Response to Reviewer 1 Comments

Xiaoguang Pang and colleagues develop an ice thickness inversion model based on laminar-flow theory that explicitly incorporates a sliding law, whereas basal sliding parameterization is often neglected or oversimplified in comparable approaches. They apply their model (ITMSL) to 16 glaciers across High Mountain Asia (HMA). First, they compare ITMSL with two other laminar- flow-based inversion methods (GV14 and GV22). Second, they compare ITMSL with four additional ice thickness inversion models that are well established in the literature.

Response: We sincerely thank the reviewer for the insightful understanding and positive recognition of our work. We particularly appreciate the key innovation you pointed out—that our model (ITMSL) explicitly incorporates a sliding law, whereas existing comparable approaches often simplify or directly neglect it. For your comments, we have carefully and thoroughly revised the manuscript point by point and updated the corresponding expressions in the text. The specific modifications are as follows:

General Comment

1. While it is natural and appropriate to compare ITMSL with the most closely related approaches (GV14 and GV22), since the new model can be viewed as an extension of these methods with an improved representation of basal sliding, the scope and added value of such a simplified approach relative to other types of models (e.g., flowline models or higher-order 3D models) must be explained and highlighted. At present, this aspect is not discussed. Section 5.4 shows that the four "established models" generally slightly outperform the laminar-flow-based models. A slightly lower performance for a simplified model is not necessarily problematic; however, the advantages and limitations of such an approach should be explicitly discussed to clarify its relevance and potential applications.

   Response: Thank you for your suggestion. Indeed, we should have described the advantages and applicable scenarios of ITMSL more clearly. ITMSL builds upon the simple assumptions regarding basal sliding made by the GV14 and GV22 ice thickness inversion models, and further refines the simulation mechanism of the basal sliding process. Compared with higher-order three-dimensional models, this simplified approach requires less stringent input data and offers higher computational efficiency, making it suitable for regional- to global-scale studies of glacier thickness and ice storage. To address this, we have revised the conclusion section to highlight the strengths of our method.

   Regarding flowline models, the authors have previously discussed the VOLTA model, which also inverts ice thickness based on centerlines. VOLTA derives ice thickness along glacier centerlines using the shallow-ice approximation, then assumes zero thickness at glacier margins and interpolates the centerline thickness to the entire glacier via inverse distance weighting[1]. However, due to complex glacier geometries, the generation of centerlines is often problematic and frequently requires manual editing. Moreover, centerlines are difficult to define for glacier tributaries. Consequently, pixels farther from the centerlines tend to have underestimated thickness, and tributaries without defined centerlines exhibit significant underestimation[2].

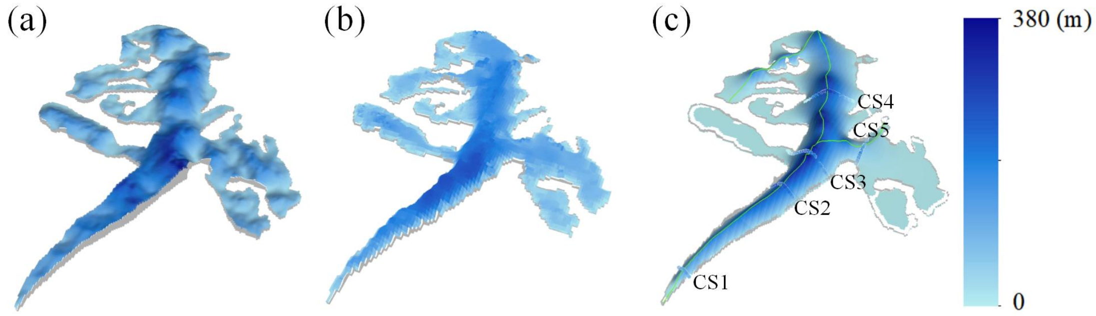


Figure 2. Ice thickness distribution of Chhota Shigri glacier modeled using GlabTop2 (a), GV22 (b), and VOLTA (c). CS1-5 in the figure show the position of the GPR measurement line, from which it can be seen whether the measured thickness corresponds to the estimated thickness.

[1] JAMES W H M, CARRIVICK J L. Automated modelling of spatially-distributed glacier ice thickness and volume [J]. Computers & Geosciences, 2016, 92: 90-103.

[2] PANG X, JIANG L, GUO R, et al. Surface Motion and Topographic Effects on Ice Thickness Inversion for High Mountain Asia Glaciers: A Comparison Study from Three Numerical Models [J]. Remote Sensing, 2023, 15(22).

2. Another important point is the absence of any reference to the ITMIX framework (Farinotti et al., 2017; 2021), in which coordinated intercomparison experiments were conducted to evaluate ice thickness inversion models. The conclusions of these studies could help to better motivate the present work. For instance, it was shown that the models by Brinkerhoff, Farinotti, Fuerst, Gantayat (= GV14), Huss, Maurer, Rabatel, Van Pelt & Leclercq, and Werder exhibit broadly similar performance. In ITMIX II, GV14 ranked among the better-performing models, making efforts to further improve it a meaningful scientific objective.

Response: Thank you very much for your valuable comments. In the original manuscript, we focused only on the discussion of ice thickness models based on the laminar-flow equation and failed to summarize the ITMIX framework, which was indeed our oversight. Following your suggestion, we have revised and supplemented the introduction section, highlighting the work of ITMIX and adding an introduction to five categories of ice thickness models. Furthermore, in the discussion section, we have conducted the analysis at the pixel scale with reference to the ITMIX framework. The revised content is as follows:

(a) Introduction Additions (P.2-3, L56-82):

To date, approximately 20 ice thickness models have been proposed, including

H-F, OGGM, GlabTop2, and ice velocity-based approaches (Huss and Farinotti, 2012; Frey et al., 2014; Gantayat et al., 2014; Farinotti et al., 2019; Maussion et al., 2019; Millan et al., 2022). To evaluate the accuracy and limitations of ice thickness inversion models, ITMIX systematically assessed 17 models that infer ice thickness from glacier surface characteristics (Farinotti et al., 2017). These models encompass different approaches, including minimization approaches, mass conservation approaches (Huss and Farinotti, 2012), shear stress-based approaches (Frey et al., 2014), ice velocity-based approaches (Gantayat et al., 2014), and convolutional neural network approaches (Jouvet et al., 2021). Different models have varying input data requirements, typically requiring two or more types of data from digital elevation models (DEMs), glacier outlines, surface velocity fields, and mass balance data (Farinotti et al., 2017; Farinotti et al., 2021). Among them, minimization approaches offer strong physical consistency and can handle complex glacier dynamics (Farinotti et al., 2021); mass conservation approaches feature a solid physical foundation and good stability (Farinotti et al., 2009); shallow ice approximation (SIA) approaches are robust and computationally fast (Ramsankaran et al., 2018); and ice velocity-based methods are particularly effective for sliding-dominated glaciers (Wu et al., 2020). ITMIX revealed that the maximum discrepancy in ice thickness estimates among models can be on the order of the actual glacier thickness itself. Substantial disparities exist among different models in their thickness estimates for the same glacier, with uncertainties reaching tens to even hundreds of meters. Weighted ensemble averaging of multiple models can effectively offset systematic biases inherent in individual models, thereby ameliorating the accuracy of ice thickness simulations (Farinotti et al., 2017). The ITMIX2 experiment evaluated the influence of the number and location of in-situ ice thickness observations on model calibration and accuracy, and found that models respond differently to data scenarios, with no single model performing best across all scenarios. At the same time, the experiment highlighted the critical role of sparse in-situ data and recommended prioritizing measurements in the thickest location of glaciers (Farinotti et al., 2021). Against this backdrop, refinements to individual ice thickness models can help improve the accuracy of the multi-model ensemble average.

