

ITMSL: an improved ice thickness inversion model integrating basal sliding dynamics for High Mountain Asia (v1.0.0)

Xiaoguang Pang^{1,2}, Liming Jiang^{2,3}, Yuxuan Wu², Xi Lu², Yi Liu^{2,3}, Xiaoen Li^{2,3}, Tingting Yao¹

¹College of Information Science and Engineering, Henan University of Technology, Henan, China
²State Key Laboratory of Precision Geodesy, Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences, Wuhan 430077, China
³College of Earth and Planetary Science, University of Chinese Academy of Sciences, Beijing 100049, China

Correspondence: Liming Jiang (jlm@whigg.ac.cn)

Highlights.

- We develop the Ice Thickness Model considering Sliding Law (ITMSL), an enhanced laminar-flow inversion with higher accuracy and physical realism.
- ITMSL enables automated joint inversion of glacier thickness and basal sliding.
- ITMSL improves ice-thickness accuracy by at least 16% over traditional ratio-assumption methods in High Mountain Asia (HMA).

Abstract. Glacier thickness plays a fundamental role in understanding ice dynamics, hydrological resources, and glacial hazards. Currently, ice thickness inversion models based on ice velocity have achieved significant progress in regional and global studies of glacier thickness and volume. However, these methods simplify the parameterization of basal sliding, introducing uncertainties and significant biases in thickness estimates. However, this method simplifies the parameterization of basal sliding, leading to uncertainties and significant biases in thickness estimates. Here, we present an improved ice thickness estimation approach through the integration of basal sliding law into laminar flow equation, termed the Ice Thickness Model considering Sliding Law (ITMSL). We apply and evaluate the model's performance and limitations across High Mountain Asia (HMA), a region characterized by complex topography and data scarcity. The model enables automated large-scale ice thickness reconstruction while simultaneously determining basal sliding velocities and subglacial topography. Validation against ground-penetrating radar (GPR) measurements on 16 glaciers shows that, compared to existing laminar flow equation-based models Gantayat (Gantayat et al., 2014) and Millan (Millan et al., 2022),

设置格式[庞校光]: 非突出显示
删除[庞校光]: Current ice thickness inversion primarily uses laminar flow theory constrained by geometric, topographic, and ice flow characteristics.
删除[庞校光]: these approaches
删除[庞校光]: over
删除[庞校光]: y
删除[庞校光]: parameterization
删除[庞校光]: substantial
删除[庞校光]: dynamics
删除[庞校光]: theory
删除[庞校光]: V14
删除[庞校光]: GV22

30 ITMSL achieves better performance, with accuracy improved by 16.2% and 28.9%, respectively. This study has demonstrated that ITMSL provides an improvement over previous methods, offering new insights for ice thickness modeling and its application in data-sparse high mountain regions.

1 Introduction

35 High Mountain Asia (HMA), encompassing the Qinghai-Tibet Plateau and surrounding regions including the Karakoram, Pamir, Himalayas, and Tien Shan (Yao et al., 2012; Chen et al., 2022; Fan et al., 2022; Wang et al., 2023), represents the most densely glacierized region outside the polar realms (Farinotti et al., 2019; Wang et al., 2023). HMA glaciers hold an estimated volume of 7000 km³, constituting the critical “Asian Water Tower”(Miles et al., 2021). This region serves as the headwater for major Asian rivers, including the Yangtze, Yellow, and Brahmaputra (Brun et al., 2017). Glacier
40 meltwater provides a vital, continuous water source for millions downstream, sustaining agriculture and domestic use—particularly in arid northwestern China (Kraaijenbrink et al., 2017; Miles et al., 2021; Li et al., 2022; Bolch et al., 2012). Glacier thickness distribution is therefore a key parameter for managing freshwater resources and projecting future glacier evolution (Fang et al., 2024; Lannutti et al., 2024; Robel et al., 2024; Wang et al., 2024). Consequently, advancing ice thickness models and
45 large-scale glacier volume estimates is essential for regional water resource management and glacier change prediction.

Glacier thickness has been estimated using both in situ measurements and modeling approaches. These include field techniques such as ground-penetrating radar (GPR), drilling, geomagnetic soundings, and seismic surveys (Wang et al., 2016; Van Tricht et al., 2021; Veitch et al., 2021; Liang and Tian, 2022),
50 as well as physical models (Farinotti et al., 2017; Farinotti et al., 2019; Millan et al., 2022) and volume-area (V-A) scaling (Radić and Hock, 2010; Grinsted, 2013). However, due to the harsh high-altitude environments and logistical difficulties, in-situ methods typically require substantial manpower and material resources. Many glaciers are remote, making substantial manpower and material resources (Wu et al., 2020; Li et al., 2022; Pang et al., 2023). While widely used, V-A scaling
55 cannot resolve the spatial distribution of glacier thickness (Bahr et al., 2015; Li et al., 2022; Liang and Tian, 2022; Pang et al., 2023). To date, approximately 20 ice thickness models have been proposed, including H-F, OGGM, GlabTop2, and ice velocity-based approaches (Huss and Farinotti, 2012; Frey

删除[庞校光]: in situ methods remain limited

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

删除[庞校光]: models based on laminar flow theory

et al., 2014; Gantayat et al., 2014; Farinotti et al., 2019; Maussion et al., 2019; Millan et al., 2022). To evaluate the accuracy and limitations of ice thickness inversion models, ITMIX systematically assessed 17 models that infer ice thickness from glacier surface characteristics (Farinotti et al., 2017). These models encompass different approaches, including minimization approaches, mass conservation approaches (Huss and Farinotti, 2012), shear stress-based approaches (Frey et al., 2014), ice velocity-based approaches (Gantayat et al., 2014), and convolutional neural network approaches (Jouvet et al., 2021). Different models have varying input data requirements, typically requiring two or more types of data from digital elevation models (DEMs), glacier outlines, surface velocity fields, and mass balance data (Farinotti et al., 2017; Farinotti et al., 2021). Among them, minimization approaches offer strong physical consistency and can handle complex glacier dynamics (Farinotti et al., 2021); mass conservation approaches feature a solid physical foundation and good stability (Farinotti et al., 2009); shallow ice approximation (SIA) approaches are robust and computationally fast (Ramsankaran et al., 2018); and ice velocity-based methods are particularly effective for sliding-dominated glaciers (Wu et al., 2020). ITMIX revealed that the maximum discrepancy in ice thickness estimates among models can be on the order of the actual glacier thickness itself. Substantial disparities exist among different models in their thickness estimates for the same glacier, with uncertainties reaching tens to even hundreds of meters. Weighted ensemble averaging of multiple models can effectively offset systematic biases inherent in individual models, thereby ameliorating the accuracy of ice thickness simulations (Farinotti et al., 2017). The ITMIX2 experiment evaluated the influence of the number and location of in-situ ice thickness observations on model calibration and accuracy, and found that models respond differently to data scenarios, with no single model performing best across all scenarios. At the same time, the experiment highlighted the critical role of sparse in-situ data and recommended prioritizing measurements in the thickest location of glaciers (Farinotti et al., 2021). Against this backdrop, refinements to individual ice thickness models can help improve the accuracy of the multi-model ensemble average.

The ice thickness inversion equation based on ice velocity is commonly referred to as the laminar flow equation (Gantayat et al., 2014; Gopika et al., 2021), or the shallow-ice approximation (SIA) (Millan et al., 2022). This equation treats surface velocity as the sum of basal sliding and ice deformation. Currently, ice thickness inversion models that rely on the ice-velocity-based approach include Gantayat

(Gantayat et al., 2014), RAAJgantayat (Farinotti et al., 2017), Gantayat-v2 (Gantayat et al., 2014), Rabatel (Farinotti et al., 2017), Millan (Millan et al., 2022), Wu (Wu et al., 2020), and others. In these models, the simulation of basal sliding parameters is grounded in surface velocity and topographic characteristics (Gantayat et al., 2014; Wu et al., 2020; Millan et al., 2022). For example, basal sliding is assumed to be a fixed ratio of surface velocity (Gantayat et al., 2014), is derived from the relationship between slope and surface velocity (Millan et al., 2022), or is taken as the difference between summer and winter surface velocities under the assumption of zero winter sliding (Wu et al., 2020). However, basal sliding is a complex process influenced by subglacial topography, basal shear stress, and ice overburden pressure (Weertman, 1957; Schoof, 2005; Cuffey and Paterson, 2010; Zoet and Iverson, 2020; Helanow et al., 2021), and the ratio of basal sliding to surface velocity exhibits significant spatial and temporal variability within glaciers, with observed values ranging widely from 0.03 to 1 (Engelhardt and Kamb, 1998; Cuffey and Paterson, 2010; Echelmeyer and Zhongxiang, 1987). These simplified parameterizations therefore risk introducing substantial biases into ice thickness estimates. With the development of artificial intelligence, the Instructed Glacier Model (IGM) ice flow model based on Convolutional Neural Network has been proposed. This approach learns the input-output mapping of the full-Stokes ice flow model under various glacier geometries and dynamic conditions, thereby becoming an efficient and high accuracy ice thickness inversion model. In this model, basal sliding is computed using the Weertman nonlinear sliding law. In glacier dynamics, simulating the basal sliding process must rely on a basal sliding law, which is the only way to describe this process (Zekollari et al., 2022; Schoof, 2005; Joughin et al., 2019; Zoet and Iverson, 2020). These laws typically assume the glacier rests on hard bedrock (Weertman, 1957; Cuffey and Paterson, 2010), with basal motion governed by basal shear stress, effective pressure, and bed roughness (Budd et al., 1979; Schoof, 2005; O. et al., 2007; Woodard et al., 2022). Compared with traditional simple assumptions, these laws more accurately capture the spatial characteristics of basal sliding and its physical relationships with other variables. Although complex ice thickness inversion models offer advantages in physical realism and inversion accuracy, their high computational cost, large number of parameters limit their application in regional and global scale glacier volume studies. Therefore, this study proposes an ice thickness inversion model that couples the basal sliding law with the laminar flow equation (ITMSL), aiming to strike a balance between physical complexity and

- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 非突出显示
- 删除[庞校光]: The laminar flow-based ice thickness mod ...
- 删除[庞校光]: Examples include:
- 删除[庞校光]: ing
- 删除[庞校光]: sliding
- 删除[庞校光]: 25% (hereafter GV14)
- 删除[庞校光]: ,
- 删除[庞校光]: ing the ratio
- 删除[庞校光]: (hereafter GV22)
- 删除[庞校光]: ,
- 删除[庞校光]: assuming zero basal sliding during winter
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 删除[庞校光]: Therefore, accurately accounting for bas ...
- 设置格式[庞校光]: 突出显示
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...
- 设置格式[庞校光]: 字体: (默认) Times New Roman, ...

computational efficiency. This model is driven by glacier topography and surface velocity. The parameters and accuracy of inversion results was validated and assessed against in situ observations. We applied ITMSL to 16 glaciers across HMA, comparing its performance with two other models (Gantayat and Millan) that are also based on the laminar flow equation. These 16 glaciers were selected because in-situ ice thickness observations are publicly available for them in HMA. In addition, we evaluated the influence of different DEMs on model performance to identify optimal datasets for glacier thickness estimation in HMA. In the subsequent sections, we will present the theoretical basis (Section 2), implementation workflow of the model (Section 3), evaluate its inversion results against existing ice thickness models (Section 4), discuss certain limitations and possible improvements (Section 5), and conclude with a summary (Section 6).

删除[庞校光]: Here, we address limitations in glacier thickness inversion under laminar flow theory by developing a novel model (ITMSL) that integrates basal sliding laws.

设置格式[庞校光]: 突出显示

删除[庞校光]: against two established models (GV14 and GV22)

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

删除[庞校光]:

2 Methods

The following section details the datasets used to drive the model and the theoretical derivation process underlying its construction.

2.1 Datasets

Glaciers in HMA are influenced by the Indian monsoon, westerlies, and East Asian monsoon (Bolch et al., 2012; Yao et al., 2012). These climatic regimes drive pronounced spatial heterogeneity in glacier distribution, mass balance, and dynamics (Hugonnet et al., 2021; Miles et al., 2021; Millan et al., 2022; Wang et al., 2023). For this study, we selected 16 glaciers spreading across the Tian Shan (7 glaciers), Himalayas (1 glacier), Qilian Shan (2 glaciers), and Karakoram (6 glaciers) (Figure 1). The sample exhibit significant diversity in area, width, and surface velocity. Notably, one glacier covers less than 1 km², while 9 exceed 5 km² (Table 1). The Urumqi No.1 Glacier (Tian Shan) bifurcated into eastern and western branches in 1993 (Wang et al., 2016); we analyze these branches as a single glacier.

