays are oriented exclusively oceanward; shoreward-directed waves are excluded because their propagation is governed by local near-source complexities and non-linear shoaling, which fall outside the scope of basin-scale transoceanic exposure screening.

A ray is terminated if it reaches land (where water depth becomes zero or negative) or enters a user-defined shallow coastal zone (500 m depth). This cutoff is imposed to exclude the complex nearshore region where nonlinear shoaling dominates and
245 where strong refraction often leads to ray crossing (caustics), rendering the standard geometrical optics approximation insufficient. Furthermore, as waves enter these shallower depths, their decreasing group velocities and shorter wavelengths would require a proportionally higher-resolution mesh. Implementing such fine resolution along global coastlines would impose prohibitive computational costs, directly defeating the primary utility of this framework as a rapid transoceanic exposure screening tool. In total, 9,000 rays were released, and the simulation was run for 24 hours, sufficient for all rays to reach a ter-
250 mination criterion. Upon termination, the spatial density of ray impacts along the coastline is calculated. ~~This metric, referred to as the~~ By utilizing a ray-density proxy rather than a classical ray-tube energy flux calculation, the model natively bypasses the mathematical singularities associated with deep-water caustics (ray crossings). In this kinematic framework, caustics simply and stably register as localized zones of high ray concentration. This normalized ray termination count (ranging from 0 to 1) ;

serves as a ~~geometric proxy for relative coastal tsunami wave amplitude~~ proxy for the relative probability of exposure (focusing potential), rather than a measure of dynamic wave amplitude or absolute energy flux, effectively mapping regions of energy convergence (focusing) and divergence (shadowing). This framework allows efficient and accurate simulation of tsunami propagation across complex global bathymetry, illustrating how basin geometry controls wave energy distribution. The simulations were performed on a 32-core Mac Studio with 512 GB of unified memory. The code is parallelized, and since each ray is treated independently, the 24-hour simulation completed in approximately 12 hours.

This ray-tracing approach captures tsunami ~~hazard exposure~~ primarily for transoceanic propagation from the source across the basin to distant locations. Hazards in the immediate vicinity of the earthquake are not considered in this study, as our focus is on the effects of ocean-basin geometry on tsunami travel; near-source tsunamis may indeed be more catastrophic, but they fall outside the scope of this work.

To systematically examine how source geometry, recurrence intervals, and bathymetric refraction shape global tsunami exposure, we designed four targeted numerical experiments. Each scenario isolates a fundamental driver of tsunami propagation: (i) fault geometry, which controls the initial azimuth and directional bias of energy release; (ii) variable bathymetry, which refracts and redirects wave trajectories across the basin; and (iii) probabilistic weighting, which modulates ~~hazard exposure~~ intensity based on earthquake return periods. Figure 3 summarizes the outcomes, where basin-scale ray density is shown using a white-to-blue scale, and coastal focusing is highlighted by a green–yellow–red scale. All results are normalized by the global maximum to enable direct comparison across experiments.

The left panels (Figures 3a-b) adopt a constant-depth approximation, forcing rays to follow great circle paths (GC) and, by design, removing any effect of bathymetric refraction. This configuration exposes the sole influence of fault geometry and position, effectively reducing the ocean basin to a two-dimensional line-of-sight system. Under these conditions, ~~we observe a striking artifact: a~~ a highly energetic corridor extending extends across the Pacific Basin, originating from the Kuril–Kamchatka and Japan trenches. This direct projection leads to ~~an exaggerated and unrealistic a broad~~ signature of exposure, producing normalized coastal exposure indices between 0.8 and 1.0 along the northeast coast of Australia, Papua New Guinea, and the Solomon Islands, as well as ~~a broad, nearly uniform zone of impact~~ along South America. Panel (a), using uniform ray release, overemphasizes the role of smaller fault systems; Panel (b) incorporates the probabilistic weights (λ) from Section 2, refining the magnitude but ~~still retaining the unrealistic~~ retaining the linear propagation patterns inherent to the constant-depth assumption, keeping exposure indices artificially high (~ 0.8 – 1.0) along these direct line-of-sight coastlines.

~~A very different picture emerges in~~ In the right panels (Figures 3c-d), where realistic bathymetry is introduced. ~~Here, refraction and scattering fundamentally reorganize the tsunami wavefield. The artificial wave-field. For example, the~~ exposure corridor toward Northeast Australia is dismantled almost entirely reduced by approximately 80% (dropping from an exposure index of 0.8–1.0 down to 0.1–0.2) as rays encounter the complex topography of the Southwest Pacific—namely the Melanesian Arc and the Solomon Rise, ~~where strong refractive scattering disrupts coherent wavefronts. This bathymetric shielding drastically reduces predicted hazard levels along the eastern Australian coastline.~~ ~ In contrast, topographic structures in the North Pacific, such as the Emperor Seamount Chain and the Mendocino Fracture Zone, act as waveguides, channeling energy toward North America. ~~Relative to the broad energy field in Panel (b), the results in Panel (d) reveal a sharpened, high-intensity~~

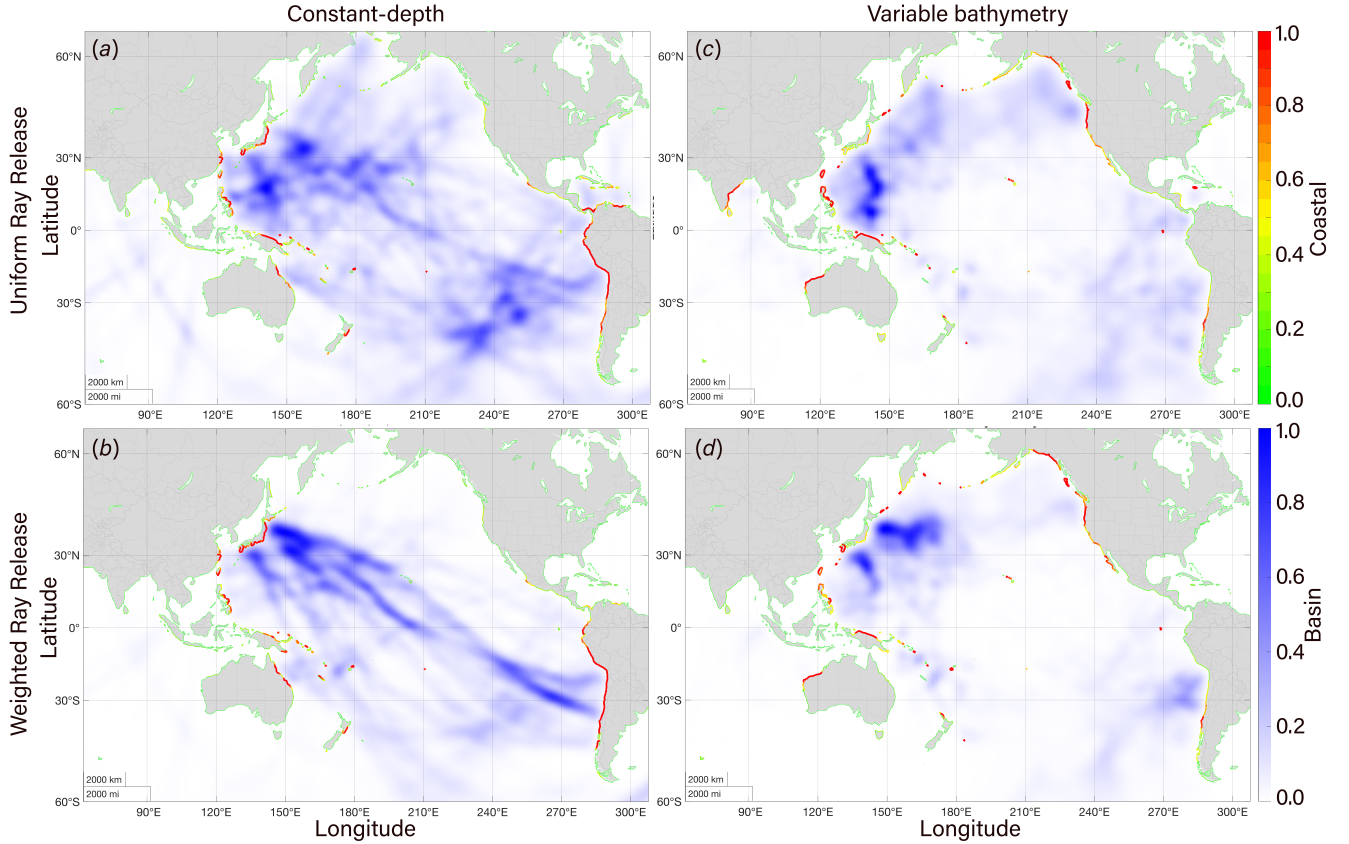


Figure 3. Basin-wide ray density and coastal tsunami hazard exposure for 9,000 tsunami rays released along potential fault zones (shown in Figure 2). Panels (a,b) represent constant-depth cases, highlighting the effects of fault location and orientation, while panels (c,d) incorporate variable bathymetry to show the combined influence of seafloor geometry and fault configuration. The top row shows uniformly released rays, and the bottom row applies probabilistic weighting (using the annual rate parameter λ from Section 2) based on return period derived from the Gutenberg–Richter relation. The ocean-floor heatmap (white to blue) illustrates ray convergence and geometric focusing, and coastline colors (green–yellow–red–) indicate relative coastal exposure using a normalized ray-count proxy (0–1) indicate relative tsunami hazard, with redder areas denoting higher exposure. Basemap created with Natural Earth data.

beam-targeting, increasing normalized coastal focusing in the Pacific Northwest and Northern California by over 50% (elevating the index from diffuse values of 0.4–0.6 up to highly localized peaks of 0.9–1.0) relative to the uniform depth case.