(b) Discussion Additions (P.22, L451-462):

Taking the CTSG glacier as an example, we compare the composite mean ice thickness (i.e., the ice thickness distribution obtained by averaging, Figure 11a) with the ice thickness spread (i.e., the spread of all model results within a pixel, Figure 11b). Figures 11c and 11d present the maximum and minimum ice thickness composites, respectively, illustrating the composition of the composite results. The models producing the most extreme results are shown in Figures 11e and 11f. For the CTSG glacier, the minimum ice thickness is predominantly provided by the OGGM model (62.2%), while the minimum values from GlabTop2 are mainly distributed over the glacier's tributaries. In contrast, the maximum ice thickness is primarily contributed by ITMSL (32.7%), Millan

(20.2%), Gantayat (19.8%), and H-F (16.2%). This pattern is consistent with the trends observed in the ITMIX results. Specifically, the maximum ice thickness from ITMSL is concentrated along the glacier margins; that from Millan is concentrated in the lower reaches and the tongue of the glacier; that from Gantayat is relatively evenly distributed; and that from the H-F model is concentrated in the upper reaches.

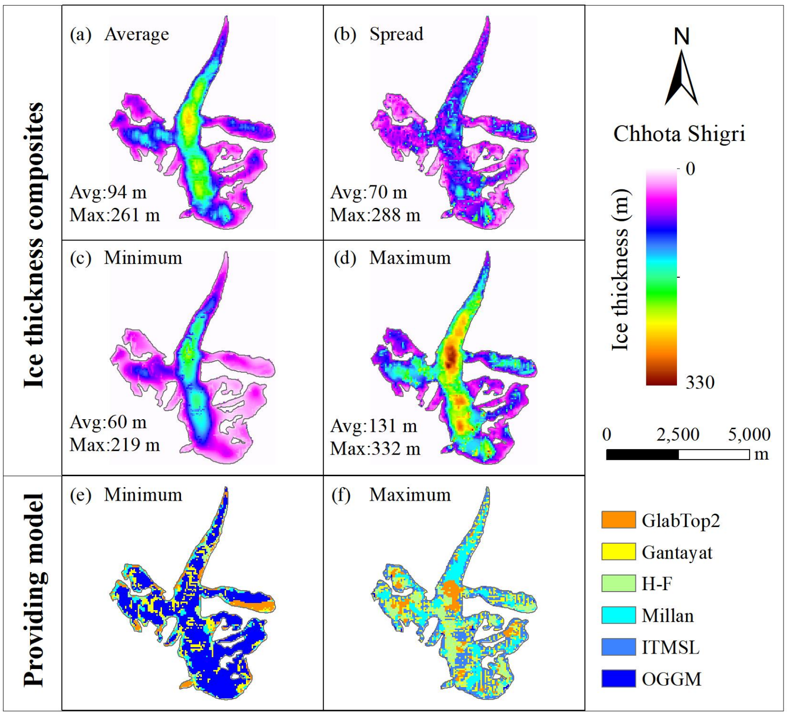


Figure 11: Comparison of simulated ice thickness among models. The first four panels present composite results from the models: (a) average, (b) spread, (c) minimal, and (d) maximal ice thickness distribution. The models corresponding to the minimum and maximum ice thickness at each pixel are shown in (e) and (f), respectively.

3. Furthermore, Farinotti et al. (2021) emphasized that no single model clearly outperforms all others, highlighting the value of multi-model ensembles over individual approaches. This suggests that model development remains an open challenge, particularly in understanding the causes of discrepancies between inverted and observed ice thickness. In this context, investigating the impact of basal sliding representation on inversion performance is indeed a relevant and timely scientific question.

Response: Thank you for your affirmation of our research. We acknowledge the significant advantages of multi-model ensembles, a point that has been addressed in the Discussion section. As you pointed out, each model has its own applicable scenarios, with respective strengths and limitations. In this work, we focused on improving the determination of basal sliding velocity within the simplified laminar-flow equation, and achieved certain progress. Meanwhile, we thoroughly analyzed the limitations of the proposed ITMSL model. Using 16 glaciers in High Mountain Asia as study targets, we found that ITMSL has deficiencies when applied to small glaciers. Section 5.1 provides a detailed discussion, and we recognize that the model still requires continuous improvement.

4. Finally, regarding the ITMIX framework, Farinotti et al.(2021) used the relative ice thickness difference (normalized by mean ice thickness) to enable comparison across glaciers of different sizes and across parts of the glaciers with varying thicknesses. Such a metric could also have been used in this study, as mentioned in the detailed comments on Figure 3.

   Response: Thank you for your suggestion. Following the ITMIX framework, we have revised Figure 3 by adjusting the vertical axis to represent RMSE divided by ice thickness and multiplied by 100% (i.e., ice-thickness normalization). The revised version is as follows **(P.14, L305)**:

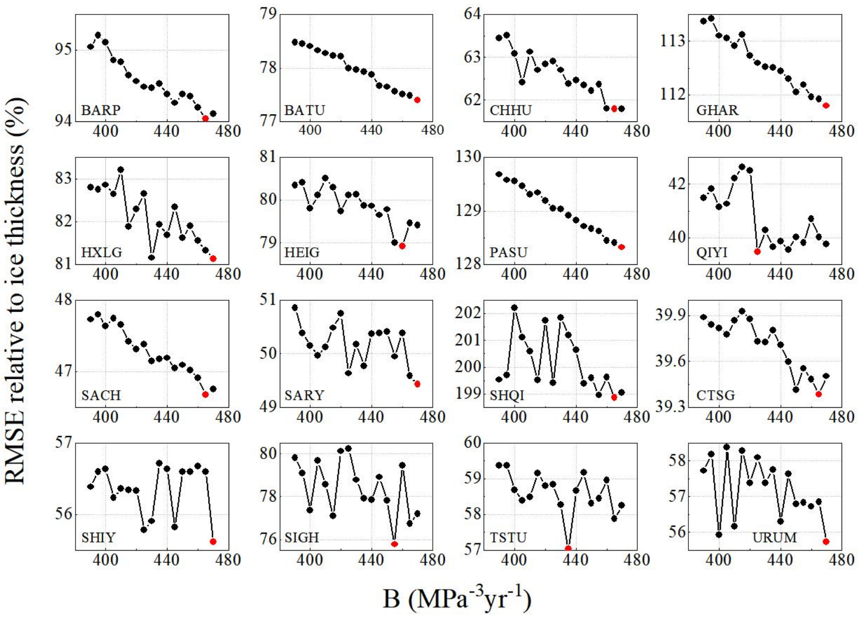


Figure 3: Ratio of the RMSE of ice thickness inversion to ice thickness for different values of parameter B in the ITMSL model.