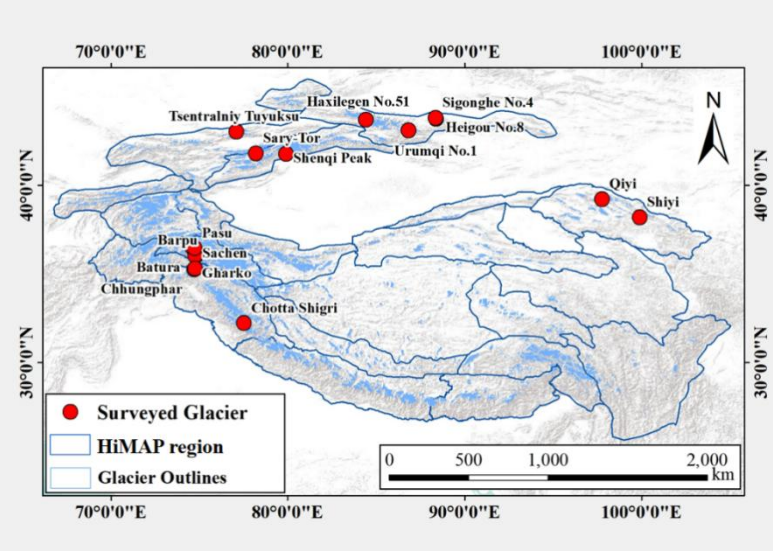


Figure 1: Distribution of HMA glaciers and the location of selected glaciers.

140 This study utilized glacier outlines from the Randolph Glacier Inventory version 6.0 (RGI 6.0; <https://www.glims.org/RGI/>) (Pfeffer et al., 2014). Topographic features essential for ice thickness inversion—including surface slope and elevation—were derived from the Copernicus DEM GLOB-30 (hereafter COPDEM30; <https://registry.opendata.aws/copernicus-dem>). COPDEM30 provides near-global coverage at 30m resolution, generated through reprocessing of World DEM™ data acquired between 2011 and 2015. The reprocessing involved resampling, void filling, and replacement of anomalous elevation values (Becek et al., 2016; González-Moradas and Viveen, 2020; Hawker et al., 2022; Li et al., 2022). The Copernicus DEM exhibits an absolute vertical accuracy of 4 m and relative vertical accuracy of 2 m (Hawker et al., 2022). For computational efficiency in ice thickness modeling, COPDEM30 data were resampled to 50m resolution.

150 Ice thickness models based on the shallow-ice approximation (SIA), estimate glacier thickness using ice motion characteristics (Gantayat et al., 2014; Millan et al., 2022). This study utilizes glacier surface velocities from Millan et al, (2022), who derived ice flow velocities for 2017-2018 by applying sub-pixel correlation techniques to remote sensing imagery (Millan et al., 2019; Millan et al., 2022). The dataset features a spatial resolution of 50 m with a reported velocity uncertainty of 10 m/yr. Moreover, this dataset includes the standard deviations of ice velocity in the east-west and north-south components, which enables the computation of the standard deviation of ice velocity for each glacier.

To evaluate model-derived ice thickness accuracy, we collected GPR measurements from 16 glaciers. Data sources include the Glacier Thickness Database version 3 (GlaThiDa v3), maintained by the

设置格式[庞校光]: 上标

删除[庞校光]: laminar flow theory

Global Terrestrial Network for Glaciers (GTN-G) through the World Glacier Monitoring Service
160 (WGMS; <https://wgms.ch/>), as well as published studies (Azam et al., 2012a; Welty et al., 2020;
Farinotti et al., 2021). GlaThiDa represents a global compilation of ice thickness measurements
obtained through GPR, drilling, and seismic methods (Welty et al., 2020). For our study glaciers,
temporal inconsistencies between ice thickness measurements and DEM/outline acquisition dates are
considered negligible (Li et al., 2022). However, the GPR measurements for the Qiyi Glacier date back
165 to 1980. After more than half a century of change, the difference between the simulated ice thickness
and the GPR measurements should be viewed with this long-term evolution in mind.

Table 1: Glaciers from RGI 6.0, survey years, and GPR data sources.

RGI-ID	Glacier (abbreviation)	Area (km ²)	u_s (m/yr)	GPR Survey Year	number of GPR points	GPR source
RGI60-14.00032	Barpu (BARP)	104.952	125.70	2015~2018	60	(Zou et al., 2021)
RGI60-14.02150	Batura (BATU)	311.419	101.34	2015~2018	23	
RGI60-14.20030	Chhungphar (CHHU)	15.136	108.40	2015~2018	17	
RGI60-14.04590	Gharko (GHAR)	30.318	81.06	2015~2018	29	
RGI60-14.03123	Pasu (PASU)	62.145	162.08	2015~2018	9	
RGI60-14.19344	Sachen (SACH)	10.307	40.90	2015~2018	55	
RGI60-13.47247	Haxilegen No.51 (HXLG)	1.099	10.70	2010	584	(Welty et al., 2020)
RGI60-13.48211	Heigou No.8 (HEIG)	6.074	24.97	2009	906	
RGI60-13.45335	Urumqi No.1 (URUM)	1.579	20.51	2014	1385	
RGI60-13.32330	Qiyi (QIYI)	2.530	8.65	1980	105	
RGI60-13.08055	Sary-Tor (SARY)	2.927	8.82	2013	1352	
RGI60-13.43165	Shenqi Peak (SHQI)	6.591	21.63	2008	823	
RGI60-13.31537	Shiyi (SHIY)	0.495	7.78	2010	294	
RGI60-13.45233	Sigonghe No.4 (SIGH)	2.641	7.01	2009	623	
RGI60-13.08624	Tsentralniy Tuyuksu (TSTU)	2.838	4.61	2013	8353	

带格式表格[庞校光]

RGI60-14.15990	Chhota Shigri (CTSG)	13.463	15.48	2009	146	(Azam et al., 2012a)
----------------	----------------------	--------	-------	------	-----	----------------------

2.2 Ice thickness model considering basal sliding law (ITMSL)

170

Laminar flow ~~equation~~ is a fundamental ~~in~~ glacier dynamics (Cuffey and Paterson, 2010). It describes the glacier surface velocity as the sum of ice deformation and basal sliding (Gantayat et al., 2014; Wu et al., 2020):

$$u_s = u_b + \frac{2A}{n+1} \tau_b^n H$$

(1)

175

where u_s and u_b (m/s) are the surface and basal sliding velocity, respectively. n is Glen’s flow law exponent, typically assumed to be 3. A is the creep parameter, dependent on ice temperature, fabric, grain size, and water content. Here we use $A = 2.4 \times 10^{-24} \text{Pa}^{-3} \text{s}^{-1}$ (Cuffey and Paterson, 2010; Gantayat et al., 2014). H (m) is the ice thickness. τ_b (Pa) is the basal shear stress, calculated as (Li et al., 2012):

$$\tau_b = f \rho g H \sin \alpha$$

(2)

180

where ρ is the ice density, taken as a constant $900 \text{ kg} \cdot \text{m}^{-3}$, g is the acceleration due to gravity ($9.8 \text{ m} \cdot \text{s}^{-2}$), α is the glacier surface slope (Farinotti et al., 2017; Farinotti et al., 2019). f is the valley shape factor, a dimensionless parameter accounting for the influence of glacier cross-sectional geometry on the relationship between driving stress and basal shear stress (Li et al., 2012; Ramsankaran et al., 2018). f is estimated using glacier width (w) and ice thickness (H):

$$f = \frac{2}{\pi} \arctan(\frac{w}{2H})$$

(3)

185

~~When using laminar flow equation to estimate glacier thickness, an assumption must be made regarding the ratio of basal sliding to surface velocity. Gantayat assumes this ratio to be 25% (Gantayat et al., 2014), whereas Millan infers that this ratio ranges from 0.1 to 0.9 based on the relationship between surface slope and ice velocity (Millan et al., 2022).~~

190

~~There are three types of basal sliding laws for glaciers (Weertman, 1957; Budd et al., 1979; Schoof, 2005). Weertman derived a basal sliding law by modelling the glacier bed as uniformly distributed regular cubic obstacles (Weertman, 1957). This law incorporates two key mechanisms: pressure melting and enhanced ice creep, establishing a nonlinear relationship between basal sliding velocity, basal shear stress, and ice bed roughness (Weertman, 1957). However, Weertman’s formulation~~

删除[庞校光]: theory

删除[庞校光]: framework

删除[庞校光]:

删除[庞校光]:

删除[庞校光]: When laminar flow theory is used to estimate glacier thickness, assumptions are made regarding the ratio of <math> to <math> (Gantayat et al., 2014; Millan et al., 2022).

设置格式[庞校光]: 字体: （默认）Times New Roman

设置格式[庞校光]: 字体: （默认）Times New Roman

设置格式[庞校光]: 字体: （默认）Times New Roman

设置格式[庞校光]: 字体: （默认）Times New Roman

删除[庞校光]:

neglects the influence of subglacial water content variations on sliding. To address this limitation, Budd

introduced a sliding law accounting for basal cavity evolution, though it retains a power-law form

(Budd et al., 1979; Fowler and Frank, 1981). Crucially, basal shear stress increases with sliding

velocity until reaching a limit controlled by subglacial hydrology (Iken, 1981). As water pressure rises,

basal drag approaches a maximum value determined by the maximum bed slope (Schoof, 2005; O. et

al., 2007; Zoet and Iverson, 2020; Helanow et al., 2021; Gilbert et al., 2023). We employ the Coulomb

sliding law to characterize this complex interaction among water pressure, shear stress, and dynamics,

which is given as follows (Helanow et al., 2021):

$$\frac{\tau_b}{N} = C \left(\frac{u_b}{u_b + A_s C^n N^n} \right)^{1/n} \quad (4)$$

where N (Pa) is the effective pressure. Effective pressure is defined as the difference between ice

overburden pressure and basal water pressure. Because subglacial hydrology is difficult to quantify

accurately, it is approximated as equivalent to the ice overburden pressure in this study

($N = \rho g H \cos \alpha$). $C = 0.84 \pm 0.02 m_{max}$ is the maximum value of τ_b/N (O. et al., 2007; Schoof,

2005). m_{max} is the maximum bedrock slope. A_s ($m Pa^{-n} yr^{-1}$) is the sliding parameter. The sliding

parameter A_s is defined as (O. et al., 2007):

$$A_s = B \lambda + \frac{-2 \times 10^{-5} + 0.013 r + 0.0262 r^2}{r^2} \quad (5)$$

where $B = 430 \pm 40 MPa^{-3} yr^{-1}$ is the ice fluidity parameter under isothermal conditions (Louis and

Lliboutry, 1987); λ is the bedrock obstacle wavelength, which is calculated from the descent gradient

of the subglacial topography using Fourier transform. r is the bedrock roughness coefficient

(Gudmundsson, 1997; Schoof, 2005; Berends et al., 2023), calculated via root mean square height

(RMSH) (Shepard et al., 2001; Berti et al., 2013):

$$r = \sqrt{\frac{1}{k-1} \sum_{i=1}^k (z_i - \bar{z})^2} \quad (6)$$

where k is the moving windows size (number of cells), z_i is the cell elevation, and $\bar{z}$ is the mean

elevation within the window (Berti et al., 2013).

2.3 Accuracy assessment

This study evaluates three ice thickness models (Gantayat, Millan, ITMSL) applied to 16 HMA glaciers.

Model performance is quantified using five statistical metrics: root mean square error (RMSE),

删除[庞校光]: r

删除[庞校光]: To ensure the sliding law accurately represents

删除[庞校光]: between

删除[庞校光]: we implement the following formulation

删除[庞校光]: ;

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

删除[庞校光]: ;

设置格式[庞校光]: 字体: (默认) Times New Roman

删除[庞校光]: GV14

删除[庞校光]: GV22

standard deviation (STD), mean error (ME), relative error (RE), and correlation coefficient (CC) (Höhle and Höhle, 2009; Liu et al., 2019; Chen et al., 2022; González-Moradas and Viveen, 2020; Pang et al., 2023). These metrics are defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (\hat{H}_i - H_i)^2}{n}} \quad (7)$$

225 $STD = \sqrt{\frac{\sum_{i=1}^n (\hat{H}_i - H_i - \frac{1}{n} \sum_{i=1}^n (\hat{H}_i - H_i))^2}{n-1}} \quad (8)$

$$ME = \frac{1}{n} \sum_{i=1}^n (\hat{H}_i - H_i) \quad (9)$$

$$RE = \frac{(\hat{H}_i - H_i)}{\hat{H}_i} \quad (10)$$

$$CC = \frac{\sum_{i=1}^n ((\hat{H}_i - \frac{1}{n} \sum_{i=1}^n \hat{H}_i)(H_i - \frac{1}{n} \sum_{i=1}^n H_i))}{\sqrt{\sum_{i=1}^n (\hat{H}_i - \frac{1}{n} \sum_{i=1}^n \hat{H}_i)^2} \sqrt{\sum_{i=1}^n (H_i - \frac{1}{n} \sum_{i=1}^n H_i)^2}} \quad (11)$$

where $\hat{H}_i$ is the ice thickness measured by GPR at point i , H_i is the modelled ice thickness at point i , and n is the total number of GPR points.

