

Thank you for addressing the concerns raised in the original review. I believe that the authors' responses to questions raised have been sufficiently addressed except for the response concerning including uncertainties on the estimates of R , μ , and σ .

In the author's response, they say

“However, not all quantities reported in the scatter-plot summaries are of the same statistical type. For fitted parameters such as the slope, uncertainty estimates are directly defined and are standard to report. For descriptive metrics such as the mean bias (μ) and spread (σ), uncertainty intervals can in principle also be estimated, but this is less standard and requires additional assumptions about statistical independence and sampling.”

I'm not sure what the authors mean by 'directly defined' and what the distinction between 'fitted parameters' and 'descriptive metrics' is. Apologies in advance if I am misunderstanding the response, but from the above sentences, I interpret the authors as implying that 'fitted parameters' such as the slope have uncertainty estimates that are not dependent on additional assumptions about the distributions of the data or their statistical independence (again, let me know if I have misinterpreted the statements above).

If I am correct in the interpretation of the author's response, the statement asserted is not correct. All uncertainty estimations rely on some sort of assumption about distributions and data independence. On page 371 of *Introduction to Probability and Statistics for Engineers and Scientists* (Ross, 2020), which examines the least square estimators for the linear equation $y = A + Bx$, states

“To specify the distribution of the estimators A and B , it is necessary to make additional assumptions about the random errors aside from just assuming that their mean is 0. The usual approach is to assume that the random errors are independent normal random variables having mean 0 and variance σ^2 .”

The 'usual approach' mentioned in Ross (2020) are the exact same assumptions most commonly used for analytic estimates of uncertainties R , μ , and σ .

For μ , not only are the standard set of assumptions identical to those for the slope of a linear regression, μ (and therefore its uncertainty) is a special case of a regression. For the general ordinary least squares problem of solving the equation

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

the best estimate of $\boldsymbol{\beta}$ is given by

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

and the variance of this estimate is given by

$$\text{Var}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}$$

where σ^2 is the variance of the residual ϵ (see https://en.wikipedia.org/wiki/Ordinary_least_squares or Chapter 9 of Ross (2020)). These expressions not only give the formulas for linear regression, but also the mean by taking \mathbf{X} as

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \end{bmatrix}$$

in which case

$$\hat{\beta} = N^{-1} \sum_{i=1}^N y_i$$
$$\text{Var}(\hat{\beta}) = \sigma^2 N^{-1}$$

where N is the number of data points. Since $\mu = \beta$ in this case, I don't know why μ would not be considered a 'fitted parameter' (although I don't think this distinction makes sense for R or σ either).

For the claim that uncertainties on R , μ , and σ are 'less standard', is that just referring to a specific software that automatically provides uncertainties for the slope but not other parameters?

The authors made changes such as

"For example, changes such as $R=0.84$ to $R=0.85$ are now described more cautiously, while greater emphasis is placed on the larger changes in bias and regression slope."

but this misses the point of the original comment. The reader cannot discern if this is a statically significant change or not without uncertainties, e.g. comparing $R = 0.84 \pm 0.01$ to $R = 0.85 \pm 0.01$ is very different from comparing $R = 0.84 \pm 0.1$ to $R = 0.85 \pm 0.1$.

In any case, adding uncertainty estimates for R , μ , and σ is essential. The standard and easily implemented method for this is assuming a normal distribution (which was most likely already assumed for the uncertainty in the slope) and using the standard analytical formulas. For μ , see https://en.wikipedia.org/wiki/Standard_error, and for σ see https://en.wikipedia.org/wiki/Variance#Sample_variance, and for R the easiest thing is to use the Fisher transformation https://en.wikipedia.org/wiki/Fisher_transformation. You can use more advanced methods if desired, but these are the standard methods.

Once the uncertainties on R , μ , and σ are calculated, if the uncertainties are many orders of magnitude less than the statistics' estimated value, then it might be sufficient to just state that the uncertainties are very small. But unless the uncertainties are extremely small, they should be attached to each value. If including the uncertainties in the plots make the images too crowded, then the uncertainties can be listed elsewhere, but should be stated somewhere in the paper. Once this is done, the authors should then add some text in the manuscript saying whether or not the changes in these statistics are statistically significant.

Ross, Sheldon M. *Introduction to probability and statistics for engineers and scientists*. Academic press, 2020.