Response to minor comments:

Aside from these points, the manuscript is generally easy to read and follow, and the methodology is clearly described. Some figures could be improved slightly, but the overall content is interesting. Additional contextualization, as discussed above, would strengthen the manuscript. Most of the analyses are relevant, though some would benefit from further refinement and deeper discussion. A few suggestions for additional analyses are provided in the detailed comments.

Response: We sincerely thank the reviewer for the encouraging comments. In response to your specific suggestions, we have carefully revised and supplemented the manuscript point by point, as detailed below:

0) Abstract :

1. "Current ice thickness inversion primarily uses laminar flow theory constrained by geometric, topographic, and ice flow characteristics" is not completely true. ITMIX papers showed that a large panel of model do inversion based on several methods and not all relying on laminar flow theory.

   Response: In the original manuscript, positioning the study as "ice thickness inversion models based on laminar-flow theory" led to potential ambiguity. We have revised this sentence as follows: "Currently, ice thickness inversion models based on ice velocity have achieved significant progress in regional and global studies of glacier thickness and volume. However, these methods simplify the parameterization of basal sliding, introducing uncertainties and significant biases in thickness estimates." **(P.1, L18-21)**

1) Introduction :

1. I recommend to extend the paragraph on inversion model complexity to better understand what are all the subtleties behind the "20 ice thickness models" mentioned at line 53. Important to introduce the ITMIX framework comparing these approaches and also the more complex approaches such as the instructed glacier model (IGM, Jouvet,2023) or Brinkerhoff in Farinotti et al.(2021). Especially since these more complex approaches also use a non-uniform sliding parametrization. Otherwise the introduction is easy to read and clear about the goals of the research.

   Response: Following your suggestion, we have revised and supplemented the Introduction section: on the one hand, we highlighted the work of the ITMIX framework and added an introduction to five categories of ice thickness models; on the other hand, we discussed the advantages and limitations of more complex ice thickness inversion methods (e.g., IGM). The revised content is as follows:

   "To evaluate the accuracy and limitations of ice thickness inversion models, ITMIX systematically assessed 17 models that infer ice thickness from glacier surface characteristics (Farinotti et al., 2017). These models encompass different approaches, including minimization approaches, mass conservation approaches (Huss and Farinotti, 2012), shear stress-based approaches (Frey et al., 2014), ice velocity-based approaches (Gantayat et al., 2014), and convolutional neural network approaches (Jouvet et al., 2021). Different models have varying input data requirements, typically requiring two or more types of data from digital elevation models (DEMs), glacier outlines, surface velocity fields, and mass balance data (Farinotti et al., 2017; Farinotti et al., 2021). Among them, minimization approaches offer strong physical consistency and can handle complex glacier dynamics (Farinotti et al., 2021); mass conservation approaches feature a solid physical foundation and good stability (Farinotti et al., 2009); shallow ice approximation (SIA) approaches are robust and computationally fast (Ramsankaran et al., 2018); and ice velocity-based methods are particularly effective for sliding-dominated glaciers (Wu et al., 2020). ITMIX revealed that the maximum discrepancy in ice thickness estimates among models can be on the order of the actual glacier thickness itself. Substantial disparities exist among different models in their thickness estimates for the same glacier, with uncertainties reaching tens to even hundreds of meters. Weighted ensemble averaging of multiple models can effectively offset systematic biases inherent in individual models, thereby ameliorating the accuracy of ice thickness simulations (Farinotti et al., 2017). The ITMIX2 experiment evaluated the influence of the number and location of in-situ ice thickness observations on model calibration and accuracy, and found that models respond differently to data scenarios, with no single model performing best across all scenarios. At the same time, the experiment highlighted the critical role of sparse in-situ data and recommended prioritizing measurements in the thickest location of glaciers (Farinotti et al., 2021). Against this backdrop, refinements to individual ice thickness models can help improve the accuracy of the multi-model ensemble average." **(P.3, L58-82)**

"These simplified parameterizations therefore risk introducing substantial biases into ice thickness estimates. With the development of artificial intelligence, the Instructed Glacier Model (IGM) ice flow model based on Convolutional Neural Network has been proposed. This approach learns the input-output mapping of the full-Stokes ice flow model under various glacier geometries and dynamic conditions, thereby becoming an efficient and high accuracy ice thickness inversion model. In this model, basal sliding is computed using the Weertman nonlinear sliding law." **(P.4, L98-104)**

"Although complex ice thickness inversion models offer advantages in physical realism and inversion accuracy, their high computational cost, large number of parameters limit their application in regional and global scale glacier volume studies. Therefore, this study proposes an ice thickness inversion model that couples the basal sliding law with the laminar flow equation (ITMSL), aiming to strike a balance between physical complexity and computational efficiency." **(P.4, L111-116)**

2) Methods: Very clear and easy to follow. A few minor comments to make it even more clear.

1. No glacier in the southern part of HMA, is this a problem in term of representation of various climatic conditions across HMA ? Is it a data scarcity issue ? This must be mentioned in the discussion section limitations, and what the potential impact of no having data from there could be. Southern part is more monsoon driven so these glaciers could behave differently.

   Response: The absence of glacier selection for analysis and validation in the southern Himalaya is indeed due to data scarcity — no in-situ ice thickness observations are available for that region. Currently, only these 16 glaciers have publicly available ice thickness measurements in High Mountain Asia. We believe that with more in-situ data, the validation and performance evaluation of the model would be more comprehensive. We have therefore supplemented the Introduction section with the reasons for selecting these 16 glaciers:

"We applied ITMSL to 16 glaciers across HMA, comparing its performance with two other models (Gantayat and Millan) that are also based on the laminar flow equation. These 16 glaciers were selected because in-situ ice thickness observations are publicly available for them in HMA." **(P.5, L118-120)**

2. Table 1 : you can eventually add the RGI-ID of the glaciers for reproducibility.

Response: We have added the RGI-ID for each glacier in Table 1. **(P.7, L167)**

Table 1: Glaciers from RGI 6.0, survey years, and GPR data sources.