RMSE quantifies the overall magnitude of errors between modelled and GPR measured ice thickness, serving as a composite measure of accuracy. STD of residuals characterizes the dispersion of model errors around their mean value, reflecting simulation stability. ME indicates systematic bias, with positive or negative values denoting overestimation or underestimation. RE expresses the absolute percentage deviation of modelled ice thickness from observations. CC measures the strength of linear relationship between modelled and measured thickness, where values approaching 1 indicate strong positive agreement. Collectively, these complementary metrics provide a comprehensive assessment of model performance in terms of accuracy, precision, bias, and consistency with observed spatial patterns.

3 Model Instructions

Model performance was validated using in situ ice thickness measurements from 16 glaciers across HMA. The experiments were driven by glacier surface velocities, topographic data, and glacier inventory. An initial simulation was conducted based on a set of assumptions, followed by iterative refinement until the results met the predefined convergence criteria. The iterative procedure for estimating glacier thickness using the ITMSL model comprises the following steps (Figure 2.):

- (1) Input Data: Provide the DEM, glacier surface velocity field, and glacier outlines.

- (2) Preprocessing: Extract the glacier mask from the outlines. Calculate the glacier surface slope (α) from the DEM, respectively. Initial u_b to 0.
- 250 (3) Initial ice thickness: The initial value of shape factor is set to 0.8, and the initial ice thickness (H_0) is calculated using the laminar flow equation (Equation 1).
- (4) Shape Factor Calculation: Determine glacier width (w) as the mean distance from the centerline to the glacier margins. Calculate the valley shape factor (f) using Equation 3, incorporating w and H_0 .
- (5) Basal shear stress: Bring f , H_0 , α , and ice density (ρ) into equation 2 to derive τ_b .
- 255 (6) Initial Bed Topography: Obtain the initial subglacial topography (Z_0) by subtracting H_0 from the surface DEM.
- (7) Sliding Law Parameters: Calculate the ice bed slope from Z_0 . Calculate the maximum slope ratio (C) and bedrock roughness (r) (Equation 6). Determine the sliding parameter (A_s) via Equation 5. Estimate effective pressure (N) using H_0 and Z_0 .
- 260 (8) Basal Sliding Velocity: Put τ_b , N , C , A_s , and n into equation 4 for iterative calculation, thus obtaining u_b .
- (9) Updated Ice Thickness: Substitute the newly-obtained u_b into Equation 1 to recompute ice thickness (H_i).
- (10) Convergence Check: Calculate the difference in glacier mean thickness between iterations: $\Delta H =$
- 265 $H_i - H_{i-1}$. If ΔH is less than the threshold, the final glacier thickness and subglacial topography are output. Else, H_i is updated to H_{i-1} and return to Step3.

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

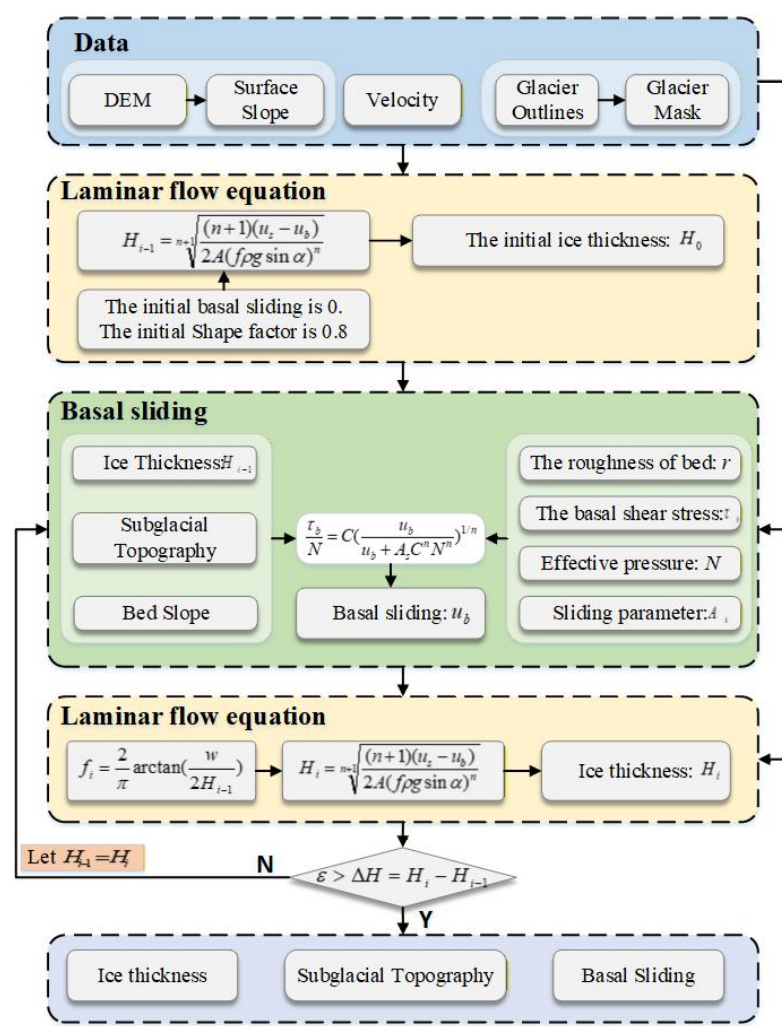


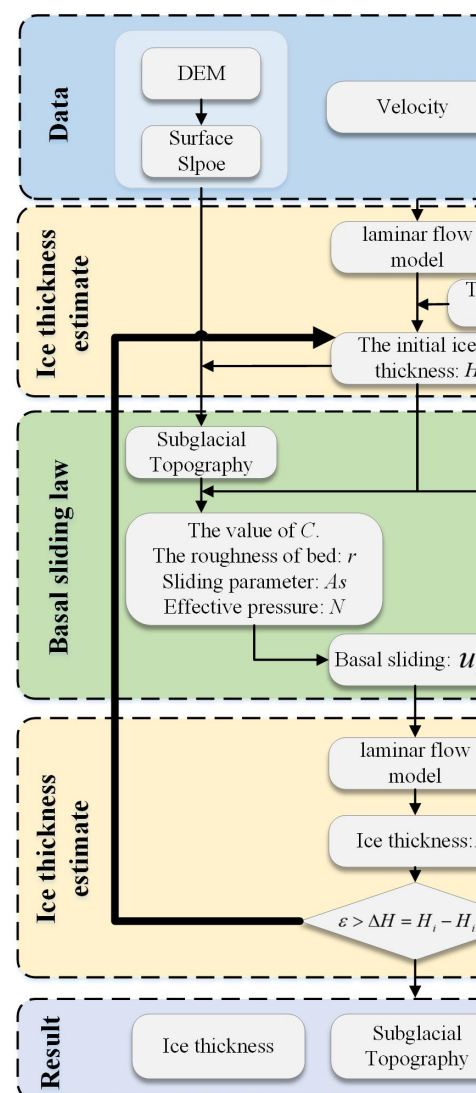
Figure 2: Flow chart of glacier thickness inversion using ITMSL model.

4 Model experiments

4.1 Calibration of ITMSL model

This study presents an ITMSL ice thickness model. To evaluate its performance, the model was applied to 16 HMA glaciers. In this study, the model parameters can be divided into two categories: those derived from glacier surface characteristics, such as shape factor and glacier width, and those derived from subglacial topography, including bed roughness, bed obstacle wavelength, and bed slope. Specifically, glacier width is calculated from glacier outline, while the shape factor is computed using both the glacier width and the initial ice thickness (Ramsankaran et al., 2018). Bed roughness is determined from subglacial topography (Berti et al., 2013), and the obstacle wavelength is obtained by applying a Fourier transform to the subglacial topography. These parameters are iteratively updated

删除[庞校光]:



设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

CHHU	465	CTSG	465
GHAR	470	SHIY	470
HXLG	470	SIGH	455
HEIG	460	TSTU	435
PASU	470	URUM	470
QIYI	425	Total	465
SACH	465		

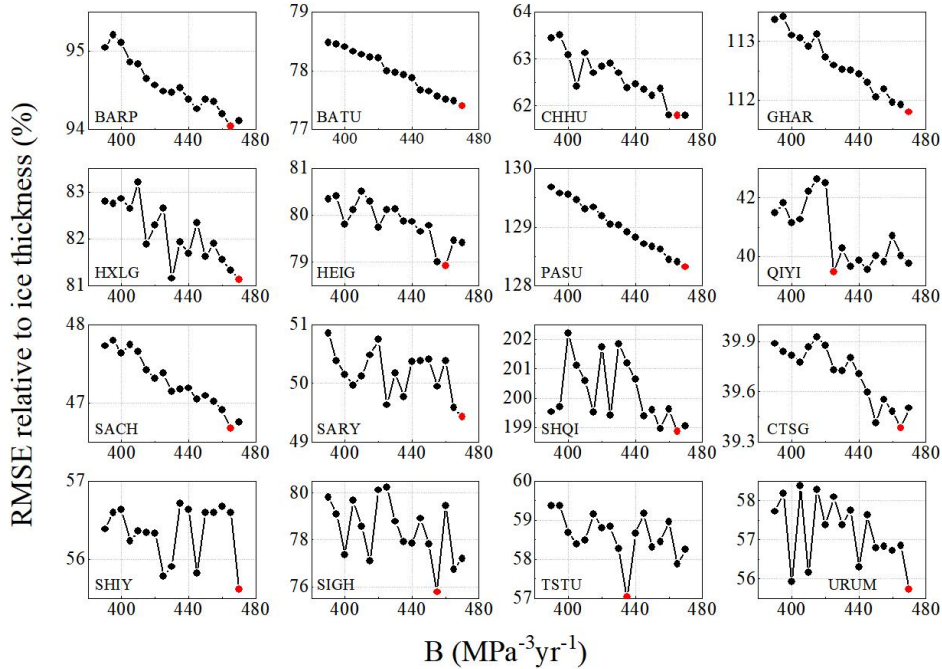


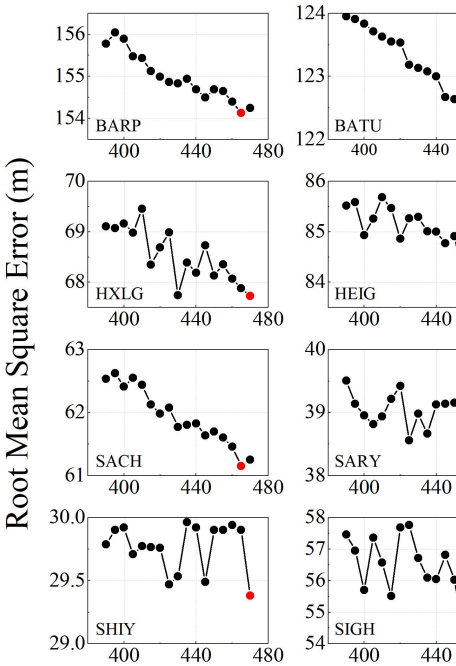
Figure 3: Ratio of the RMSE of ice thickness inversion to ice thickness for different values of parameter B in the ITMSL model.

4.2 Ice thickness distribution and subglacial topography

Using calibrated parameters, we applied the ITMSL model to estimate thickness distributions across the 16 study glaciers. For comparison, we also referred to the ice thickness estimates derived from the laminar flow equation-based Gantayat, and Millan, models (Gantayat et al., 2014; Millan et al., 2022).

Among the 16 glaciers, CTSG is a widely studied glacier frequently used for ice thickness model validation (Frey et al., 2014; Azam et al., 2012b). Therefore, this study takes CTSG as an example to conduct a detailed analysis of ice thickness, subglacial topography, and basal sliding; the same procedure is applied to the remaining 15 glaciers. At CTSG glacier, GPR measurements across

validation points indicated a mean ice thickness of 172.88 m. The ITMSL model produced a mean



删除[庞校光]:

删除[庞校光]: **RMSE values of ice thickness inversion when the ITMSL model takes different B values.**

删除[庞校光]: theory

删除[庞校光]: GV14

删除[庞校光]: GV22

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

删除[庞校光]: Figure 4 illustrates modeled thickness patterns for CTSG glacier under the three models. Figure 5 details CTSG's subglacial topography, surface velocity, basal sliding velocity, and the ratio of basal velocity to surface velocity. Parallel analyses performed on the remaining 15 glaciers yielded comprehensive ice thickness, bed topography, and basal sliding datasets (Table 4).

thickness of 114.63 m and a total glacier volume of 1.5277 km³. Spatially, all three models (ITMSL, [Gantayat](#), [Millan](#)) consistently captured characteristic thickness patterns: maximum along the central flowline and minimum in tributaries and marginal zones. [In terms of mean ice thickness](#), ITMSL (114.63 m), [Millan](#) (104.90 m), and [Gantayat](#) (98.39 m) [differ from one another](#), with [Gantayat being the lowest](#). Crucially, ITMSL exhibited closest agreement with GPR data, indicating superior accuracy relative to the comparison models.