Panel (d) represents the most complete hazard-exposure characterization, combining realistic bathymetry with the probabilistic source weighting (λ) derived in Section 2. By modulating the intensity of each ray based on its source’s annual occurrence rate (λ), this configuration filters out low-probability contributions seen in Panel (c) and exposes the dominant corridors of trans-oceanic energy. Notably, it reveals a striking-reciprocal-hazard-reciprocal focusing pathway between South America and East Asia: just-as the Kuril–Kamchatka system preferentially feeds energy toward Chile, ~~(index > 0.8), and~~ the Peru–Chile trench returns a concentrated ray field toward Japan and the Philippines. Additionally, the model identifies a pronounced shadow zone in the Southeast Pacific—west of the East Pacific Rise—where energy is notably absent. This suggests that the ridge system acts as a large-scale defocusing boundary. Finally, unlike the uniform scenario, the ambient basin ray densities of 0.3–0.4 (seen in Panel b) are reduced to near zero (< 0.05). Finally, the probabilistic approach intensifies the signal over the Bay of Bengal and the Indonesian archipelago, elevating the normalized exposure index from background levels (~ 0.3 – 0.5 in Panel c) to peak values exceeding 0.8, where high recurrence rates of tsunamigenic earthquakes lead to elevated hazard levels that would otherwise be underrepresented exposure levels.

4 M8.8 Kamchatka Peninsula event

On July 29, 2025, at 23:24 UTC, a moment magnitude (Mw) 8.8 earthquake struck off the eastern coast of Russia’s Kamchatka Peninsula. The epicenter was located at 52.51°N, 160.32°E, approximately 136 km east-southeast of Petropavlovsk-Kamchatsky, at a depth of 35 km, according to the United States Geological Survey (USGS). The earthquake resulted from thrust faulting along the Kuril–Kamchatka subduction zone, where the Pacific plate subducts beneath the North American (Okhotsk) plate. This event ranks among the largest earthquakes globally since 2011 and is one of the six largest recorded in the modern instrumental era.

The earthquake generated a trans-Pacific tsunami, with maximum run-up heights of about 3 m in Severo-Kurilsk, Russia, and up to 1.2 m in Hawaii (NGDC/WDS, 2024; NOAA, 2025a). While initial warnings indicated potentially higher impacts, the remote location and rapid alert system limited casualties and damage. One indirect fatality was reported in Japan, and several injuries occurred. In Kamchatka Krai, moderate damage was observed, including power outages and structural impacts in Petropavlovsk-Kamchatsky (NGDC/WDS, 2024; NOAA, 2025b; USGS, 2025). This event underscores the potential of large subduction zone earthquakes in the North Pacific to generate widespread trans-oceanic tsunami hazards and highlights the importance of early warning systems for coastal risk mitigation.

The initial condition for the tsunami propagation model FUNWAVE is the residual vertical seafloor displacement field generated using the elastic half-space model of Okada (1985). For the M8.8 Kamchatka Peninsula event, this study utilizes two distinct source models, hereinafter referred to as the NOAA and USGS scenarios. The NOAA scenario is not derived from direct seismic measurements, but from an inversion of tsunami observations recorded at DART buoys. This approach uses the Method of Splitting Tsunami (MOST) model, which involves matching real-time observations to a linear combination of

pre-computed tsunami waveforms from unit sources (Titov et al., 2005). The resulting source model for this event consists of nine sub-faults, with characteristics shown in Table 1. In contrast, the USGS scenario is based on seismological inversion using ground motion data and finite-fault analysis, resulting in a more complex source defined by 345 sub-faults (Hayes, 2017). The NOAA solution commonly performs better for tsunami propagation because it assimilates offshore DART waveforms. The vertical coseismic displacement fields for both scenarios are illustrated in Figure 4.

To quantitatively compare these sources and estimate key tsunami parameters, a standardized method is applied to each deformation field. First, the primary generation area is identified by a polygon enclosing all points where the absolute displacement exceeds a 10% threshold of the maximum deformation. An oriented bounding box is then fitted to this polygon, yielding the source’s effective length (L_{eff}), effective width (W_{eff}), and average strike. Subsequently, a single characteristic length (L_c) is derived as the hypotenuse of the effective dimensions ($L_c = \sqrt{L_{eff}^2 + W_{eff}^2}$). This geometric proxy serves as a robust estimate for the dominant initial wavelength, explicitly linking the spatial extent of the seafloor uplift to the frequency content of the generated wave. This characteristic length, combined with the average ocean depth across the source region, is then applied to the linear dispersion relation to calculate the tsunami’s characteristic wave period (Geist, 2002). Table 2 summarizes these effective fault and tsunami parameters for both scenarios. For validation, these model-derived periods are later compared against the wave period determined from an analysis of the observed DART buoy recordings for this event.

Table 1. Fault parameters for NOAA scenario, with length 100 km, width 50 km, and rake 90°

Lon ($^\circ$)	Lat ($^\circ$)	Depth (km)	Strike ($^\circ$)	Dip ($^\circ$)	Slip (m)
160.2771	52.2889	38.2502	218	29	3.9036
158.8796	51.8229	62.4902	218	29	8.3453
158.5230	50.8438	38.2502	214	29	11.4335
159.0456	50.6201	15.5655	214	25	1.0997
158.0060	51.0634	62.4902	214	29	4.1837
157.7104	50.1261	40.5760	218	31	6.2835
158.1906	49.8844	16.3498	218	27	1.8474
157.2360	50.3631	66.3260	218	31	5.9568
156.8407	49.4333	40.5760	220	31	5.7094

4.1 High fidelity modeling (FUNWAVE)

For the simulation of tsunami wave propagation, we use the Boussinesq-type wave model, FUNWAVE, in spherical coordinates (Kirby et al., 2013). The model has been validated against a large set of analytical, laboratory, and field benchmarks as part of the development of tsunami hazard maps for the US East Coast (Tehranirad et al., 2011). An optimized MPI parallel implementation of FUNWAVE was developed, which has highly scalable algorithms, with a typical parallel scaling efficiency of more than 90% of the number of cores in a typical medium-sized computer cluster. This makes it possible to run the model over large ocean basin-scale grids with a sufficiently fine resolution.

Table 2. Comparison of NOAA and USGS fault scenarios

Parameter	NOAA	USGS
No. of sub-faults	9	345
sub-faults dimensions (km)	100×50	30×15
Fault effective length (km)	520	450
Fault effective width (km)	140	123
Characteristic length (km)	540	465
Mean depth (m)	2880	2960
Average strike (°)	217	218
Wave period (s)	3200	2700