RGI-ID	Glacier (abbreviation)	Area (km^2)	u_s (m/yr)	GPR Survey Year	number of GPR points	GPR source
RGI60-14.00032	Barpu (BARP)	104.952	125.70	2015~2018	60	(Zou et al., 2021)
RGI60-14.02150	Batura (BATU)	311.419	101.34	2015~2018	23	
RGI60-14.20030	Chhungphar (CHHU)	15.136	108.40	2015~2018	17	
RGI60-14.04590	Gharko (GHAR)	30.318	81.06	2015~2018	29	
RGI60-14.03123	Pasu (PASU)	62.145	162.08	2015~2018	9	
RGI60-14.19344	Sachen (SACH)	10.307	40.90	2015~2018	55	
RGI60-13.47247	Haxilegen No.51 (HXLG)	1.099	10.70	2010	584	(Welty et al., 2020)
RGI60-13.48211	Heigou No.8 (HEIG)	6.074	24.97	2009	906	
RGI60-13.45335	Urumqi No.1 (URUM)	1.579	20.51	2014	1385	
RGI60-13.32330	Qiyi (QIYI)	2.530	8.65	1980	105	
RGI60-13.08055	Sary-Tor (SARY)	2.927	8.82	2013	1352	
RGI60-13.43165	Shenqi Peak (SHQI)	6.591	21.63	2008	823	
RGI60-13.31537	Shiyi (SHIY)	0.495	7.78	2010	294	
RGI60-13.45233	Sigonghe No.4 (SIGH)	2.641	7.01	2009	623	
RGI60-13.08624	Tsentralniy Tuyuksu (TSTU)	2.838	4.61	2013	8353	
RGI60-14.15990	Chhota Shigri (CTSG)	13.463	15.48	2009	146	(Azam et al., 2012a)

3. Table 1: It should be interesting to have the number of GPR measurements per glacier to better understand the values of the aggregated statistics in the following figures (under label Total in fig.6-10-11).

Response: We have added the number of GPR measurement points for each glacier in Table 1, as detailed in comment 2 above.

4. Line 137 :Write explicitly the assumption on the ratio u_b to u_s done in GV14 and GV22.

Response: We have supplemented this passage, and the revised version is as follows **(P.8, L185-188)**:

When using laminar flow equation to estimate glacier thickness, an assumption must be made regarding the ratio of basal sliding to surface velocity. Gantayat assumes this ratio to be 25% (Gantayat et al., 2014), whereas Millan infers that this ratio ranges from 0.1 to 0.9 based on the relationship between surface slope and ice velocity (Millan et al., 2022).

5. **5.** Where does equation 4 come from ? How do you compute the effective pressure N ?

   Response: Equation 4 is taken from a reference, in which the effective stress is calculated based on the ice body and the slope of the ice bed. We have added references in the manuscript to indicate their sources. Regarding the concept and calculation of effective pressure, we have provided supplementary explanations in the manuscript.

   (a) Source of Equation (4) (P.9, L199-202)

   We employ the Coulomb sliding law to characterize this complex interaction among water pressure, shear stress, and dynamics, which is given as follows (Helanow et al., 2021):

   $$\frac{\tau_b}{N} = C\left(\frac{u_b}{u_b + A_s C^n N^n}\right)^{1/n} \quad (4)$$

   [1] HELANOW C, IVERSON N R, WOODARD J B, et al. A slip law for hard-bedded glaciers derived from observed bed topography [J]. Science Advances, 2021, 7(20): eabe7798.

   (b) Calculation method of effective stress (P.9, L203-206)

   where N (Pa) is the effective pressure. Effective pressure is defined as the difference between ice overburden pressure and basal water pressure. Because subglacial hydrology is difficult to quantify accurately, it is approximated as equivalent to the ice overburden pressure in this study ($N = \rho g H cos\alpha$).

6. **6.** Line 158 : How do you compute lambda ?

   Response: λ is the bedrock obstacle wavelength, derived from the simulated ice bed using Fourier transform. We have provided an explanation in the main text, and the revised content is as follows **(P.9, L211-212)**:

   λ is the bedrock obstacle wavelength, which is calculated from the descent gradient of the subglacial topography using Fourier transform.

   For the detailed calculation steps, please refer to the source code.

7. **7.** Equations 7-11 : I suggest to remove indexes ‘GPR’ and ‘mod’ for readability. You could choose $\hat{H}$ and H for GPR measurement and inverted ice thickness respectively for instance.

   Response: Following your suggestion, we have revised the subscripts of Equations (7) to (11) in the main text to improve their readability **(P.10, L224-228)**.

   3) Model instructions:

1. Line 194 : What is the value of the shape factor f when computing the initial ice thickness ? Eventually adapt Figure 2 with initial value of f there too.

   Response: The shape factor used for calculating the initial ice thickness takes an empirical value of 0.8. We have supplemented the statements in the manuscript, and the revised main text and flow chart are as follows:

   (3) Initial ice thickness: The initial value of shape factor is set to 0.8, and the initial ice thickness (H_0) is calculated using the laminar flow equation (Equation 1). **(P.11, L249-250)**

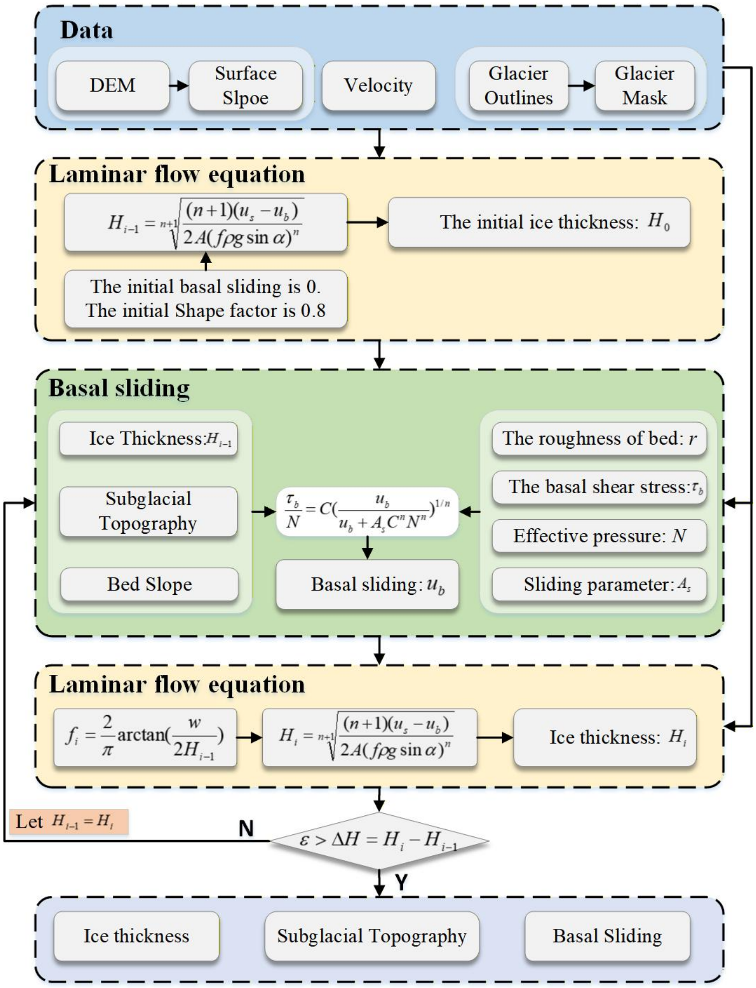


Figure 2: Flow chart of glacier thickness inversion using ITMSL model.