GPR surveys delineated 5 cross-sections (CS1-CS5) at CTSG glacier (Figure 4). We evaluated ice thickness accuracy at each CS using RMSE for three models ([Figure 5](#)). ITMSL demonstrated superior performance at CS1, CS4, and CS5 with RMSE values of 34.4m, 47.4m, and 24.5m respectively, outperforming [Gantayat](#) (40.5m, 65.6m, 33.6m) and [Millan](#) (98.2m, 69.9m, 64.8m). Conversely, [Millan](#) achieved lower RMSE at central flowline sections CS2 (29.4m) and CS3 (42.1m), compared to ITMSL (40.9m, 57.4m) and [Gantayat](#) (51.7m, 70.4m).

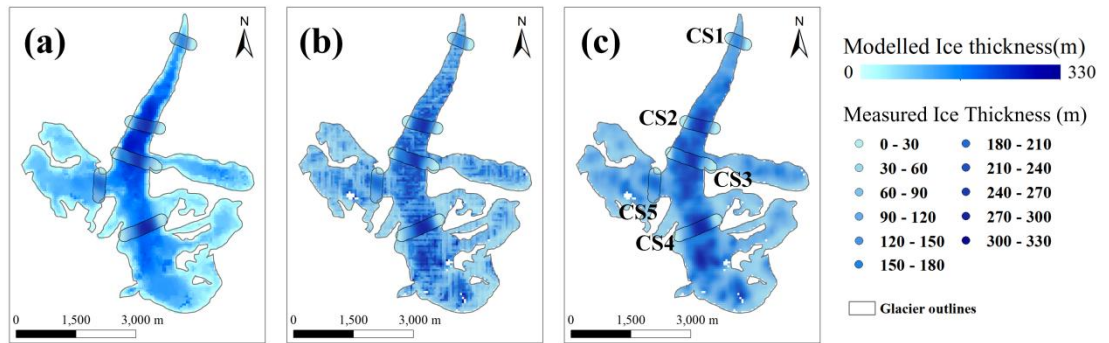


Figure 4: CTSG glacier thickness estimated by the three models. (a) [Millan et al., 2022](#), (b) [Gantayat et al., 2014](#), (c) ITMSL.

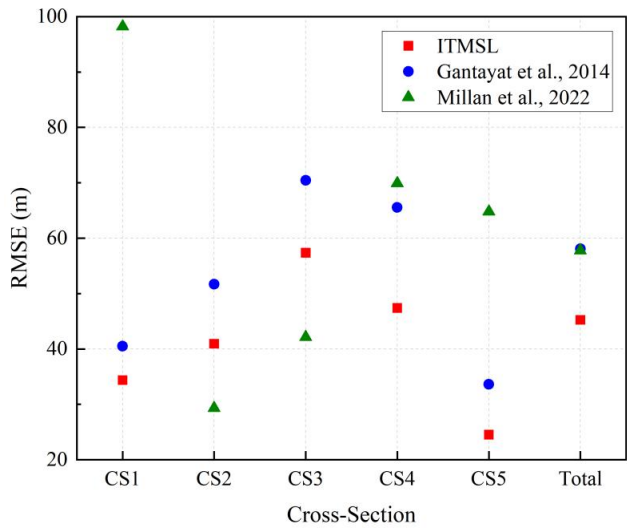
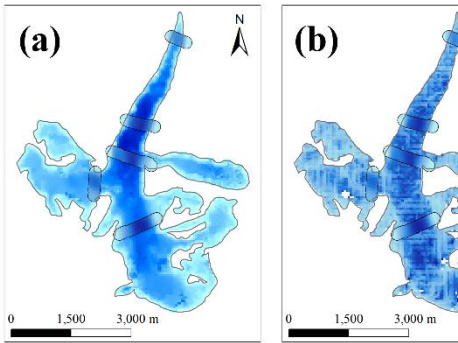


Figure 5: RMSE of CTSG glacier 5 cross-sections.

删除[庞校光]: GV14
删除[庞校光]: GV22
删除[庞校光]: Quantitatively
删除[庞校光]: GV22
删除[庞校光]: GV14
删除[庞校光]: showed divergent mean estimates
删除[庞校光]: GV14
删除[庞校光]: producing systematically lower values
删除[庞校光]: Table
删除[庞校光]: 3
删除[庞校光]: GV14
删除[庞校光]: GV22
删除[庞校光]: GV22
删除[庞校光]: GV14



删除[庞校光]:
删除[庞校光]: GV22
删除[庞校光]: GV14

删除[庞校光]: Table 3: Accuracy of ice thickness estimates for five cross-sections of the CTSG glacier.

RMSE
STD

Figure 6a reconstructs the subglacial topography of CTSB glacier by subtracting modelled ice thickness from the DEM-derived surface elevation. The accuracy of this reconstructed bed topography is contingent upon the precision of both the ice thickness estimates and the input DEM. Analysis of basal sliding velocity reveals a glacier-wide mean of 5.53 m/yr ($u_b/u_s = 39.9\%$). The quantified u_b/u_s ratio further provides critical constraints on glacial dynamical behavior, underscoring its essential role in advancing ice thickness inversion methodologies.

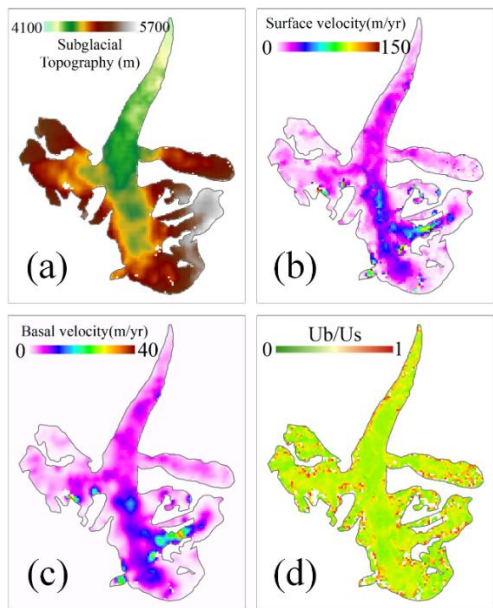


Figure 6: (a) CTSB glacier subglacial topography, (c) basal sliding, and (d) the ratio of sliding velocity to surface velocity inverted by ITMSL; (b) surface velocity used as input to ITMSL.

We applied the ITMSL model to estimate ice thickness and basal sliding velocity across the selected glaciers (Table 3). Modeled ice thickness values represents point-specific averages collocated with GPR measurements. Results indicate systematic overestimation for 12 glaciers and underestimation for four. This bias arises from inherent limitations in laminar flow equation parameterization. The derived u_b/u_s ratio ranges from 0.342 to 0.457—significantly exceeding Gantayat’s assumed 0.25, which induces substantial thickness overestimation. Consequently, incorporating the physically-based sliding law enhances laminar flow equation by explicitly representing subglacial processes, thereby improving region ice thickness estimation accuracy.

Table 3: The ice thickness and motion characteristics of the 16 glaciers are mean values calculated from inversion results extracted at the GPR point locations.

Glacier	GPR (m)	Gantayat (m)	Millan (m)	ITMSL (m)	u_b (m/yr)	u_b/u_s
---------	---------	--------------	------------	-----------	--------------	-----------

BARP	120.95	143.86	191.74	163.90	42.35	0.350
BATU	127.06	142.37	211.63	157.94	36.35	0.342
CHHU	103.93	113.63	142.55	127.08	44.92	0.391
GHAR	152.94	114.62	153.15	138.11	31.08	0.366
HXLG	43.96	83.91	42.60	83.47	4.25	0.457
HEIG	114.87	94.88	68.75	106.43	9.65	0.385
PASU	145.55	156.36	185.24	163.58	59.89	0.368
QIYI	77.81	63.58	53.26	59.14	3.34	0.401
SACH	121.79	123.40	185.03	131.01	16.56	0.388
SARY	75.52	77.80	81.28	77.68	3.32	0.415
SHQI	38.68	71.31	80.88	71.16	7.92	0.354
CTSG	172.88	104.90	98.39	114.63	5.53	0.399
SHIY	39.47	49.18	30.09	52.83	2.95	0.398
SIGH	68.93	71.99	42.28	72.0	2.63	0.399
TSTU	51.63	64.63	63.55	60.21	1.78	0.422
URUM	40.05	78.45	45.61	83.85	7.67	0.415

4.3 Accuracy assessment of the glacier thickness

Comprehensive evaluation of the ITMSL model against Gantayat, Millan, using 5 statistical metrics applied to GPR-validated thickness data (Figure 7) demonstrates significant improvements: ITMSL achieves higher correlation coefficients than Millan in 8 of 16 glaciers and surpasses Gantayat in 12 of 16, indicating stronger linear relationships with observations; reduces STD to 41.31m (28.87% lower than Millan's 58.08m and 16.17% below Gantayat's 49.28m), with particularly notable decreases at BATU (down 60.5%) and HXLG (down 37.9%); and achieves 17% lower RMSE (56.72m) than Millan (68.34m) and 12.6% reduction versus Gantayat (64.92 m). Although 4 glaciers (SHIY, HXLG, URUM, HEIG) exhibit higher deviation in ice thickness inversion using ITMSL compared to Millan et al., 2022, and the changes in mean and relative errors are limited, the improvement in accuracy still confirms the reliability of ITMSL for ice thickness inversion over most glaciers. Regarding RE, except for an outlier on the SHQI glacier, the RE values of the remaining 15 glaciers are relatively similar: -2.48–0.2 for

删除[庞校光]: GV14
删除[庞校光]: GV22
删除[庞校光]: 6
删除[庞校光]: GV22
删除[庞校光]: GV14
删除[庞校光]: GV22
删除[庞校光]: GV14
删除[庞校光]: GV22
删除[庞校光]: GV14
删除[庞校光]: than GV22—likely due to complex subglacial topography—
删除[庞校光]: /
删除[庞校光]: show limited changes
删除[庞校光]: consistent precision enhancements confirm ITMSL's superior reliability across most glaciers
设置格式[庞校光]: 字体: (默认) Times New Roman

365 Millan et al., 2022, -1.89–0.02 for ITMSL, and -1.85–0.02 for Gantayat et al., 2014. The overall mean
RE values for these 16 glaciers are -1.81, -1.58, and -1.72, respectively. In terms of ME, the ranges of
ME for the 16 glaciers inverted by the three models are -277.69–19.72 (Millan et al., 2022), -193.23–
9.88 (ITMSL), and -135.42–7.26 (Gantayat et al., 2014), with overall mean ME values of -36.01 m,
-38.87 m, and -42.27 m, respectively. Both ME and RE indicate that the inversion results from the three
370 models are generally higher than the GPR-measured ice thickness, suggesting that the model input data
and parameter calibration still have room for improvement. Correlation analysis between the
model-inverted thickness and GPR-measured ice thickness yields overall correlation coefficients of
0.29 (Millan et al., 2022), 0.52 (ITMSL), and 0.42 (Gantayat et al., 2014). These advances stem
directly from the physically constrained basal sliding parameterization, which effectively mitigates
375 Gantayat's problematic u_b/u_s assumption of 0.25 that systematically overestimates thickness. ITMSL
thus represents a significant advancement for ice thickness estimation, while also highlighting the
limitations of the ITMSL model requiring further refinement.

Despite inherent limitations in input data quality and model parameterization, the ITMSL model
demonstrates robust performance across diverse HMA glaciers (Figure 7), though localized
380 discrepancies persist in topographically complex regions. Inter-model variations primarily arise from
uncertainties in foundational datasets, particularly DEM precision, surface velocity accuracy, and slope
derivation errors. Crucially, the convergence of ITMSL with established models (Gantayat and Millan)
validates its utility as a physically constrained framework for regional glacier thickness distribution
mapping. Estimation accuracy can be further enhanced through integration of multi-temporal velocity
385 products, optimization of sliding law parameters, and assimilation of subglacial topography constraints.