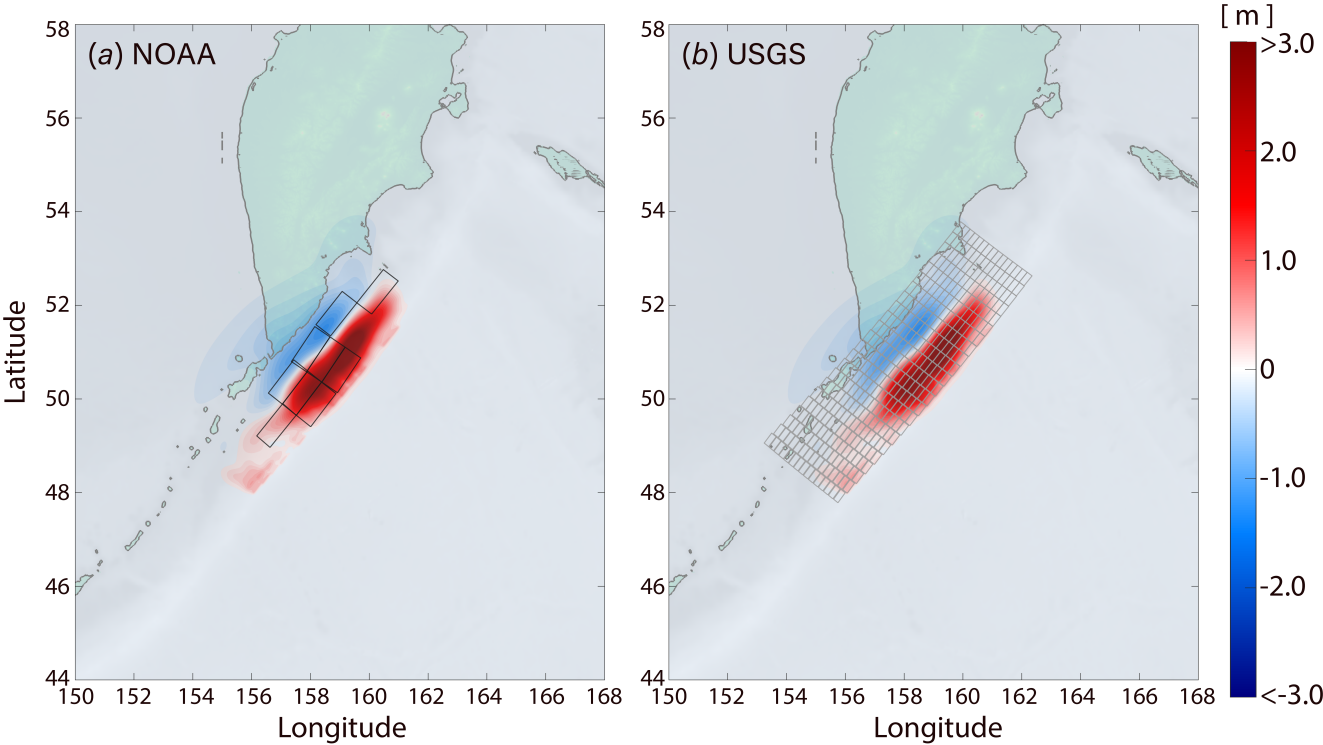


Figure 4. Residual vertical component of bed deformation for two scenarios based on inversion performed by a) NOAA with 9 sub-faults with 50×100 km dimension and b) USGS with 345 sub-faults with 15×30 km dimension.

The grid used in FUNWAVE simulations was developed from GEBCO (2025), which was downscaled from a resolution of 1-arc-second to 15 arc-seconds to 4 arc-minutes and cropped to focus on the Pacific Ocean, resulting in a final rectilinear grid comprising over 5 million points. Simulations were executed on an AMD EPYC 7H12 processor with a 2.6 GHz 64-core (128-thread) architecture, requiring approximately 50 minutes of real time to simulate 32 hours of tsunami propagation.

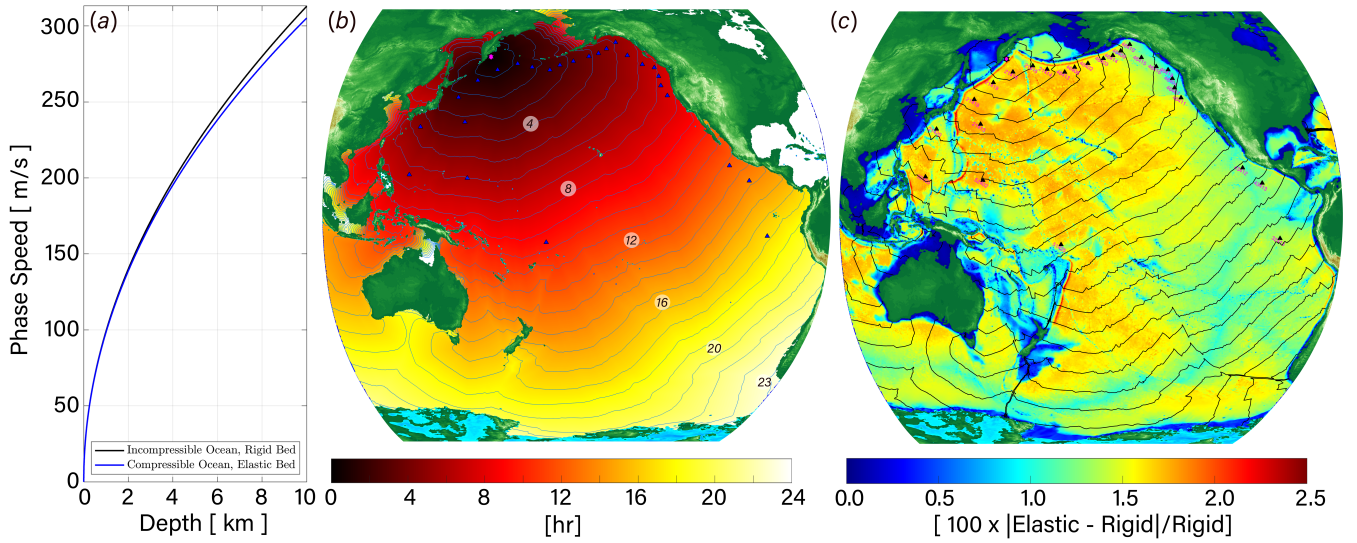


Figure 5. (a) Phase speed from the classic dispersion relation for an incompressible ocean over a rigid bottom (black) and for a compressible ocean with an elastic bottom and background density (blue); (b) Surface gravity wave travel time derived from the blue curve in panel (a); black contours show travel time at 1-hour intervals; (c) Percentage reduction in phase speed between the black and blue curves in panel (a); black contours show the difference in travel time between the two dispersion relations at 1-minute intervals.

To mitigate wetting and drying numerical artifacts along the coastline while preserving the steep bathymetric gradients of the Kuril-Kamchatka subduction zone essential for accurate wave generation, a spatially selective smoothing approach was adopted. A ramp function (Figure 6) was constructed around the 10-meter depth contour to linearly transition from the original bathymetry in the deep ocean to a Gaussian-smoothed grid in shallow coastal waters. This approach effectively ensures numerical stability in the nearshore region without compromising the bathymetric fidelity at the earthquake epicenter. Additionally, sensitivity simulations conducted on ~~higher-lower~~ resolution 8 and 16 arc-minute grids yielded negligible differences in results, confirming that the chosen 4 arc-minute resolution is adequate for resolving the initial seafloor displacement and modeling transoceanic tsunami propagation.

For both cases, NOAA and USGS, FUNWAVE simulations were performed using the bed deformation as the initial free surface water height. Figure 7 depicts the time history of generation and propagation of tsunami waves in the computational domain for the NOAA case. Figure 7 depicts the time history of generation and propagation of tsunami waves in the computational domain for the NOAA case with zero initial velocities. Simulations initialized with velocities from the elastic half-space model overestimate wave heights at DART stations for both the NOAA and USGS cases. In contrast, simulations with zero initial velocities provide more accurate predictions of wave heights at the DART locations (disregarding the phase lag), exhibiting minimal over-prediction or under-prediction of wave heights. This result indicates that within the framework of Boussinesq-type models like FUNWAVE, the initial bed-deformation is sufficient for effectively modeling tsunami wave propagation accurately.

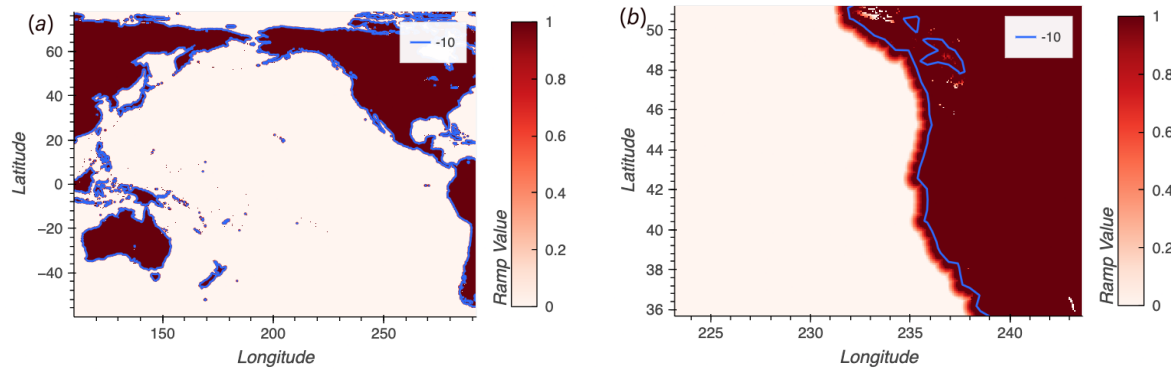


Figure 6. (a) Ramp function used to smooth transition the original grid to the Gaussian smooth grid. (b) Enlarged view of the ramp function along the United State's West Coast. Blue contour line denotes the 10 meter depth contour in both panels.

365 4.1.1 Phase lag correction

In FUNWAVE, the ocean is represented as incompressible, without background density, and

Standard Boussinesq models assume an incompressible ocean over a rigid bottom. This assumption produces faster phase speeds bed, causing theoretical phase speed overestimations and earlier arrivals, with the effect amplified at distant stations. Without correction, this theoretical limitation results in large temporal errors, representing a major drawback of standard incompressible models for far-field hazard assessment. To correct this bias, the lag was we quantified and applied a kinematic time-shift to the model outputs. Crucially, this correction is implemented strictly as a kinematic time-shift to align arrival times; it does not adjust, without adjusting the hydrodynamic wave amplitudes calculated by the model.