4) Model experiments:

1. You obtain optimal value for B thanks to calibration against measurement for individual glacier. At line 221, you write that “These calibrated parameters provide reference values for regional ice thickness inversions in unmeasured HMA glacier (Table 2). ” but you have 6 values of B for the 16 glaciers. A discussion on the choice of B for unmeasured glaciers seems missing. Fig.3 shows that the sensitivity of the model to parameter B is relatively small and

considering the “TOTAL” values (465) may be a good option maybe, especially knowing that most of the 16 glaciers have B =465 or 470 as optimal parameter.

Response: We indeed overlooked the discussion of glaciers in the Indian monsoon influenced regions of the Himalaya and southeastern Tibet, primarily because we could not obtain in-situ ice thickness observations in these areas. To address this issue, we have supplemented the manuscript with an explanation of how to select the B value for unmeasured glaciers. The revised content is as follows **(P.13, L297-301)**:

“These calibrated parameters provide reference values for regional ice thickness inversions in unmeasured HMA glacier (Table 2). It should be noted that the 16 glaciers investigated in this study are mainly distributed in the Tien Shan and Karakoram regions, with limited validation in the Indian monsoon-dominated Himalaya and southeastern Tibetan Plateau. Moreover, as shown in Figure 3, the RMSE for some glaciers decreases monotonically with increasing B values, whereas other glaciers display irregular fluctuations. This indicates that the B value range is not universally suitable for all glaciers, potentially due to differences in glacier characteristics and climatic conditions. Therefore, caution is required when applying B=465 $\mathrm{MPa}^{-3}\mathrm{yr}^{-1}$ to study glacier thickness and volume in these regions.”

2. Table 2 : How do you compute “TOTAL”?

   Response: By comparing the simulated ice thicknesses with the GPR observations for the 16 glaciers, the simulation result with the minimum RMSE is regarded as the overall optimum. We have supplemented the manuscript with the calculation procedure for the Total B value **(P.13, L290-293)**:

   The Total B value for the 16 glaciers is determined by pooling all GPR measurement points together, calculating RMSE of the simulated ice thickness for different B values, and then selecting the B value that minimizes the RMSE as the global optimal value.

3. To highlight how sensitive is the model to parameter B, displaying the RMSE as a percentage of the mean ice thickness of each glacier may be interesting. It seems that it never exceeds 2m which is very small with respect to glacier ice thickness.

   Response: Following the ITMIX framework, we have revised Figure 3 by adjusting the vertical axis to represent RMSE divided by ice thickness and multiplied by 100% (i.e., ice-thickness normalization). The revised version is as follows **(P.14, L305)**:

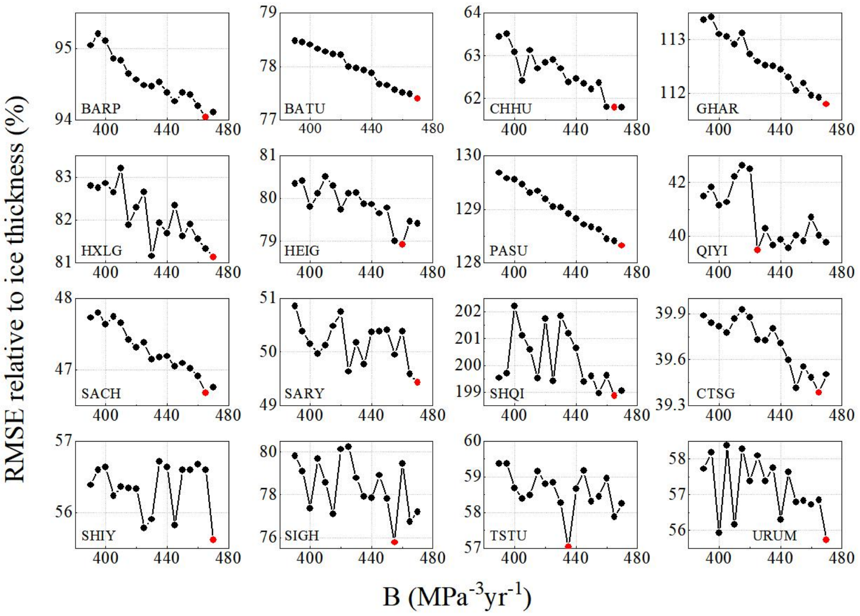


Figure 3: Ratio of the RMSE of ice thickness inversion to ice thickness for different values of parameter B in the ITMSL model.

4. Line 234-238 "Figure 4 illustrates … sliding datasets (Table 4)." can be removed, the figure are again introduced in the flow of the text later. It can be replaced by a short sentence motivating why you focus first on one glacier only (CTSG).

   Response: We have simplified this sentence, and the revised version is as follows:

   Among the 16 glaciers, CTSG is a widely studied glacier frequently used for ice thickness model validation (Frey et al., 2014; Azam et al., 2012b). Therefore, this study takes CTSG as an example to conduct a detailed analysis of ice thickness, subglacial topography, and basal sliding; the same procedure is applied to the remaining 15 glaciers.

5. Line 242-244 : "Quantitatively, ITMSL (114.63 m), GV22 (104.90 m), and GV14 (98.39 m) showed divergent mean estimates, with GV14 producing systematically lower values" from which table/figure do you conclude this statement in bold ?

   Response: This conclusion is drawn based on the ice thickness inverted by GV14 being lower than that from the other two models. We apologize for not expressing ourselves clearly, which may have caused you confusion. We have made corresponding corrections accordingly **(P.15, L319-321)**:

   In terms of mean ice thickness, ITMSL (114.63 m), Millan (104.90 m), and Gantayat (98.39 m) differ from one another, with Gantayat being the lowest.

6. Fig4 : Reproduce it for the 15 other glaciers in supplementary material. It could also be interesting to have a 2D map of the difference between GPR measurement and inverted ice thickness for the 3 models.

   Response: Thank you for your interest in our study. The thicknesses of the 16 glaciers and the two-dimensional distribution of the differences between simulated ice thickness and GPR measurements have been added to the appendix. Only partial results are shown here; the rest can be found in the appendix **(Supplement Figure S1-S4)**.