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman

删除[庞校光]: GV14

删除[庞校光]: 6

删除[庞校光]: GV14

删除[庞校光]: GV22

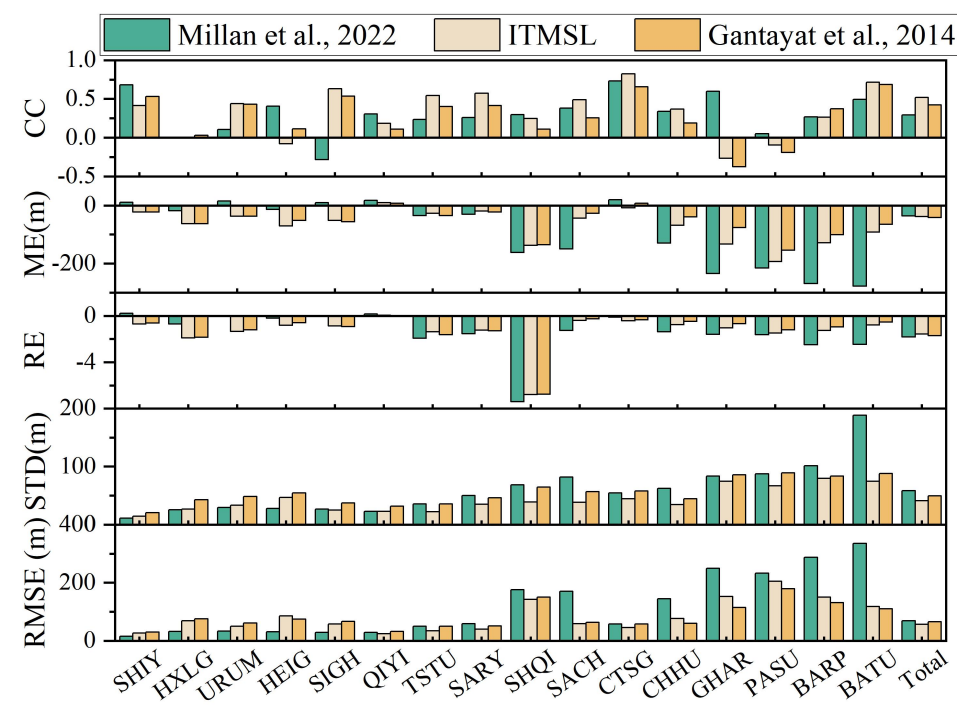
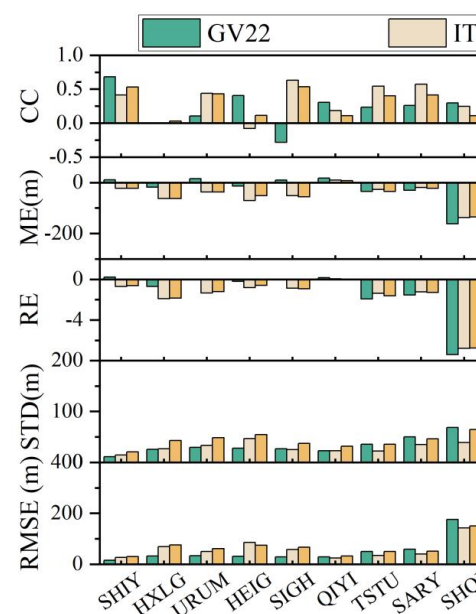


Figure 7: Accuracy of glacier thickness estimated by three ice thickness models based on SIA.



删除[庞校光]:

删除[庞校光]: 6

删除[庞校光]: laminar flow theory

5 Discussion

5.1 Analysis of factors affecting the accuracy of simulated ice thickness

390 Using glacier-specific characteristics, we iteratively computed key parameters, including the valley shape factor (f), effective stress (N), basal sliding velocity (u_b), sliding parameter (A_s), and bedrock roughness (r), to derive ice thickness distributions across the 16 glaciers. Although ITMSL demonstrated improvements, localized accuracy deficits at 4 glaciers (SHIY, HXLG, URUM, HEIG) necessitate targeted analysis. Ice thickness model accuracy is governed by 4 factors: 1) input data quality (DEM, surface velocity, glacier outline); 2) model limitations; 3) parameterization uncertainties; and 4) glacier characteristics (slope, aspect, bedrock topography) (Frey et al., 2014; Pang et al., 2023; Wang et al., 2023). For URUM glacier, analyzed as an integrated system incorporating both eastern and western branches (Wang et al., 2016), we quantified performance using STD.

Statistical analysis of glacier geometry and dynamics (Figure 8) revealed distinct characteristics for 4 glaciers that showed limited improvement in accuracy: SHIY (length: 926m, area: 0.495km², width: 534m), HXLG (length: 1499m, area: 1.099km², width: 733m), and the eastern branches (URUME; length: 2012m, area: 1.03km², width: 513m) and western branches (URUMW; length: 1694m, area:

删除[庞校光]: i

删除[庞校光]: 7

0.55km², width: 322m) of the URUM glacier. While sharing comparable elevation, velocity, and slope characteristics with other glaciers, their small size exacerbates four key limitations. First, reduced glacier dimensions challenges ice thickness inversion due to insufficient grid resolution. Second, spatial resolution mismatches between modelled ice thickness (50m cells) and GPR point spacing (1.5-5m) amplify errors. Third, velocity monitoring for small glaciers is less reliable, and basal sliding is weak or absent, hindering effective simulation. Finally, inherent model uncertainties in ice density parameterization, sliding coefficient calibration, and bedrock roughness estimation contribute to inaccuracies. Consequently, the thickness results obtained from these four small glaciers also highlight the limitations and applicable conditions of the ITMSL. Addressing these multiscale challenges, spanning data acquisition, model structure, and parameter sensitivity, remains essential for enhancing ITMSL’s reliability across diverse glacial regimes.

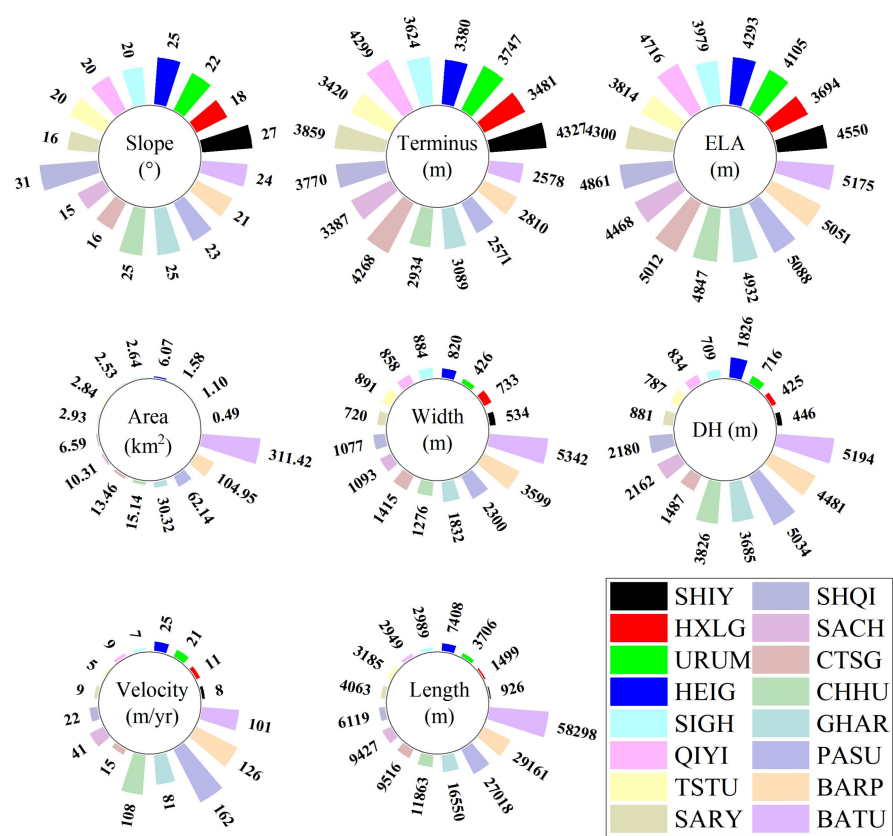


Figure 8: Characteristics of glacier surface slope, terminus elevation, equilibrium line altitude, area, mean width, difference in height (DH), ice velocity, length.

5.2 Influence of glacier basal sliding on the ice thickness

This study combines the basal sliding law with the laminar flow equation to develop the ITMSL model (Schoof, 2005; Gantayat et al., 2014; Zoet and Iverson, 2020; Millan et al., 2022). At each GPR measurement point, we computed the u_b/u_s ratio using the ITMSL model. Based on this ratio, we binned the points into two regimes: 0.1–0.4 and 0.4–1.0. The comparative analysis at these GPR points (Figure 9) reveals distinct performance regimes: for the 0.1–0.4 bin, the magnitude of ice thickness deviations follows Gantayat > ITMSL > Millan; for the 0.4–1.0 bin, ITMSL reduces the absolute deviation relative to both Gantayat and Millan. Crucially, all models exhibit peak biases within the 0.2–0.5 u_b/u_s , where ITMSL achieves 57.4% and 46.9% lower deviations than Gantayat and Millan, demonstrating how basal drag constraints fundamentally improve thickness estimation.

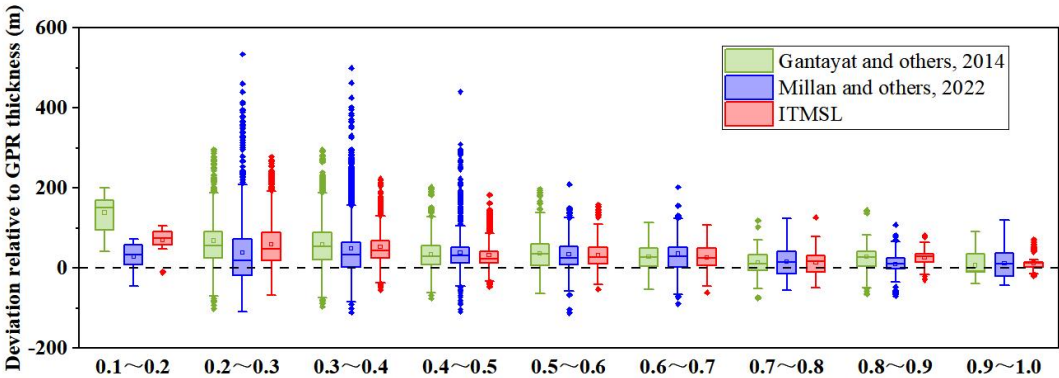


Figure 9: Comparative analysis of ice thickness estimation deviations for different ratios of basal sliding to surface velocity.

Beyond model intercomparison, we analyze the effects of basal sliding velocity on ice thickness estimation (Figure 10). The correlation coefficients between ice thickness bias and u_b are 0.2417 for Gantayat, 0.2802 for Millan, and 0.4711 for ITMSL. ITMSL can reasonably capture the spatial variation of basal sliding and thereby improve the physical consistency of ice thickness estimates, whereas other models that ignore sliding cannot reflect this key dynamical process. This enhanced physical responsiveness validates the efficacy of integrating basal drag constraints into laminar flow equation. Consequently, this integration significantly improves the robustness of thickness estimation across diverse glacial conditions.

删除[庞校光]: Building upon established glaciological

删除[庞校光]: , this study develops the ITMSL model by integrating basal sliding law into laminar flow theory

设置格式[庞校光]: 字体: 10 磅, 字体颜色: 自动设置

删除[庞校光]: Comparative analysis at GPR measurement points (Figure 8) reveals distinct performance regimes tied to u_b/u_s : For u_b/u_s ranging from 0.1 to 0.4, ice thickness deviations follow GV14 > ITMSL > GV22, whereas in the 0.4–1.0 range ITMSL reduces absolute deviation relative to GV14 and GV22, respectively.

删除[庞校光]: GV14

删除[庞校光]: GV22

删除[庞校光]: 8

删除[庞校光]: 9

删除[庞校光]: GV14

删除[庞校光]: GV22

删除[庞校光]: theory

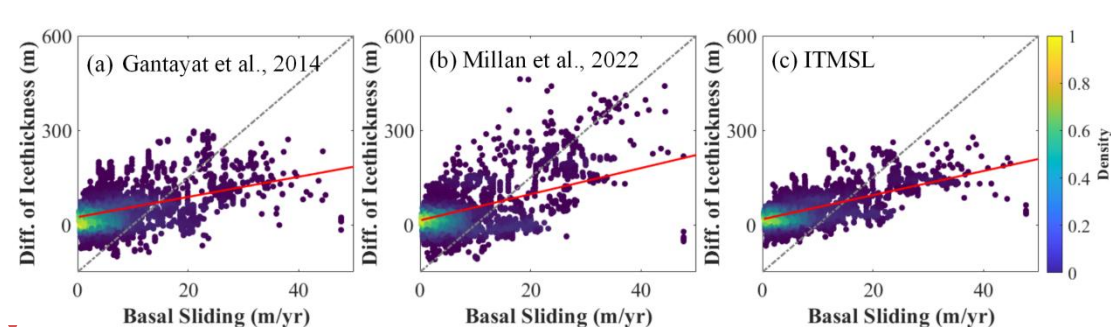


Figure 10: Deviations of ice thickness estimates from three laminar flow models at different basal sliding. (a) Gantayat; (b) Millan; (c) ITMSL.