Tsunami periods were first estimated from the characteristic fault length (465–540 km) and mean depth (~3000 m), yielding values of 2700–3200 s for the NOAA and USGS scenarios (Table 2). To validate these estimates, a 12-h observational window at DART stations was analyzed (Fig. 8b), indicating an average period of 3600 s. Phase speeds were then calculated using (i) the classical dispersion relation under incompressibility and rigid-bottom assumptions, and (ii) a compressible-ocean formulation with elastic bed and background density (Abdolali et al., 2019), as shown in Figure 5a. We utilize the formulation by Abdolali et al. (2019) and Abdolali et al. (2026), which incorporates water compressibility, Earth elasticity, and background density to calculate accurate phase speeds. These values were incorporated into the a shortest-path framework to compute arrival (Kadri et al., 2025) to compute precise arrival times. Figure 5 illustrates this correction, displaying the travel times (Kadri et al., 2025).

Figure 5 b presents tsunami travel times derived from Dijkstra's algorithm (Kadri et al., 2025), which operates on graphs with non-negative edge weights, employing a greedy strategy to update shortest-path distances from the source node while maintaining a set of visited nodes and a priority queue. The travel times from solution (ii) are shown with 1-h contour intervals across the Pacific basin; DART buoy locations are indicated by rectangles. Figure 5c shows the phase speed from the full solution (ii) in the background, with contours representing the difference in arrival times between the two formulations (in

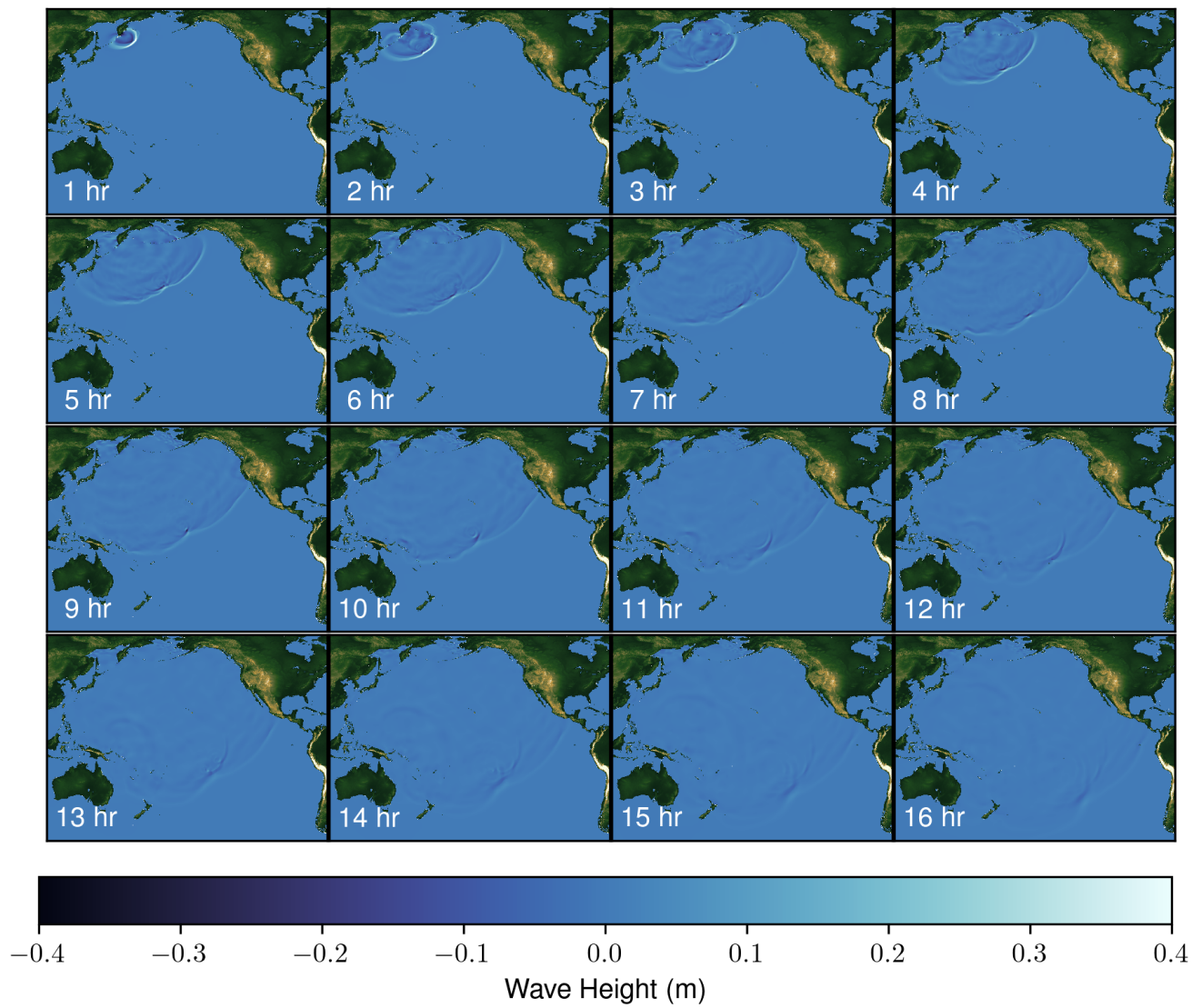


Figure 7. Free surface height (η) from FUNWAVE simulation initialized with bed deformation from NOAA scenario. Simulation results are shown at 1-hour intervals across the panels.

~~minutes)~~and the calculated time lag between the standard and compressible-ocean formulations. These differences (listed in 8c) at the DART sites were used to shift the FUNWAVE outputs, aligning modeled arrivals with observations (Fig. 8a).

Statistical analysis of phase-lag–corrected FUNWAVE outputs at DART stations demonstrates that the model effectively captures the tsunami signals generated by both source scenarios, with superior performance achieved using the NOAA fault configuration. The NOAA-forced simulation yields a Root Mean Square Error (RMSE) of 0.057, approximately 35% lower than the USGS RMSE of 0.088, indicating a substantially smaller overall prediction error. Furthermore, the NOAA solution exhibits higher model fidelity, characterized by a Skill Factor of 0.955 and a low Scatter Index (SI) of 0.151, compared to the USGS scenario, which produced a Skill Factor of 0.915 and a significantly larger SI of 0.380.

4.2 Spatial ~~Hazard~~Exposure Assessment

With the hydrodynamic model successfully validated against DART observations, we now evaluate the spatial distribution of tsunami energy across the Pacific basin. This step bridges the probabilistic and geometric frameworks outlined in Sections 2 and 3 with the deterministic, high-fidelity simulation of the 2025 Kamchatka Peninsula event.

To evaluate the predictive capability of the ray-tracing framework, the model was initialized using the fault parameters of the Kamchatka event (NOAA scenario) listed in Table 2. ~~Rays were released along the fault line using a characteristic wave period of 3200 s and a fault length of 540 km, with a strike of 217°.~~A total of 270 rays were emitted perpendicular to the 540 km, with a strike of 217°.~~Rays were emitted perpendicular to the fault orientation, with a 3200 s (53 minutes) characteristic period.~~

The resulting normalized ray termination counts are shown in Figure 9a. Note that ray-based shadow zones appear sharper because diffractive spreading is absent in geometric optics. The simulation reveals clear corridors of energy focusing. In the ~~near-field~~near-field, high ray densities emerge along the Aleutian Islands and Alaska, primarily governed by the fault orientation and source proximity. In the ~~far-field~~far-field, bathymetric refraction organizes tsunami propagation into a dominant southeastward beam that first targets the Hawaiian Islands and then continues into the Southeast Pacific, ultimately forming a coherent refractive corridor that extends southward toward the coast of Chile. A second beam projects eastward toward the North American coast, notably targeting Washington, Oregon, and Northern California.

These geometric patterns are strongly supported by the full hydrodynamic simulation. Figure 9b shows the maximum wave amplitude envelope computed by FUNWAVE. The spatial distribution closely aligns with the convergence zones identified in panel a, while also capturing additional physical processes absent in the ~~ray-tracing~~ray-tracing model. First, the ray-based simulation produces sharp shadow zones behind island chains (e.g., southeast of Hawaii), an expected consequence of its geometrical optics approximation. In contrast, FUNWAVE shows softer gradients, where diffraction allows partial energy penetration into these shadow regions. Second, the hydrodynamic solution reveals faint concentric ripples radiating from the continental margins—reflected energy that is naturally captured in FUNWAVE but purposely omitted in the ray-tracing model, where rays terminate upon coastal impact.