S1 Ice thickness results derived from model inversion

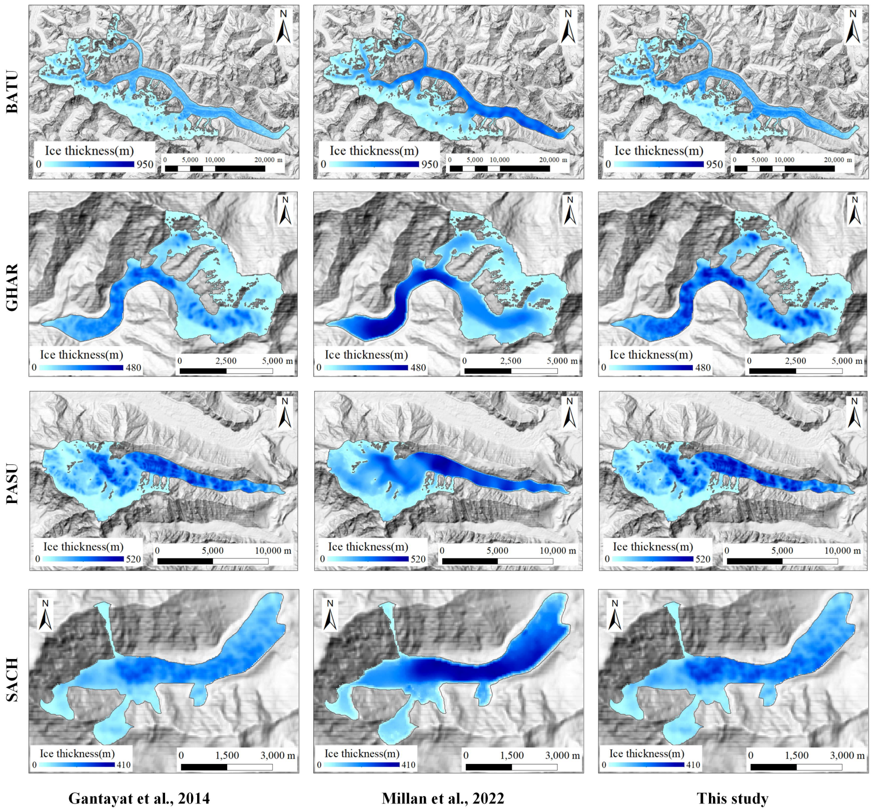


Figure S1: Glacier thickness inverted by the Gantayat, Millan, and ITMSL models.

S2 Difference between simulated ice thickness and GPR measurements

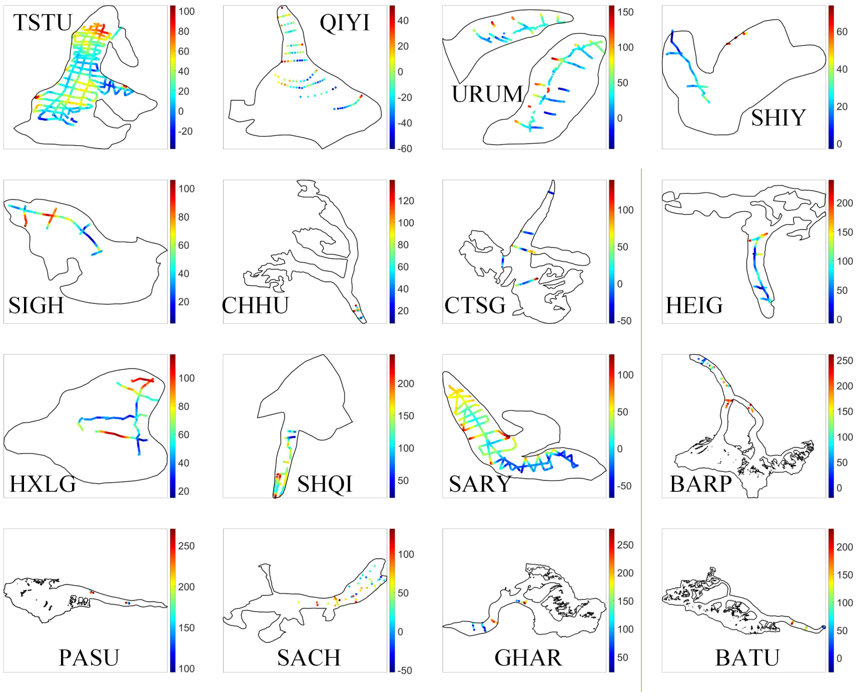


Figure S4: Glacier thickness inverted by ITMSL (unit: m).

7. Table 3 : Only RMSE is mentioned in the main text. Are the other information not useful for the analysis ? You have to mention the other values or to remove them to avoid losing the reader in a mass of information.

Response: Other indicators can help analyze the model results from different perspectives. However, for the sake of conciseness, only RMSE is explained in the main text. Since too much information can be confusing, we have presented RMSE as a figure and moved Table 3 to the appendix **(Supplement Table S1)**.

Table S1: Accuracy of ice thickness estimates for five cross-sections of the CTSG glacier.

	Root Mean Square Error		
	ITMSL	Gantayat et al., 2014	Millan et al., 2022
CS1	34.38	40.50	98.20
CS2	40.95	51.69	29.35
CS3	57.37	70.45	42.14
CS4	47.41	65.56	69.92
CS5	24.53	33.60	64.81
Total	45.25	58.06	57.77

8. This table is hard to read. I advise to make a figure from the data contained in the table with the table as supplementary material. One panel per statistic. CS_x as x-axis and values as y-axis with one color per model.

Response: Thanks. We have plotted the RMSE distribution for each profile to improve readability. And moved the accuracy metrics table for the CTSG glacier profiles to the appendix **(P.15, L333)**.

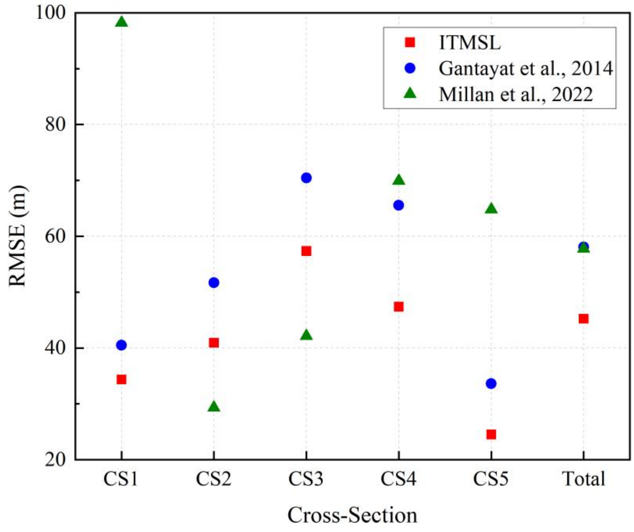


Figure 5: RMSE of CTSG glacier 5 cross-sections.

9. Fig5 : Caption : [… surface velocity] modeled by ITMSL.

Response: We have revised the figure caption as follows **(P.16, L342)**:

Figure 6: (a) CTSG glacier subglacial topography, (c) basal sliding, and (d) the ratio of sliding velocity to surface velocity inverted by ITMSL; (b) surface velocity used as input to ITMSL.

10. Line 271-272 : From equation 1 and 2, I would expect thicker ice with a smaller u_b/u_s ratio so the opposite of your diagnostic. Do you know more about it ?

Response: As you pointed out, we carefully examined the ratio of basal sliding velocity to surface flow velocity for the 16 glaciers and found that some glaciers do exhibit the opposite pattern you mentioned. Based on the analysis of the simulated ice thickness, we believe the reasons are as follows:

1) Limitations of the method: we found that pixels with large ratios are mainly distributed in the marginal areas of the glaciers.

2) The valley shape factor parameter is iteratively updated during the ice thickness simulation, which changes the ice thickness values, differing from Gantayat’s approach where a constant value is used.

3) The simulation of basal sliding lacks constraints and may deviate from true values; this needs to be improved in future work.