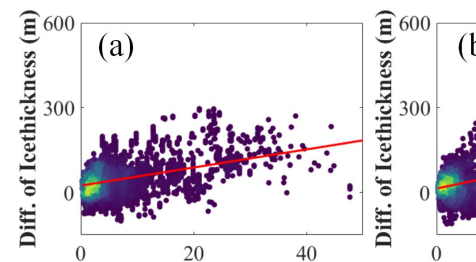
5.3 Comparison with existing ice thicknesses

In this study, we evaluate the accuracy of the ITMSL model within the framework of the ITMIX. First, compare the ice thickness results calculated by each model. To place the ITMSL, Gantayat, and Millan models within a broader glaciological model framework, we compared their thickness estimates with

four established models, namely Composite, GlabTop2, H-F, and OGGM (Huss and Farinotti, 2012; Frey et al., 2014; Farinotti et al., 2019; Maussion et al., 2019). The thickness data for these models are derived from Farinotti et al. (2019), whose methodological foundations include the mass conservation approach (OGGM, H-F), the shallow ice approximation theory (GlabTop2), and the weighted multi-model ensemble (Composite).

Taking the CTSG glacier as an example, we compare the composite mean ice thickness (i.e., the ice thickness distribution obtained by averaging, Figure 11a) with the ice thickness spread (i.e., the spread of all model results within a pixel, Figure 11b). Figures 11c and 11d present the maximum and minimum ice thickness composites, respectively, illustrating the composition of the composite results.

The models producing the most extreme results are shown in Figures 11e and 11f. For the CTSG glacier, the minimum ice thickness is predominantly provided by the OGGM model (62.2%), while the minimum values from GlabTop2 are mainly distributed over the glacier's tributaries. In contrast, the maximum ice thickness is primarily contributed by ITMSL (32.7%), Millan (20.2%), Gantayat (19.8%), and H-F (16.2%). This pattern is consistent with the trends observed in the ITMIX results. Specifically, the maximum ice thickness from ITMSL is concentrated along the glacier margins; that from Millan is concentrated in the lower reaches and the tongue of the glacier; that from Gantayat is relatively evenly distributed; and that from the H-F model is concentrated in the upper reaches.



删除[庞校光]:

删除[庞校光]: 9

删除[庞校光]: GV14

删除[庞校光]: GV22

删除[庞校光]: 5.3 Influence of DEMs on the ice thickness

Contemporary DEMs derive primarily from microwave (InSAR) and optical (stereophotogrammetry) sensors (González-Moradas and Viveen, 2020; Chen et al., 2022; Hawker et al., 2022), typically referenced to EGM2008 ge

删除[庞校光]: 4

设置格式[庞校光]: 字体: (中文) Times New Roman

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

设置格式[庞校光]: 字体: (默认) Times New Roman

设置格式[庞校光]: 非突出显示

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

设置格式[庞校光]: 字体: (默认) Times New Roman, (中文) 宋体, 英语(美国), (中文) 中文(简体)

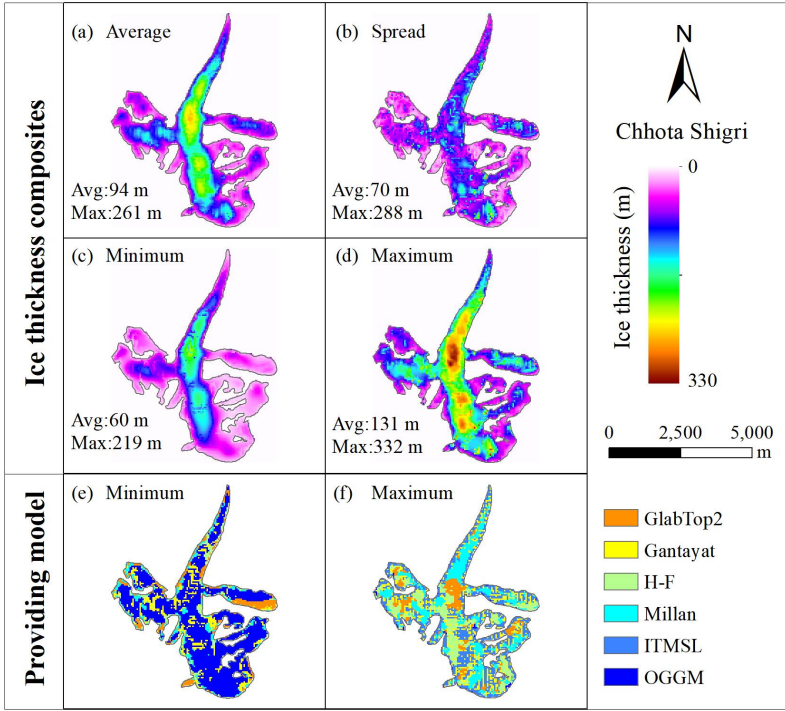


Figure 11: Comparison of simulated ice thickness among models. The first four panels present composite results from the models: (a) average, (b) spread, (c) minimal, and (d) maximal ice thickness distribution. The models corresponding to the minimum and maximum ice thickness at each pixel are shown in (e) and (f), respectively.

To evaluate the absolute accuracy of the ITMSL model, we collected GPR ice thickness data from 16 glaciers in HMA and conducted an accuracy assessment based on these data (Figure 12). Systematic analysis reveals consistent ice thickness overestimation across all seven models for at least 8 of the 16 glaciers, with SIA-based models (ITMSL, Gantayat, Millan) exhibiting particularly pronounced deviations—exceeding other models' estimates for 9 glaciers. Millan uniquely overestimated SACH, CHHU, GHAR, BARP, and BATU glaciers, while ITMSL and Gantayat dominated overestimation at SHIY, HXLG, and URUM. Comparison with the GPR ice thickness (61.24 m) shows that the thickness is estimated to be 100.65 m (64.4%) for ITMSL, 104.05 m (69.9%) for Gantayat, 97.46 m (59.1%) for Millan, while it is 84.91 m (38.7%) for Composite, 81.32 m (32.8%) for GlabTOP2, 80.13 m (30.8%) for H-F, and 75.55 m (23.4%) for OGGM. This suggests that current ice thickness models have been overestimating glacier thickness.

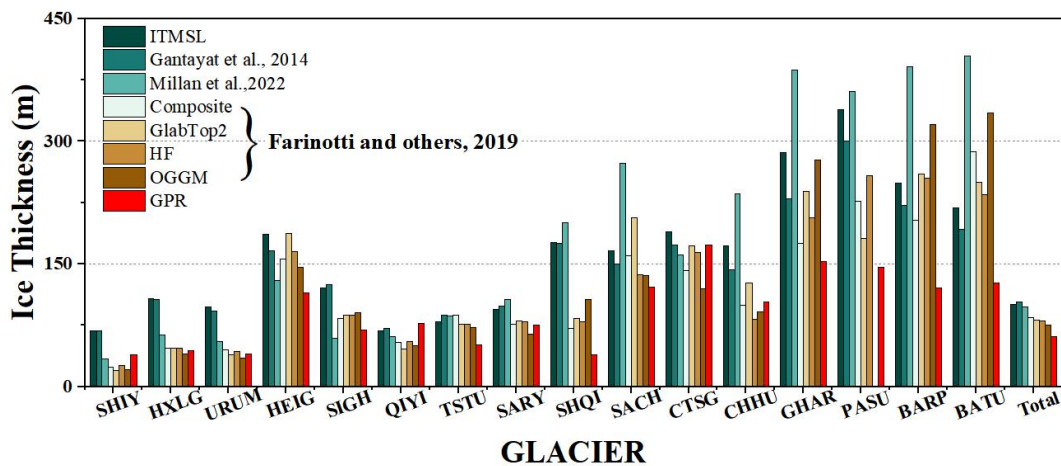
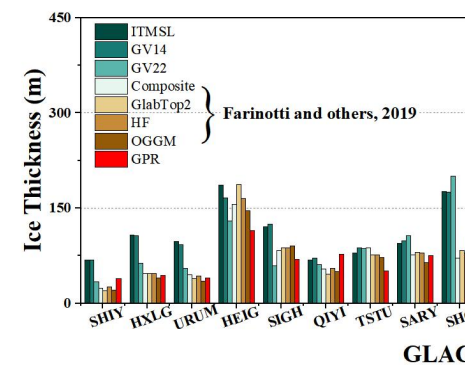
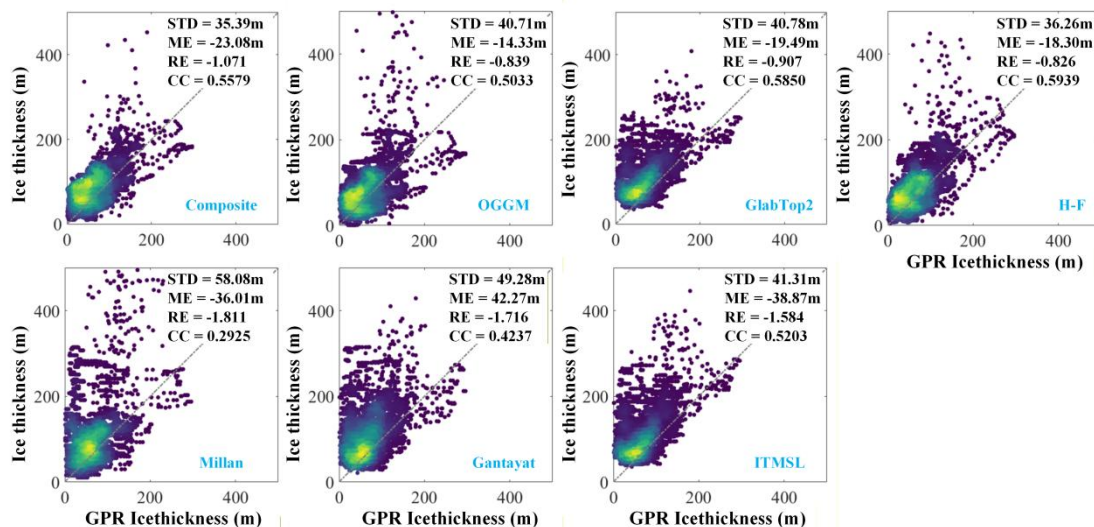


Figure 12: Comparative analysis of the ice thickness of 16 glaciers estimated by 7 models (ITMSL, Gantayat, Millan, Composite, GlabTop2, H-F, and OGGM) and GPR measurements.

Comprehensive accuracy assessment reveals consistent ice thickness overestimation across all models, though ITMSL effectively constrains Millan's excessive deviations as evidenced by thickness distributions (Figure 13). The ME quantification confirms a systematic positive bias. Farinotti et al. (2021) emphasized that no single model clearly outperforms all others, highlighting the advantage of multi - model ensembles over individual approaches. Compared to OGGM, the Composite model reduces thickness errors by 13.1%; by 13.2% relative to GlabTop2; by 2.4% relative to H-F; by 39.1% relative to Millan; by 28.2% relative to Gantayat; and by 14.3% relative to ITMSL. These findings validate multi-model ensemble approaches for leveraging complementary strengths through error compensation mechanisms, though absolute accuracy requires further enhancement given persistent overestimation trends.



删除[庞校光]:

删除[庞校光]: 1

删除[庞校光]: GV14

删除[庞校光]: GV22

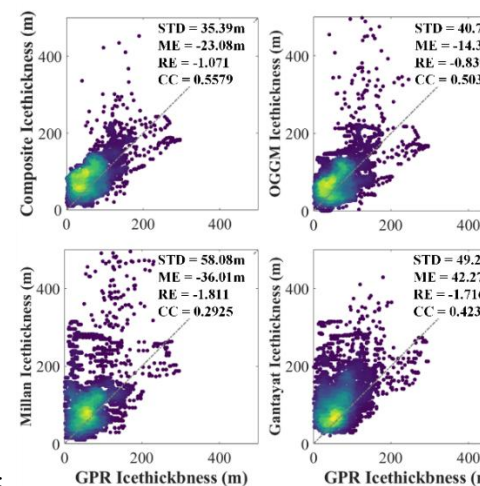
删除[庞校光]: GV22

删除[庞校光]: 2

设置格式[庞校光]: 字体: (默认) Times New Roman

删除[庞校光]: ME quantification confirms systematic positive bias, with the Composite demonstrating relative improvements—reducing thickness error by 13.1% versus OGGM, 13.2% against GlabTop2, 2.4% compared to H-F, 39.1% relative to GV22, 28.2% below GV14, and 14.3% less than ITMSL.

设置格式[庞校光]: 突出显示



删除[庞校光]:

Figure 13: Evaluation of model-inverted ice thickness accuracy using GPR-measured ice thickness.