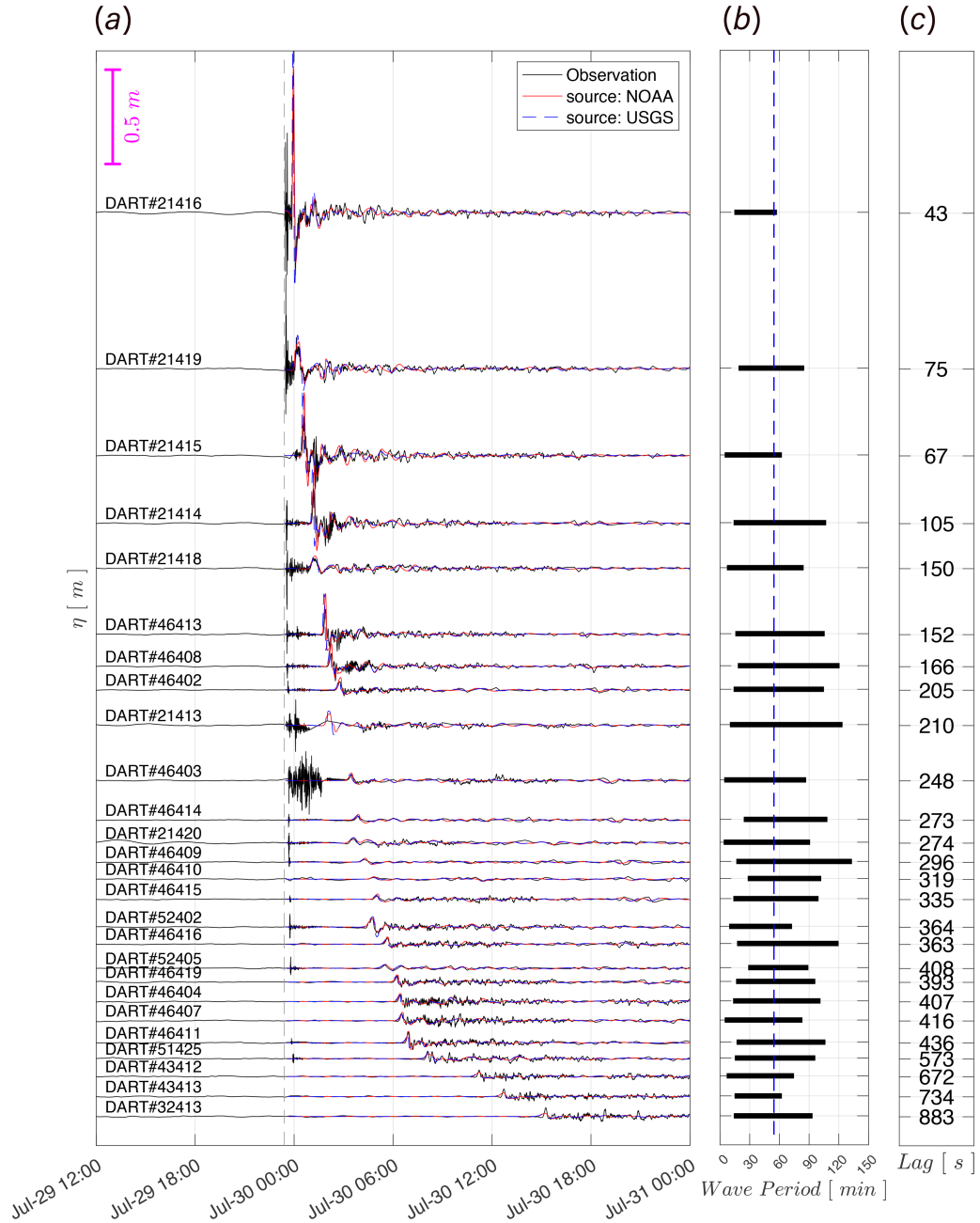


Figure 8. (a) Time series at DART observatories (black), with FUNWAVE simulations forced by the NOAA fault scenario (red) and the USGS scenario (blue). Stations are ordered by distance from the epicenter (magenta star in Fig. 5). The locations of the DART buoys are shown in Fig. 5. The vertical dashed line marks the earthquake origin time. (b) Estimated range of tsunami wave periods at all stations. The vertical dashed blue line indicates the overall mean period (~ 60 min), used in the phase speed calculation. (c) Time lag between computed arrivals based on the standard dispersion relation and those incorporating ocean compressibility, Earth elasticity, and background density. The FUNWAVE results in panel (a) are shifted according to the values in panel (c).

420 While a strong qualitative agreement is visually evident in the spatial alignment of energy corridors shown in Figure 9, we further evaluated the model's performance statistically. To ~~quantify the consistency between the geometric proxy and the hydrodynamic simulation, we extracted~~ evaluate far-field predictive performance against the fully coupled FUNWAVE hydrodynamic simulations, a Spearman rank correlation was calculated for spatially co-located ~~pairs of maximum wave amplitude from the FUNWAVE simulation and normalized ray density along the transoceanic coastlines, excluding the near-field~~ deformation zone where finite-fault effects dominate. Spearman's rank correlation coefficient (ρ) was selected to evaluate the monotonic relationship between the geometric proxy and the physical wave heights, assessing the model's ability to correctly rank hazard intensities rather than assuming linear proportionality. The analysis yields a correlation of $\rho = 0.66$, indicating a substantial positive relationship. This metric confirms that, despite the omission of diffraction and non-linear integration in the geometric optics approximation, the ray-tracing framework reliably captures the dominant spatial hierarchy of transoceanic ~~energy distribution~~ coastal amplitude distributions. To isolate far-field refractive propagation from localized static coseismic deformation, 161 coastal observation points were retained after explicitly excluding all points within a 1,000 km radius of the epicenter, yielding a baseline correlation of $\rho = 0.65$.

435 Despite these differences, both models consistently identify the primary energy corridors toward Hawaii and the U.S. West Coast. It is worth emphasizing that while FUNWAVE resolves extreme wave heights near the source (e.g., Russia and ~~Because~~ standard significance testing ignores the spatial autocorrelation of coastal tsunami amplitudes, a spatial block-bootstrap resampling scheme was implemented to estimate unbiased confidence bounds. The remaining coastline was partitioned into contiguous 500 km segments along the coastal trajectory, which were randomly sampled with replacement over 10,000 iterations to preserve local spatial dependence. This methodology yielded an unbiased 95% confidence interval of [0.35, 0.83]. Because the confidence bounds remain strictly positive, the ~~Kuril Islands~~), correlation is statistically significant ($p < 0.01$). Sensitivity ~~analyses across 300 to 800 km block sizes confirmed that while confidence interval widths scale inversely with effective degrees of freedom, the lower bounds remain confidently above zero, confirming that the correlation is robust and independent of the selected bin size. Ultimately, despite underpredicting amplitudes in diffraction-dependent coastal shadow zones, the spatially adjusted analysis confirms a highly significant, physical monotonic relationship between the rapid screening predictions and high-resolution hydrodynamic benchmarks in the ray-tracing approach does not attempt to capture near-field, finite-fault deformation effects; its strength lies in resolving transoceanic propagation pathways with high computational efficiency~~ far-field.

445 Finally, these modeled hotspots are corroborated by in-situ observations. Figure 10 presents the maximum water levels recorded at NOAA tide gauge stations. The observational record exhibits strong agreement with both the ~~ray-tracing~~ ray-tracing and hydrodynamic models. The highest amplitudes (red and orange markers) occur precisely where both frameworks predicted energy focusing—along the Hawaiian Islands and the U.S. West Coast. Importantly, the secondary southern beam identified in both the ray-tracing solution (Figure 9a) and the hydrodynamic envelope (Figure 9b) is confirmed by an elevated water level at a gauge in the Southeast Pacific (Figure 10), demonstrating the model's ability to resolve narrow, refractive transoceanic pathways over great distances.