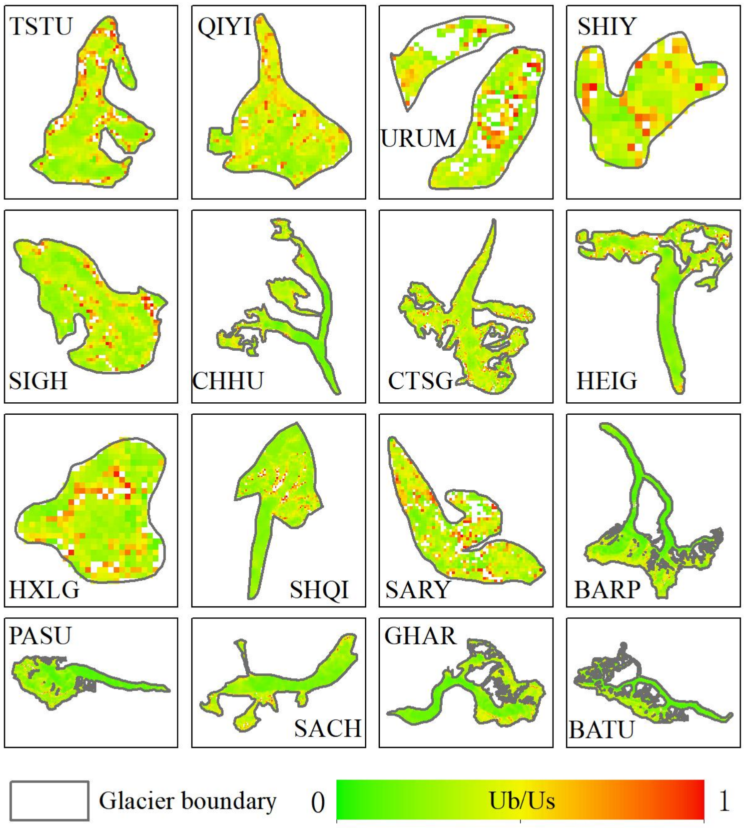


Figure S5: Ratio of basal sliding velocity to surface velocity simulated by the ITMSL model for glaciers.

11. Table 4 : Caption miss the information that the values in the table are averaged over each glacier.

Response: We have revised the title of Table 4, and the table number has been changed to Table 3 in the revised manuscript **(P.17, L352)**.

Table 3: The ice thickness and motion characteristics of the 16 glaciers are mean values calculated from inversion results extracted at the GPR point locations.

12. Line 284 : "likely due to complex subglacial topography" sounds like an empty argument. Is this established in the literature that these glaciers have a more complex subglacial topography and why is this a reason that ITMSL performs less good than GV22 ?

Response: This is an inaccuracy in our expression. The subglacial topography is unknown; it is currently obtained by subtracting the simulated ice thickness from the surface DEM. We implicitly assumed that the subglacial topography is complex. We have revised the relevant passage to make it more rigorous **(P.17, L361-364)**.

"Although 4 glaciers (SHIY, HXLG, URUM, HEIG) exhibit higher deviation in ice thickness inversion using ITMSL compared to Millan et al., 2022, and the changes in mean and relative errors are limited, the improvement in accuracy still confirms the reliability of ITMSL for ice thickness inversion over most glaciers."

13. Line 287: You assumed that overestimation of ice thickness by GV14 was due to u_b/u_s ratio. GV14 also used different velocity input and probably other DEM. Could this be another reason for the bias ?

Response: In this study, all ice thickness inversion methods used the glacier surface velocity data published by Millan et al. (2022). We subsequently evaluated the impact of different DEMs on the accuracy of ice thickness inversion and finally adopted the GLO-30 DEM uniformly. Therefore, the input data are consistent across all methods.

14. Line 288-289 : last sentence of this paragraph is not clear too me.

Response: We have revised this sentence to make its logic more coherent **(P.18, L377-378)**.

"Compared with other ice thickness models also based on the laminar flow equation, ITMSL achieves improved accuracy in glacier thickness inversion studies."

15. Fig6 : You displayed ME and RE in the plot but do not discuss it in the text while it gives interesting information about the negative bias of the three models.

Response: We have supplemented the description of the relevant accuracy metrics in the main text **(P.18, L364-374)**.

"Regarding RE, except for an outlier on the SHQI glacier, the RE values of the remaining 15 glaciers are relatively similar: -2.48-0.2 for Millan et al., 2022, -1.89-0.02 for ITMSL, and -1.85-0.02 for Gantayat et al., 2014. The overall mean RE values for these 16 glaciers are -1.81, -1.58, and -1.72, respectively. In terms of ME, the ranges of ME for the 16 glaciers inverted by the three models are -277.69-19.72 (Millan et al., 2022), -193.23-9.88 (ITMSL), and -135.42-7.26 (Gantayat et al., 2014), with overall mean ME values of -36.01 m, -38.87 m, and -42.27 m, respectively. Both ME and RE indicate that the inversion results from the three models are generally higher than the GPR-measured ice thickness, suggesting that the model input data and parameter calibration still have room for improvement. Correlation analysis between the model-inverted thickness and GPR-measured ice thickness yields overall correlation coefficients of 0.29 (Millan et al., 2022), 0.52 (ITMSL), and 0.42 (Gantayat et al., 2014).”

5) Discussion

1. Line 305 : locailzed --> localized

   Response: The issue has been corrected **(P.19, L394)**.

   “Although ITMSL demonstrated improvements, localized accuracy deficits at 4 glaciers (SHIY, HXLG, URUM, HEIG) necessitate targeted analysis.”

2. Line 316: You write :”particularly sub 1km2” to highlight common characteristics to the four glaciers badly handled by ITMSL while only 1 is smaller than 1km2.

   Response: A clear applicability boundary for the ITMSL model is difficult to quantify; we can only summarize it based on geometric or kinematic characteristics. We have revised the relevant statement to make it more logical **(P.20, L404-405)**:

   “While sharing comparable elevation, velocity, and slope characteristics with other glaciers, their small size exacerbates four key limitations.”

3. Line 320 : You write “Third, data gaps in remote sensing, derived velocity fields propagate through thickness calculations.” . This phrase should be rephrases to be understandable. And a reference to literature or a supplementary figure should be provided.

   Response: The input data are identical across all models, so the differences in results mainly reflect the varying sensitivity of the models to glacier types. We have revised this sentence to make it more reasonable **(P.20, L408-409)**.

   “Third, the reliability of velocity monitoring for small glaciers is low, and basal sliding is weak or absent, making effective simulation difficult.”

4. Fig7 : Caption need to be rewritten, describing all the variable correctly. What is DH ? You should also highlight the four glaciers that the analysis is interested in (opaque versus transparent color for instance). This analysis can be very interesting so it is important to have the plot the clearest possible.