Subglacial topography was reconstructed by subtracting modelled ice thickness from GLOB30 surface elevations, with validation against GPR-derived bed measurements (Figure 14). The modeled and observed ice beds show excellent agreement, with correlation coefficients exceeding 0.97 across all seven models and relative errors below 0.4%. The high-fidelity reproduction of basal geometry—achieved despite complex subglacial terrain—confirms the robustness of inversion methodologies for applications requiring precise bed mapping, particularly potential glacial lake research.

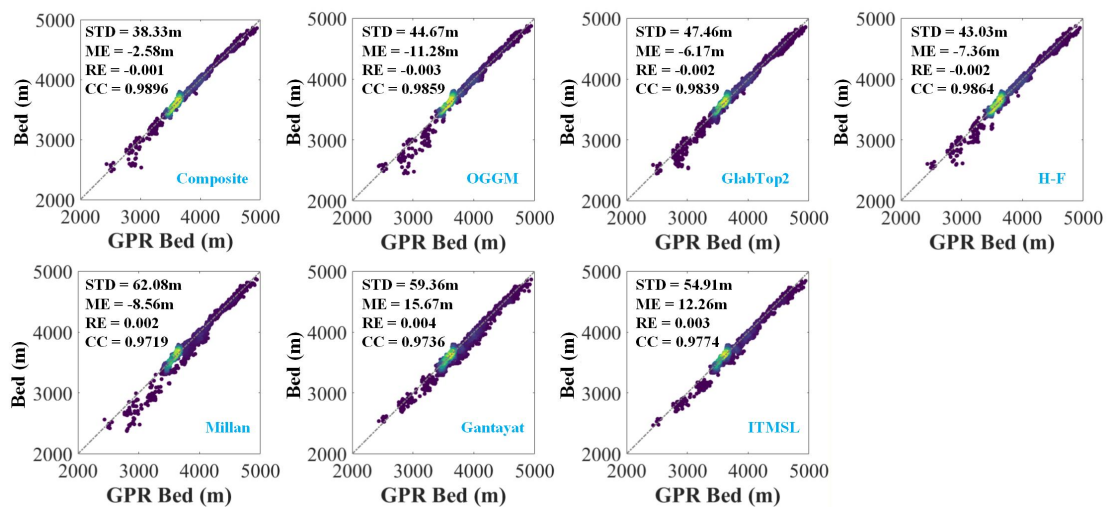


Figure 14: Evaluation of model-inverted subglacial topography accuracy using ice-bed measurements.

6 Conclusions and perspectives

This study presents a novel glacier thickness inversion model, ITMSL, which enhances traditional laminar flow equation by incorporating a physically-based basal sliding law. Applied to 16 glaciers across High Mountain Asia (HMA), ITMSL model autonomously estimates basal sliding velocity, ice thickness distribution, and subglacial topography with significantly improved accuracy—reducing STD by 16.2% compared to Gantayat and 28.9% versus Millan. This study highlighted the importance of considering glacier basal sliding in ice thickness estimates, particularly when u_b/u_s is greater than 0.4. Although ITMSL exhibits robust performance on glaciers larger than 1 km², its performance degrades on smaller glaciers. Previous studies have demonstrated that multi-model ensemble approaches offer significant advantages in ice thickness inversion. Therefore, ITMSL should be used as a member of such an ensemble, rather than as a replacement for other methods.

With the growing availability of high-resolution remote sensing data, ITMSL shows great potential for glacier thickness estimation. However, the ITMSL does not account for the effects of glacier temperature and surface debris on ice thickness estimation, and lacked verification of modeled glacier sliding measurements. Despite these limitations, ITMSL model establishes a promising foundation for estimating ice thickness in complex mountain environments, particularly where traditional ~~SIA~~ assumptions prove inadequate.

删除[庞校光]: laminar flow

Code and data availability

The current version of ITMSL is available from the project website <https://github.com/pxgxhcxs/ITMSL> (last access: 24 November 2025) under the GNU General Public License (GPL) v3.0. The exact version of the model used to produce the results used in this paper is archived on Zenodo under <https://doi.org/10.5281/zenodo.18267363> (Pang et al., 2025), as are input data and scripts to run the model and produce the plots for all the simulations presented in this paper (Pang et al., 2025).

Author contribution

XP is the main ITMSL developer and wrote most of the paper. LJ conceived and designed the research. YW tested the model and assisted with design and debugging. XL carried out the model validation and accuracy assessment with contributions from YL and XL. TY revised the article.

Competing interests

The contact author has declared that none of the authors has any competing interests.

Acknowledgments

We acknowledge the open-access glacier inventory and GlaThiDa dataset from the National Snow and Ice Data Center (NSIDC), the Copernicus DEM from ESA, and the glacier velocity data from Millan et al (2022). This research was funded by the National Natural Science Foundation of China (Grant No.

42174046), the Second Qinghai-Tibet Plateau Scientific Expedition and Research Program, No.2019QZKK0905.

References

- 540 Azam, M. F., Wagnon, P., Ramanathan, A., Vincent, C., Sharma, P., Arnaud, Y., Linda, A., Pottakkal, J. G., Chevallier, P., and Singh, V. B.: From balance to imbalance: a shift in the dynamic behaviour of Chhota Shigri glacier, western Himalaya, India, *Journal of Glaciology*, 58, 315-324, doi:10.3189/2012JoG11J123, 2012a.
- 545 Azam, M. F., Wagnon, P., Ramanathan, A., Vincent, C., Sharma, P., Arnaud, Y., Linda, A., Pottakkal, J. G., Chevallier, P., Singh, V. B., and Berthier, E.: From balance to imbalance: a shift in the dynamic behaviour of Chhota Shigri glacier, western Himalaya, India, *Journal of Glaciology*, 58, 315-324, 10.3189/2012JoG11J123, 2012b.
- Bahr, D. B., Pfeffer, W. T., and Kaser, G.: A review of volume-area scaling of glaciers, *Rev. Geophys.*, 53, 95-140, 10.1002/2014rg000470, 2015.
- 550 Becek, K., Koppe, W., and Kutoğlu, Ş.: Evaluation of Vertical Accuracy of the WorldDEM™ Using the Runway Method, *Remote Sensing*, 8, 10.3390/rs8110934, 2016.
- Berends, C. J., van de Wal, R. S. W., van den Akker, T., and Lipscomb, W. H.: Compensating errors in inversions for subglacial bed roughness: same steady state, different dynamic response, *The Cryosphere*, 17, 1585-1600, 10.5194/tc-17-1585-2023, 2023.
- 555 Berti, M., Corsini, A., and Daehne, A.: Comparative analysis of surface roughness algorithms for the identification of active landslides, *Geomorphology*, 182, 1-18, 10.1016/j.geomorph.2012.10.022, 2013.
- Bolch, T., Kulkarni, A., Kaab, A., Huggel, C., Paul, F., Cogley, J. G., Frey, H., Kargel, J. S., Fujita, K., Scheel, M., Bajracharya, S., and Stoffel, M.: The state and fate of Himalayan glaciers, *Science*, <https://doi.org/10.1093/9780197802601.003.0002>, 2012.
- 560 Brun, F., Berthier, E., Wagnon, P., Kaab, A., and Treichler, D.: A spatially resolved estimate of High Mountain Asia glacier mass balances from 2000 to 2016, *Nature Geoscience*, 10, 668-+, 10.1038/ngeo2999, 2017.
- Budd, W. F., Keage, P. L., and Blundy, N. A.: Empirical studies of ice sliding, *Journal of Glaciology*, 23, 157-170, doi:10.3189/S0022143000029804, 1979.
- 565 Chen, W. F., Yao, T. D., Zhang, G. Q., Li, F., Zheng, G. X., Zhou, Y. S., and Xu, F. L.: Towards ice-thickness inversion: an evaluation of global digital elevation models (DEMs) in the glacierized Tibetan Plateau, *Cryosphere*, 16, 197-218, 10.5194/tc-16-197-2022, 2022.
- Cuffey, K. and Paterson, W.: *The Physics of Glaciers* Fourth Edition, 2010.
- Echelmeyer, K. and Zhongxiang, W.: Direct Observation of Basal Sliding and Deformation of Basal Drift at Sub-Freezing Temperatures, *Journal of Glaciology*, 33, 83-98, 10.3189/S0022143000005396, 570 1987.
- Engelhardt, H. and Kamb, B.: Basal sliding of Ice Stream B, West Antarctica, *Journal of Glaciology*, 44, 223-230, <https://doi:10.3189/S0022143000002562>, 1998.
- Fan, Y., Ke, C.-Q., Zhou, X., Shen, X., Yu, X., and Lhakpa, D.: Glacier mass-balance estimates over High Mountain Asia from 2000 to 2021 based on ICESat-2 and NASADEM, *Journal of Glaciology*, 575 1-13, 10.1017/jog.2022.78, 2022.

- Fang, K., Mei, Z., Wu, H., Zhou, F., Seppä, H., and Guo, Z.: Indian summer monsoon drives synchronous interdecadal hydroclimate changes in the Tibetan Plateau and surroundings, *Global and Planetary Change*, 234, 10.1016/j.gloplacha.2024.104379, 2024.
- Farinotti, D., Huss, M., Bauder, A., Funk, M., and Truffer, M.: A method to estimate the ice volume and ice-thickness distribution of alpine glaciers, *Journal of Glaciology*, 55, 422-430, 10.3189/002214309788816759, 2009.
- Farinotti, D., Huss, M., Fürst, J. J., Landmann, J., Machguth, H., Maussion, F., and Pandit, A.: A consensus estimate for the ice thickness distribution of all glaciers on Earth, *Nature Geoscience*, 12, 168-173, 10.1038/s41561-019-0300-3, 2019.
- Farinotti, D., Brinkerhoff, D. J., Furst, J. J., Gantayat, P., Gillet-Chaulet, F., Huss, M., Leclercq, P. W., Maurer, H., Morlighem, M., Pandit, A., Rabatel, A., Ramsankaran, R., Reerink, T. J., Robo, E., Rouges, E., Tamre, E., van Pelt, W. J. J., Werder, M. A., Azam, M. F., Li, H. L., and Andreassen, L. M.: Results from the Ice Thickness Models Intercomparison eXperiment Phase 2 (ITMIX2), *Frontiers in Earth Science*, 8, 21, 10.3389/feart.2020.571923, 2021.
- Farinotti, D., Brinkerhoff, D. J., Clarke, G. K. C., Fürst, J. J., Frey, H., Gantayat, P., Gillet-Chaulet, F., Girard, C., Huss, M., Leclercq, P. W., Linsbauer, A., Machguth, H., Martin, C., Maussion, F., Morlighem, M., Mosbeux, C., Pandit, A., Portmann, A., Rabatel, A., Ramsankaran, R., Reerink, T. J., Sanchez, O., Stentoft, P. A., Singh Kumari, S., van Pelt, W. J. J., Anderson, B., Benham, T., Binder, D., Dowdeswell, J. A., Fischer, A., Helfricht, K., Kutuzov, S., Lavrentiev, I., McNabb, R., Gudmundsson, G. H., Li, H., and Andreassen, L. M.: How accurate are estimates of glacier ice thickness? Results from ITMIX, the Ice Thickness Models Intercomparison eXperiment, *The Cryosphere*, 11, 949-970, 10.5194/tc-11-949-2017, 2017.
- Fowler, A. C. and Frank, F. C.: A theoretical treatment of the sliding of glaciers in the absense of cavitation, *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 298, 637-681, doi:10.1098/rsta.1981.0003, 1981.
- Frey, H., Machguth, H., Huss, M., Huggel, C., Bajracharya, S., Bolch, T., Kulkarni, A., Linsbauer, A., Salzmann, N., and Stoffel, M.: Estimating the volume of glaciers in the Himalayan–Karakoram region using different methods, *The Cryosphere*, 8, 2313-2333, 10.5194/tc-8-2313-2014, 2014.
- Gantayat, P., Kulkarni, A. V., and Srinivasan, J.: Estimation of ice thickness using surface velocities and slope: case study at Gangotri Glacier, India, *Journal of Glaciology*, 60, 277-282, 10.3189/2014JoG13J078, 2014.
- Gilbert, A., Gimbert, F., Gagliardini, O., and Vincent, C.: Inferring the Basal Friction Law From Long Term Changes of Glacier Length, Thickness and Velocity on an Alpine Glacier, *Geophysical Research Letters*, 50, 10.1029/2023gl104503, 2023.
- González-Moradas, M. d. R. and Viveen, W.: Evaluation of ASTER GDEM2, SRTMv3.0, ALOS AW3D30 and TanDEM-X DEMs for the Peruvian Andes against highly accurate GNSS ground control points and geomorphological-hydrological metrics, *Remote Sensing of Environment*, 237, 111509, <https://doi.org/10.1016/j.rse.2019.111509>, 2020.
- Gopika, J. S., Kulkarni, A. V., Prasad, V., Srinivasalu, P., and Raman, A.: Estimation of glacier stored water in the Bhaga basin using laminar flow and volume-area scaling methods, *Remote Sensing Applications: Society and Environment*, 24, 10.1016/j.rsase.2021.100656, 2021.
- Grinsted, A.: An estimate of global glacier volume, *The Cryosphere*, 7, 141-151, 10.5194/tc-7-141-2013, 2013.