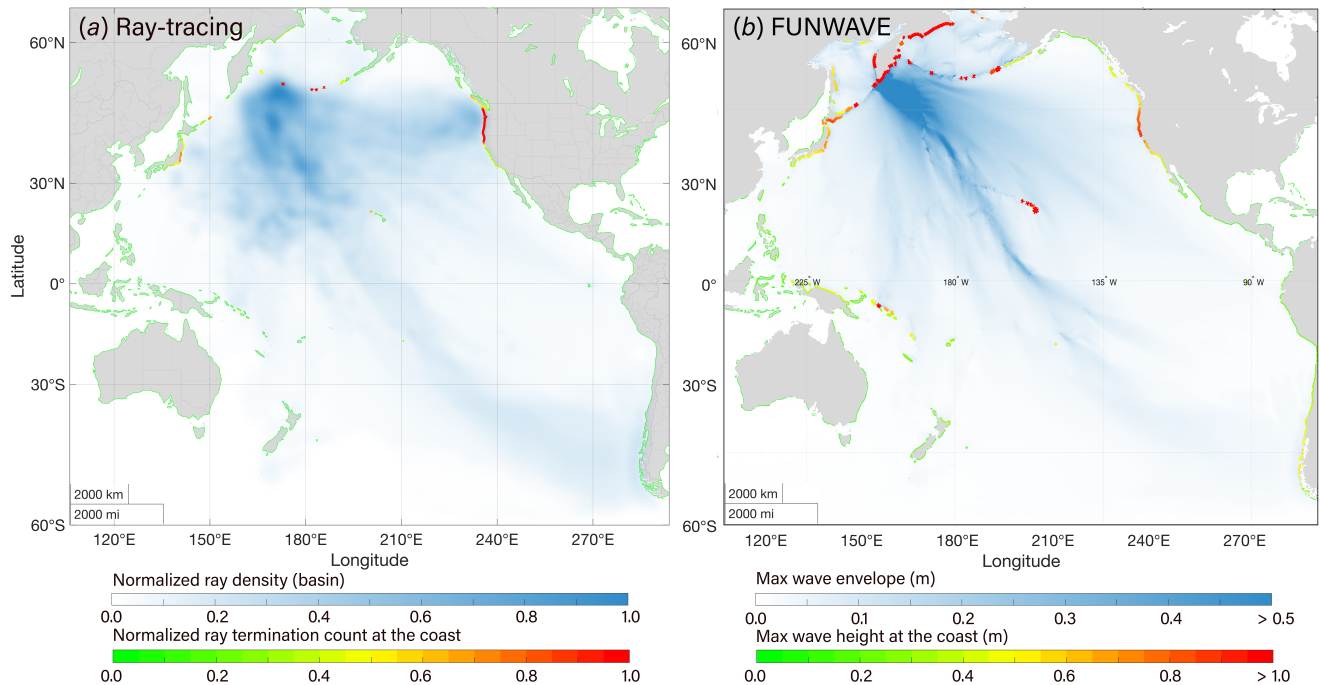


Figure 9. (a) Basin-wide ray density and coastal tsunami ~~hazard exposure~~ derived from the ray-tracing model, where the coastline metric is a normalized ray-count proxy (0–1); (b) Maximum free-surface elevations from FUNWAVE simulations for the 2025 Kamchatka Peninsula event, with the basin-wide field shown using the white–blue colormap and coastline segments using the green–yellow–red colormap. Basemap created with Natural Earth data.

Collectively, these results show that when informed by accurate source mechanics, the ray-tracing framework serves as a robust and computationally efficient proxy for identifying far-field coastal exposure and quantifying basin-scale tsunami ~~hazard exposure~~.

5 2011 Tohoku-oki Tsunami

As a second case study, we apply the proposed ray-tracing framework to the March 11, 2011, M_w 9.0 Tohoku-oki megathrust earthquake to evaluate its performance under a different fault rupture geometry and source configuration. This event generated one of the largest trans-Pacific tsunamis ever recorded, characterized by highly directional energy radiation and pronounced far-field energy beaming across the Pacific Ocean (Kirby et al., 2013; Abdolali and Kirby, 2017). Figure 11 shows snapshots of the simulated free-surface elevation during the first 16 hours of propagation, illustrating the strong directivity of the tsunami wave field as it traverses the Pacific basin.

Time series analyses of deep-ocean observations indicate a dominant mean wave period of approximately 35 minutes across the Pacific DART buoy network (Abdolali et al., 2019). Using the phase-lag correction procedure described in Section 4.1.1,

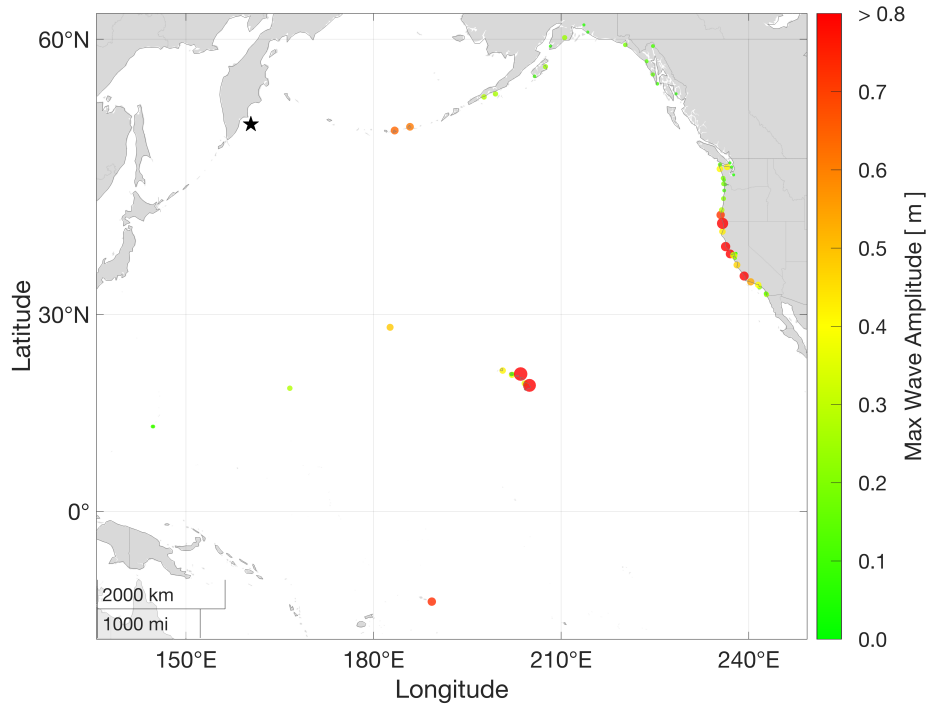


Figure 10. Maximum observed water levels at NOAA tide gauge stations along the coastline. Basemap created with Natural Earth data.

465 this dominant wave period was used to correct the high-fidelity FUNWAVE simulations. The phase correction successfully aligned the modeled tsunami arrivals with the real-time de-tided observations recorded at DART stations throughout the Pacific (Figure 12).

We next evaluate the spatial exposure using the geometric ray-density metric described in Section 4.2. Figure 13 compares the normalized ray-count distribution with the maximum free-surface elevation obtained from the baseline FUNWAVE simulation.

470 The ray-tracing framework successfully reproduces the primary regions of tsunami energy concentration and coastal exposure associated with this event. The spatial block-bootstrap analysis resulted in a correlation coefficient of $\rho = 0.631$ (95% CI: 0.36–0.77, $p < 0.01$), demonstrating statistically significant agreement between the rapid geometric proxy and the full hydrodynamic solution. These results further confirm that the proposed unstructured ray-tracing approach provides a computationally efficient and robust framework for rapid tsunami exposure assessment across different fault geometries and ocean-basin scales.

475 6 Conclusions

This study introduced a hybrid probabilistic–geometric framework, intended for rapid ~~farfield-hazard~~ far-field exposure screening rather than local inundation modeling, that bridges the gap between statistical seismic ~~hazard~~ exposure assessment and physically grounded tsunami propagation modeling. By integrating earthquake recurrence statistics derived from the Guten-

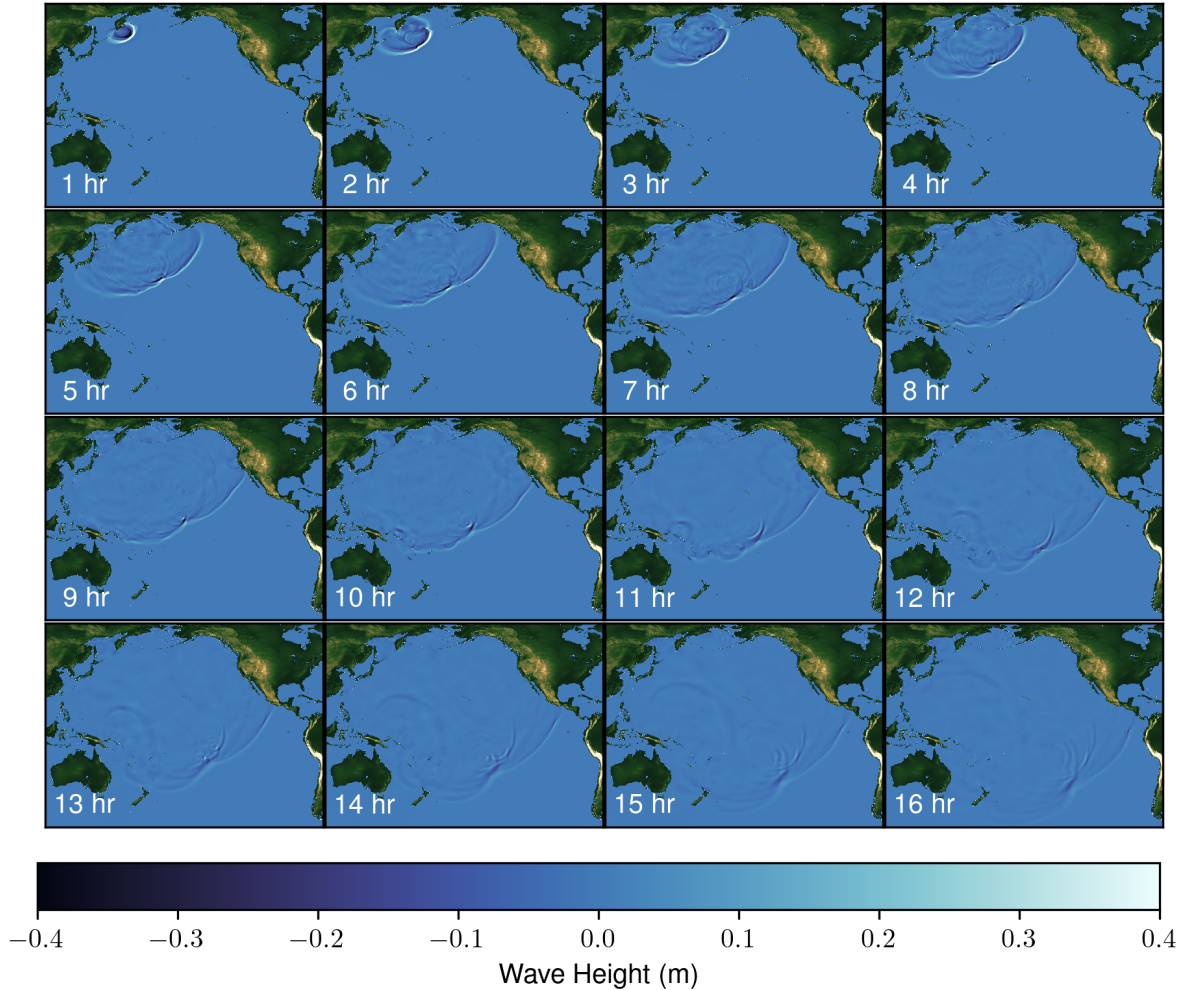


Figure 11. Spatiotemporal snapshots of the free surface height (η) simulated in FUNWAVE at 1-hour intervals, illustrating the highly directional far-field energy beaming across the Pacific Ocean (Kirby et al., 2013).

berg–Richter relation with basin-scale ~~ray-tracing~~ray-tracing, the method efficiently maps global patterns of energy conver-
480 gence and identifies coastlines most susceptible to transoceanic tsunami exposure. This approach provides a rapid, computa-
tionally scalable alternative to full hydrodynamic ensemble modeling, making it suitable for global screening and early-stage
~~hazard~~exposure prioritization.

The framework was rigorously validated using the 2025 M8.8 Kamchatka Peninsula earthquake as a benchmark case. Phase-
corrected FUNWAVE simulations showed strong agreement with in-situ DART and tide-gauge observations, confirming that
485 the hydrodynamic model, when corrected for compressibility, earth elasticity and background density, reproduced the key
timing and amplitude characteristics of the event. Building on this foundation, the ray-tracing model, initialized with the

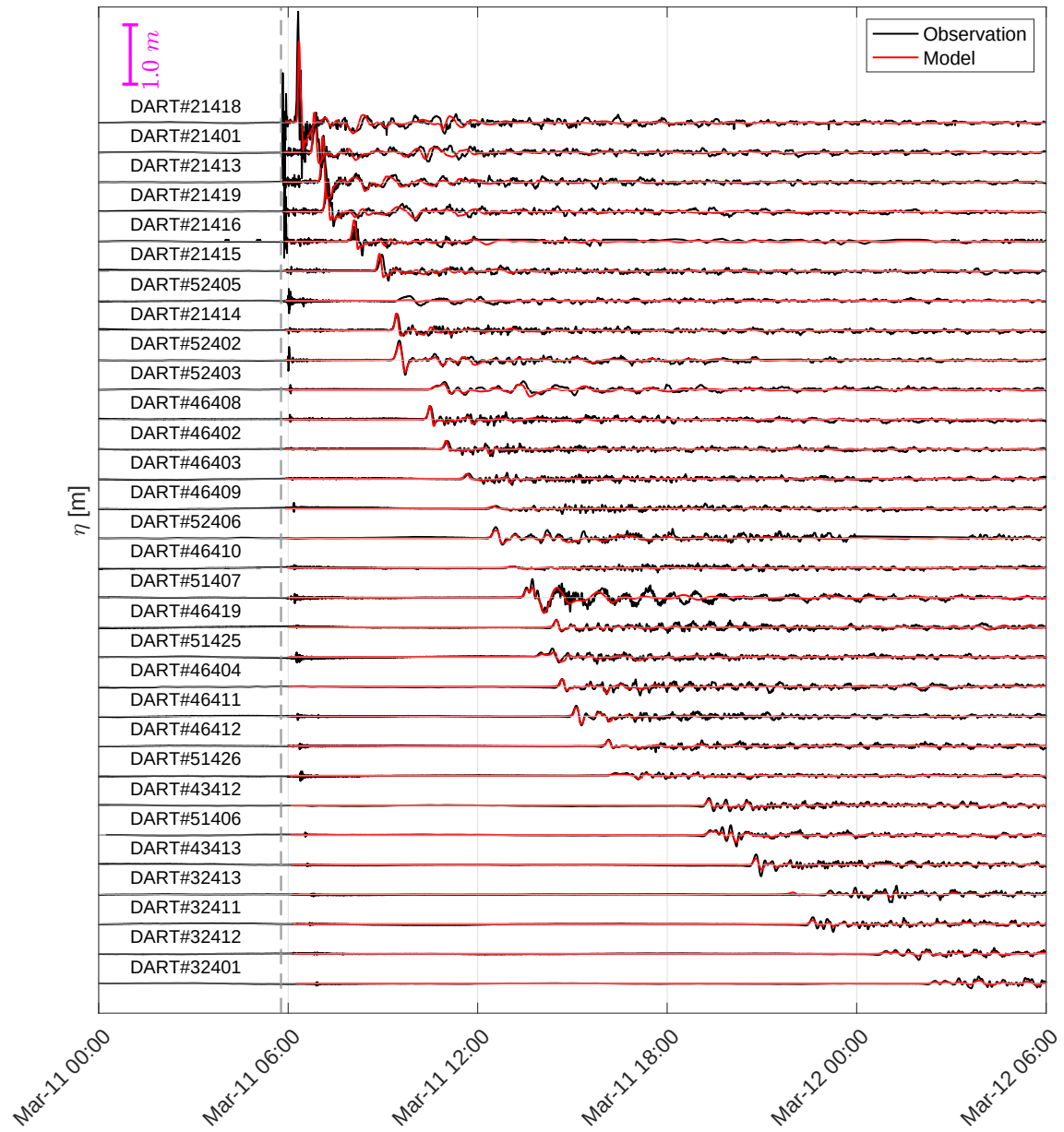


Figure 12. [Validation of phase-lag-corrected FUNWAVE hydrodynamic simulations \(red\) against de-tided observations \(black\) recorded at deep-ocean Pacific DART stations \(Abdolali et al., 2019\).](#)

optimized NOAA source parameters, successfully resolved the dominant transoceanic energy trajectories. Both the [ray-tracing](#) and hydrodynamic models identified the primary focusing pathway toward Hawaii and the U.S. West Coast, as well as the refraction-driven beam directed toward Chile.

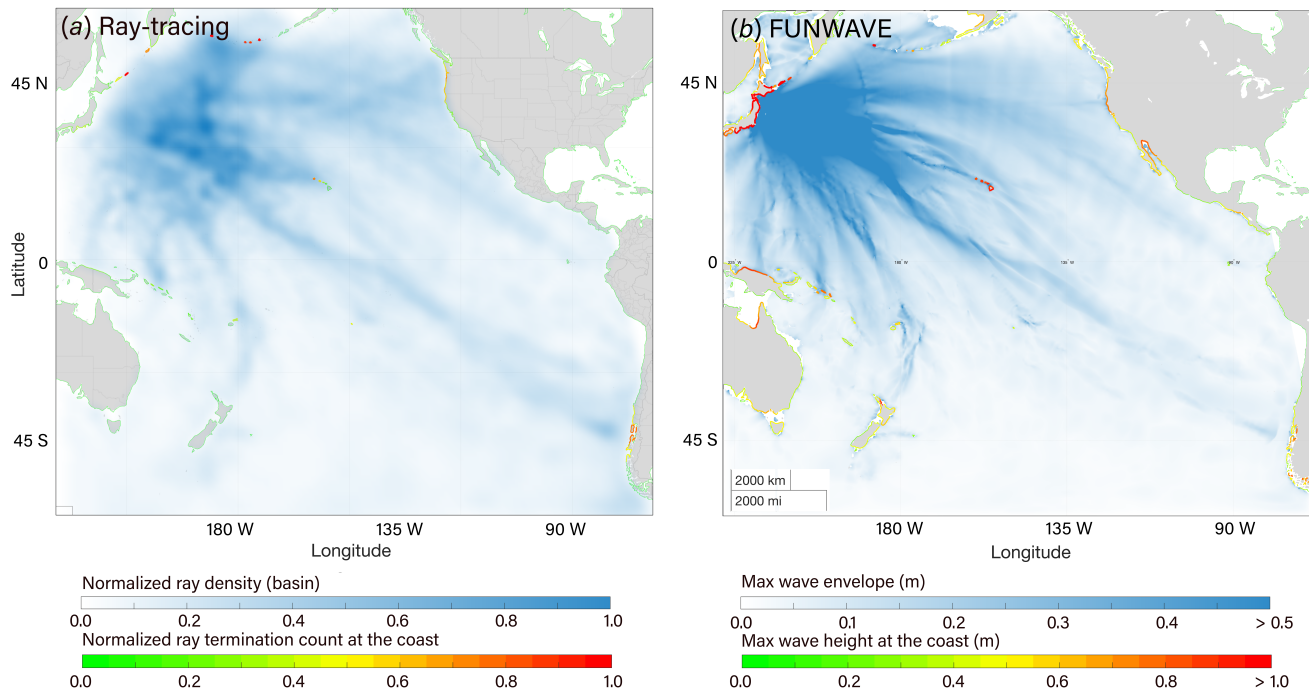


Figure 13. Comparison of the 2011 Tohoku-oki event showing (a) normalized ray-count proxy (0–1) and (b) the baseline FUNWAVE maximum wave envelope (η_{max}). Basemap created with Natural Earth data.