- Gudmundsson, G. H.: Basal-flow characteristics of a non-linear flow sliding frictionless over strongly undulating bedrock, *Journal of Glaciology*, 43, 80-89, 10.3189/S0022143000002835, 1997.
- Hawker, L., Uhe, P., Paulo, L., Sosa, J., Savage, J., Sampson, C., and Neal, J.: A 30 m global map of elevation with forests and buildings removed, *Environmental Research Letters*, 17, 10.1088/1748-9326/ac4d4f, 2022.
- Helanow, C., Iverson, N. R., Woodard, J. B., and Zoet, L. K.: A slip law for hard-bedded glaciers derived from observed bed topography, *Science Advances*, 7, eabe7798, doi:10.1126/sciadv.abe7798, 2021.
- Höhle, J. and Höhle, M.: Accuracy assessment of digital elevation models by means of robust statistical methods, *ISPRS Journal of Photogrammetry and Remote Sensing*, 64, 398-406, 10.1016/j.isprsjprs.2009.02.003, 2009.
- Hugonnet, R., McNabb, R., Berthier, E., Menounos, B., Nuth, C., Girod, L., Farinotti, D., Huss, M., Dussaillant, I., Brun, F., and Kaab, A.: Accelerated global glacier mass loss in the early twenty-first century, *Nature*, 592, 726-731, 10.1038/s41586-021-03436-z, 2021.
- Huss, M. and Farinotti, D.: Distributed ice thickness and volume of all glaciers around the globe, *Journal of Geophysical Research: Earth Surface*, 117, n/a-n/a, 10.1029/2012jf002523, 2012.
- Iken, A.: The Effect of the Subglacial Water Pressure on the Sliding Velocity of a Glacier in an Idealized Numerical Model, *Journal of Glaciology*, 27, 407-421, doi:10.3189/S0022143000011448, 1981.
- Joughin, I., Smith, B. E., and Schoof, C. G.: Regularized Coulomb Friction Laws for Ice Sheet Sliding: Application to Pine Island Glacier, Antarctica, *GEOPHYSICAL RESEARCH LETTERS*, 46, 4764-4771, 10.1029/2019GL082526, 2019.
- Jouvet, G., Cordonnier, G., Kim, B., Lüthi, M., Vieli, A., and Aschwanden, A.: Deep learning speeds up ice flow modelling by several orders of magnitude, 2021.
- Kraaijenbrink, P. D. A., Bierkens, M. F. P., Lutz, A. F., and Immerzeel, W. W.: Impact of a global temperature rise of 1.5 degrees Celsius on Asia's glaciers, *Nature*, 549, 257-260, 10.1038/nature23878, 2017.
- Lannutti, E., Lenzano, M. G., Vacaflor, P., Rivera, A., Moragues, S., Gentile, M., and Lenzano, L.: Ice thickness distribution and stability of three large freshwater calving glaciers on the eastern side of the Southern Patagonian Icefield, *Cold Reg. Sci. Tech.*, 221, 10.1016/j.coldregions.2024.104158, 2024.
- Li, F., Maussion, F., Wu, G., Chen, W., Yu, Z., Li, Y., and Liu, G.: Influence of glacier inventories on ice thickness estimates and future glacier change projections in the Tian Shan range, Central Asia, *Journal of Glaciology*, 1-15, 10.1017/jog.2022.60, 2022.
- Li, H. L., Ng, F., Li, Z. Q., Qin, D. H., and Cheng, G. D.: An extended "perfect-plasticity" method for estimating ice thickness along the flow line of mountain glaciers, *Journal of Geophysical Research Earth Surface*, 117, -, <https://doi.org/10.1029/2011JF002104>, 2012.
- Liang, P.-B. and Tian, L.-D.: Estimation of glacier ice storage in western China constrained by field ground-penetrating Radar surveys, *Advances in Climate Change Research*, <https://doi.org/10.1016/j.accr.2022.04.002>, 2022.
- Liu, K., Song, C., Ke, L., Jiang, L., and Ma, R.: Global open-access DEM performances in Earth's most rugged region High Mountain Asia: A multi-level assessment, *Geomorphology*, 338, <https://doi.org/10.1016/j.geomorph.2019.04.012>, 2019.
- Louis and Lliboutry: Realistic, yet simple bottom boundary conditions for glaciers and ice sheets, *Journal of Geophysical Research*, <https://doi.org/10.1029/JB092iB09p09101>, 1987.

- Maussion, F., Butenko, A., Champollion, N., Dusch, M., Eis, J., Fourteau, K., Gregor, P., Jarosch, A. H., Landmann, J., Oesterle, F., Recinos, B., Rothenpieler, T., Vlug, A., Wild, C. T., and Marzeion, B.: The Open Global Glacier Model (OGGM) v1.1, *Geoscientific Model Development*, 12, 909-931, 10.5194/gmd-12-909-2019, 2019.
- Miles, E., McCarthy, M., Dehecq, A., Kneib, M., Fugger, S., and Pellicciotti, F.: Health and sustainability of glaciers in High Mountain Asia, *Nat. Commun.*, 12, 10, 10.1038/s41467-021-23073-4, 2021.
- Millan, R., Mouginot, J., Rabatel, A., and Morlighem, M.: Ice velocity and thickness of the world's glaciers, *Nature Geoscience*, 15, 124-129, 10.1038/s41561-021-00885-z, 2022.
- Millan, R., Mouginot, J., Rabatel, A., Jeong, S., Cusicanqui, D., Derkacheva, A., and Chekki, M.: Mapping Surface Flow Velocity of Glaciers at Regional Scale Using a Multiple Sensors Approach, *Remote Sensing*, 11, 10.3390/rs11212498, 2019.
- O., Gagliardini, D., Cohen, P., Råback, and, T., and Zwinger: Finite-element modeling of subglacial cavities and related friction law, *Journal of Geophysical Research Earth Surface*, <https://doi.org/10.1029/2006JF000576>, 2007.
- Pang, X., Jiang, L., Guo, R., Xu, Z., Li, X., and Lu, X.: Surface Motion and Topographic Effects on Ice Thickness Inversion for High Mountain Asia Glaciers: A Comparison Study from Three Numerical Models, *Remote Sensing*, 15, 10.3390/rs15225378, 2023.
- Pang, X., Jiang, L., Wu, Y., Lu, X., Liu, Y., Li, X., and Yao, T.: ITMSL: an improved ice thickness inversion model integrating basal sliding dynamics for High Mountain Asia (v1.0.0), *Zenodo*, <https://doi.org/10.5281/zenodo.18267363>, 2025.
- Pfeffer, W. T., Arendt, A. A., Bliss, A., Bolch, T., Cogley, J. G., Gardner, A. S., Hagen, J.-O., Hock, R., Kaser, G., Kienholz, C., Miles, E. S., Moholdt, G., Mölg, N., Paul, F., Radić, V., Rastner, P., Raup, B. H., Rich, J., and Sharp, M. J.: The Randolph Glacier Inventory: a globally complete inventory of glaciers, *Journal of Glaciology*, 60, 537-552, 10.3189/2014JoG13J176, 2014.
- Radić, V. and Hock, R.: Regional and global volumes of glaciers derived from statistical upscaling of glacier inventory data, *Journal of Geophysical Research*, 115, 10.1029/2009jf001373, 2010.
- Ramsankaran, R., Pandit, A., and Azam, M. F.: Spatially distributed ice-thickness modelling for Chhota Shigri Glacier in western Himalayas, India, *Int. J. Remote Sens.*, 39, 3320-3343, 10.1080/01431161.2018.1441563, 2018.
- Robel, A. A., Ultee, L., Ranganathan, M., and Nash, M.: For whom and by whom is glaciology?, *Journal of Glaciology*, 1-11, 10.1017/jog.2024.29, 2024.
- Schoof, C.: The effect of cavitation on glacier sliding, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, 461, 609-627, <https://doi.org/10.1098/rspa.2004.1350>, 2005.
- Shepard, M. K., Campbell, B. A., Bulmer, M. H., Farr, T. G., Gaddis, L. R., and Plaut, J. J.: The roughness of natural terrain: A planetary and remote sensing perspective, *J. Geophys. Res.-Planets*, 106, 32777-32795, 10.1029/2000je001429, 2001.
- Van Tricht, L., Huybrechts, P., Van Breedam, J., Furst, J. J., Rybak, O., Satylkanov, R., Ermenbaiev, B., Popovnin, V., Neyns, R., Paice, C. M., and Malz, P.: Measuring and inferring the ice thickness distribution of four glaciers in the Tien Shan, Kyrgyzstan, *Journal of Glaciology*, 67, 269-286, 10.1017/jog.2020.104, 2021.

- 705 Veitch, S. A., Karplus, M., Kaip, G., Gonzalez, L. F., Amundson, J. M., and Bartholomaus, T. C.: Ice thickness estimates of Lemon Creek Glacier, Alaska, from active-source seismic imaging, *Journal of Glaciology*, 67, 824-832, 10.1017/jog.2021.32, 2021.
- Wang, L., Liu, H., Bhlon, R., Chen, D., Long, J., and Sherpa, T. C.: Modeling glacio-hydrological processes in the Himalayas: A review and future perspectives, *Geography and Sustainability*, 5, 179-192, 10.1016/j.geosus.2024.01.001, 2024.
- 710 Wang, P., Li, Z., Li, H., Yao, H., Xu, C., Zhou, P., Jin, S., and Wang, W.: Analyses of recent observations of Urumqi Glacier No. 1, Chinese Tianshan Mountains, *Environmental Earth Sciences*, 75, 10.1007/s12665-016-5551-3, 2016.
- Wang, R.-J., Ding, Y.-J., Shangguan, D.-H., Liu, S.-Y., Guo, W.-Q., Han, H.-D., Li, Y.-J., Song, M., and Zheng, Z.-Q.: Spatial differences of ice volume across High Mountain Asia, *Advances in Climate Change Research*, 14, 511-521, 10.1016/j.accre.2023.08.004, 2023.
- 715 Weertman, J.: On the Sliding of Glaciers, *Journal of Glaciology*, 3, 33-38, doi:10.3189/S0022143000024709, 1957.
- Welty, E., Zemp, M., Navarro, F., Huss, M., Fürst, J. J., Gärtner-Roer, I., Landmann, J., Machguth, H., Naegeli, K., Andreassen, L. M., Farinotti, D., and Li, H.: Worldwide version-controlled database of glacier thickness observations, *Earth Syst. Sci. Data*, 12, 3039-3055, 10.5194/essd-12-3039-2020, 2020.
- 720 Woodard, J. B., Zoet, L. K., Iverson, N. R., and Helanow, C.: Inferring forms of glacier slip laws from estimates of ice-bed separation during glacier slip, *Journal of Glaciology*, 69, 324-332, 10.1017/jog.2022.63, 2022.
- 725 Wu, K., Liu, S., Zhu, Y., Liu, Q., and Jiang, Z.: Dynamics of glacier surface velocity and ice thickness for maritime glaciers in the southeastern Tibetan Plateau, *Journal of Hydrology*, 590, 125527, <https://doi.org/10.1016/j.jhydrol.2020.125527>, 2020.
- Yao, T., Thompson, L., Yang, W., Yu, W., Gao, Y., Guo, X., Yang, X., Duan, K., Zhao, H., Xu, B., Pu, J., Lu, A., Xiang, Y., Kattel, D. B., and Joswiak, D.: Different glacier status with atmospheric circulations in Tibetan Plateau and surroundings, *Nat. Clim. Chang.*, 2, 663-667, 10.1038/nclimate1580, 2012.
- 730 Zekollari, H., Huss, M., Farinotti, D., and Lhermitte, S.: Ice-Dynamical Glacier Evolution Modeling—A Review, *Rev. Geophys.*, 60, 10.1029/2021rg000754, 2022.
- 735 Zoet, L. K. and Iverson, N. R.: A slip law for glaciers on deformable beds, *Science*, 368, 76-78, doi:10.1126/science.aaz1183, 2020.
- Zou, X. J., Gao, H. F., Zhang, Y. S., Ma, N., Wu, J. F., and Bin Farhan, S.: Quantifying ice storage in upper Indus river basin using ground-penetrating radar measurements and glacier bed topography model version 2, *Hydrol. Process.*, 35, 14, 10.1002/hyp.14145, 2021.

740 |