490 The comparative analysis highlighted both the utility and the physical limitations of the geometric approach. While the ray-tracing model accurately predicted the location of far-field hotspots, it produced sharp shadow zones that neglected the diffraction effects captured by the weakly dispersive hydrodynamic model. Despite this approximation, the spatial alignment of maximum energy envelopes across observations, FUNWAVE simulations, and ray clusters, quantified by a Spearman correlation of $\rho = 0.66$, demonstrates that bathymetric refraction is the primary mechanism governing basin-scale tsunami intensity

495 distribution.

Ray-tracing Ray-tracing, weighted by recurrence-based probability, directly addresses the need for scalable **hazard-exposure** assessment. By filtering theoretical energy exposure through the lens of recurrence likelihood, the model provides a prioritized map of coastal regions where tsunami impacts are both physically focused and statistically probable. This combination offers a robust practical utility for coastal resilience, as consistently identified along high-energy refraction pathways can be flagged

500 for focused vulnerability assessment and the allocation of high-fidelity modeling resources.

To date, many new construction initiatives along the coastlines require design analysis against both the extreme weather hazards and seismic-generated wave flooding events. Frameworks like those described in this manuscript can give rise to improved operational utility and creation of a single pane of glass, or a system of systems for hazard mitigation. In summary, this work demonstrates that a probabilistic–geometric approach serves as a defensible and efficient proxy for global tsunami **hazard**

505 [exposure](#) assessment, guiding decision-makers toward the communities facing the highest likelihood of future transoceanic impact.

Code availability. The FUNWAVE-TVD model code used in this study is publicly available on GitHub at <https://github.com/fengyanshi/FUNWAVE-TVD>. The current version of Okeanos-Ray, including the code and input files to produce the results shown in this paper, can be accessed from the Zenodo archive (<https://doi.org/10.5281/zenodo.21461088>) under GNU GPL v3.0 Licensing.

510 **Appendix A:** [Supplementary figures](#)

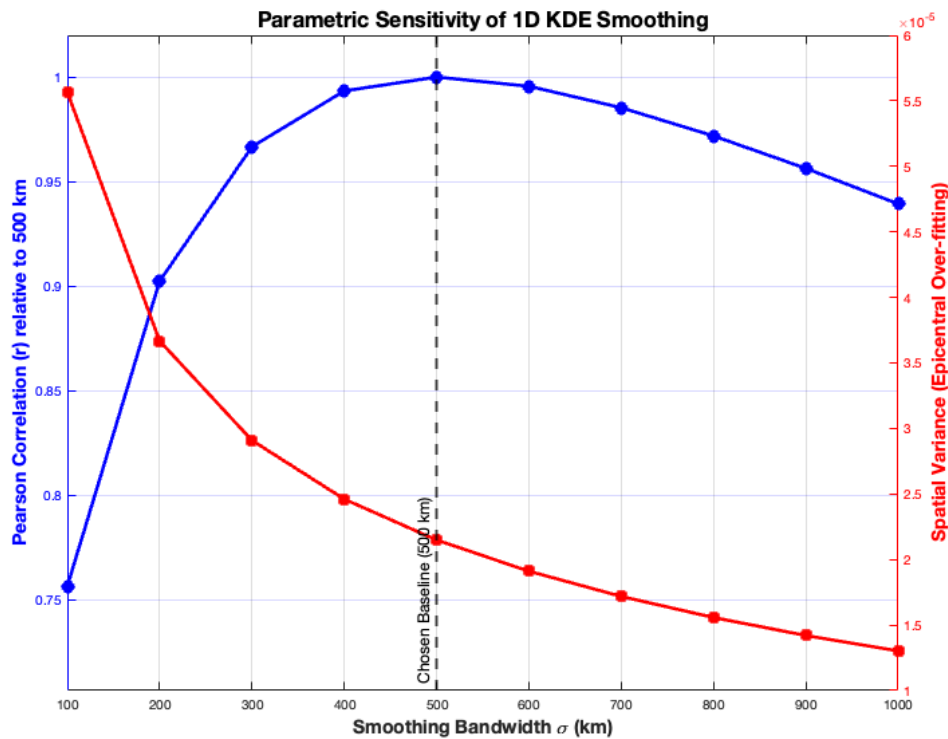


Figure A1. [Parametric sensitivity of the 1D Kernel Density Estimation \(KDE\) smoothing bandwidth \(\$\sigma\$ \).](#) The red curve (right axis) shows [spatial variance](#); values stabilize at the 500 km baseline, consistent with characteristic rupture lengths of great (M_w 8.8–9.0) megathrust events. The blue curve (left axis) shows the Pearson correlation coefficient ($r > 0.95$) relative to the 500 km baseline, demonstrating [geographic stability across the bandwidth spectrum](#).

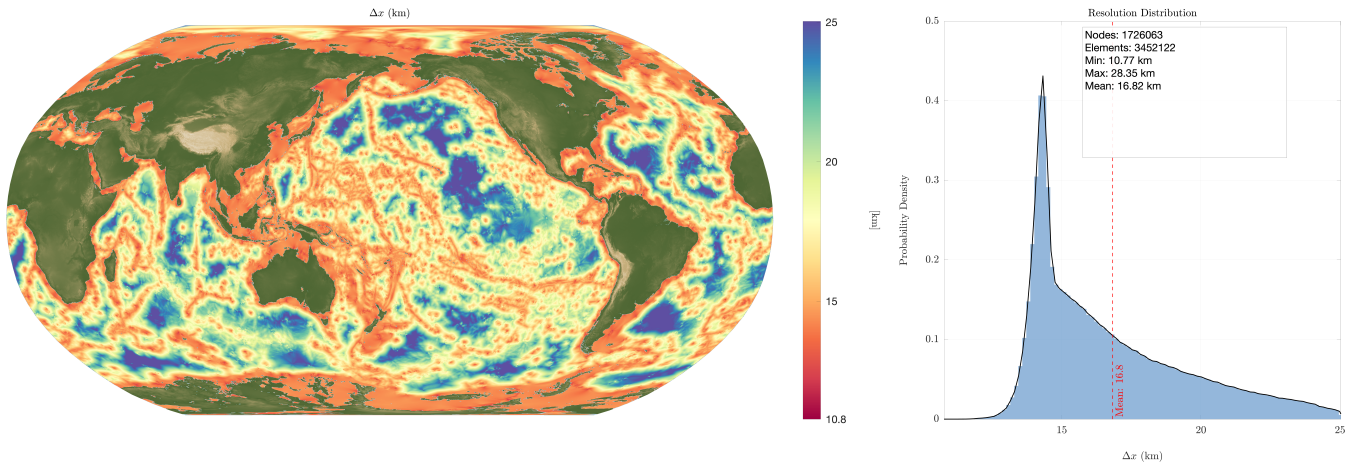


Figure A2. Spatial distribution and statistical properties of the variable-resolution unstructured ray-tracing mesh. (Left) Global map of the element sizes (Δx in km) demonstrating the feature-tracking nodal generation. Finer resolutions (red/yellow, ~ 10.8 – 15 km) are explicitly forced along high-gradient bathymetric features such as seamount chains, mid-ocean ridges, and subduction trenches to preserve critical refractive boundaries. Coarser resolutions (blue, up to 28.35 km) are restricted to relatively flat abyssal plains. (Right) Probability density distribution of the element sizes, detailing the specific mesh geometry of $1,726,063$ nodes and $3,452,122$ elements with a mean resolution of 16.82 km.

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Author contributions. AA: Conceptualization, Methodology, Software (Ray-tracing model development), Validation, Formal analysis, Investigation, Data Curation, Supervision, Visualization, and Writing – original draft. ML: Investigation, FUNWAVE simulation, Visualization, and Writing – review & editing. UK: Methodology, Investigation and Writing – review & editing. MM: Investigation, FUNWAVE simulation and Writing – review & editing. MF: Investigation, Software (Tsunami Travel Time/GREAT) and Writing – review & editing. FS: Investigation, Software (FUNWAVE model development) and Writing – review & editing.

Competing interests. The authors declare that they have no competing interests

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