Response: We apologize. DH refers to the glacier elevation range. For the sake of clarity, we have redrawn this figure to highlight the four glaciers of interest in the analysis, and we have also clarified the meaning of DH (Difference in Height). The revised figure and caption are as follows **(P.20, L415)**:

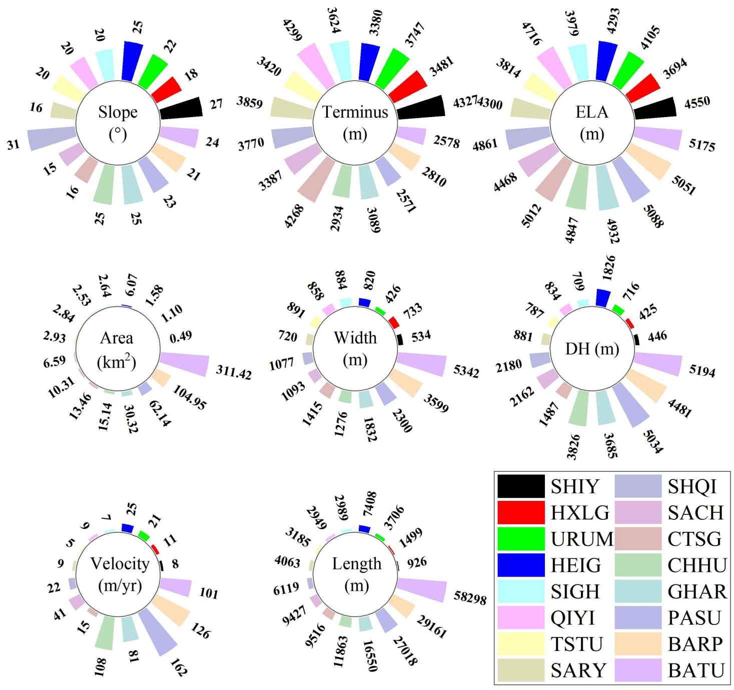


Figure 7: Characteristics of glacier surface slope, terminus elevation, equilibrium line altitude, area, mean width, difference in height (DH), ice velocity, length.

5. Line 329 : "Building upon established glaciological, this study ...", glaciological is an adjective, a noun is missing.

   Response: We have rewritten this sentence **(P.20, L419)**.

   "This study combines the basal sliding law with the laminar flow equation to develop the ITMSL model (Schoof, 2005; Gantayat et al., 2014; Zoet and Iverson, 2020; Millan et al., 2022)."

6. Line 332 : "reveals distinct performance regimes tied to u_b/u_s" ... ratio model led by ITMSL. To make it more clear that you made a binning of the ice thickness based on this ratio, computed on this given model, at the location of the GPR measurement.

   Response: We have reorganized and rewritten this sentence accordingly **(P.20-21, L420-425)**.

   "At each GPR measurement point, we computed the u_b/u_s ratio using the ITMSL model. Based on this ratio, we binned the points into two regimes: 0.1–0.4 and 0.4–1.0. The comparative analysis at these GPR points (Figure 9) reveals distinct performance regimes: for the 0.1–0.4 bin, the magnitude of ice thickness deviations follows Gantayat > ITMSL > Millan; for the 0.4–1.0 bin, ITMSL reduces the absolute deviation relative to both Gantayat and Millan.”

7. Fig9: A colorbar of the density plot is missing. Could be nice to add the name of each model as title of each panel.

   Response: We have added a color bar and included the name of each model in the subplots. The revised version is as follows **(P.21, L440)**:

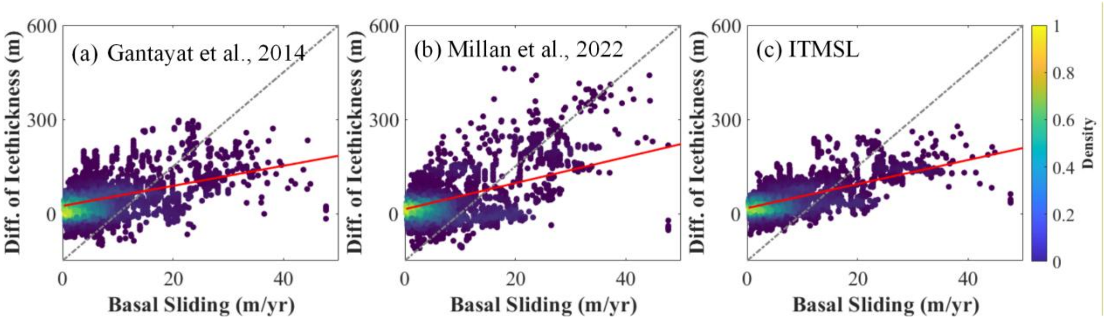
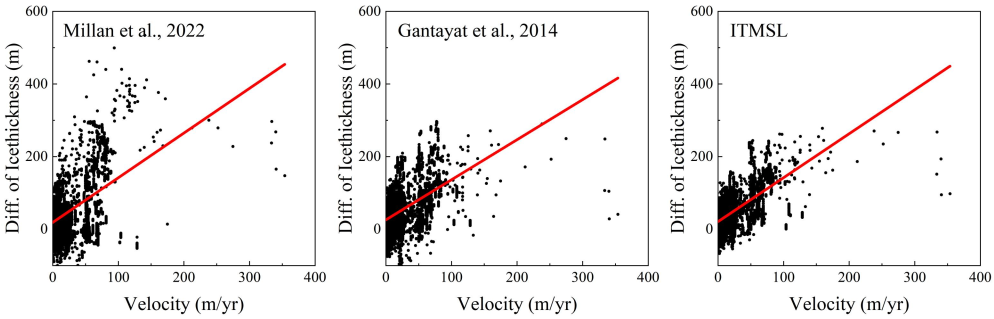


Figure 9: Deviations of ice thickness estimates from three laminar flow models at different basal sliding. (a) GV14; (b) GV22; (c) ITMSL.

8. Section 5.3: Nice to analyze the sensitivity to DEM model. Did you check how the model react to other surface velocity input ?

   Response: For the 16 glaciers investigated in this study, we compared the ice flow velocity and the deviation of ice thickness simulated by three models from the GPR measurements. According to the laminar flow equation, it can be inferred that ice thickness increases with increasing sliding velocity, which is consistent with physical expectations, and the results indeed confirm our inference. Therefore, no additional analysis has been added to the main text, and we kindly ask for your understanding.



Furthermore, another reviewer considered that the impact of different DEMs on the accuracy of ice thickness models is only weakly related to the construction and validation of the ice thickness model in this study, and suggested removing this section. Therefore, in the revised manuscript, we have deleted the original Section 5.3 concerning the influence of DEM on ice thickness.

9. Line 362: You write that Chen et al.(2022) preferred NASADEM for their inversion. What kind of inversion did they perform. This could be a valuable value to mention here.

   Response: We have supplemented the manuscript with a description of the work by Chen et al. (2022).

   Chen et al., (2022) using ICESat-2 laser altimetry data as a reference, systematically evaluated the vertical accuracy of six global digital elevation models (AW3D30, SRTM-GL1, NASADEM, TanDEM-X, SRTM v4.1, and MERIT) for glacier ice thickness inversion, and found that NASADEM performed the best.

   However, in the revised manuscript, Section 5.3 has been removed.

10. Fig10, 12 and 13: I'm note sure if this is a correct phrasing for a caption. Caption should describe what is in the figure, no interpretation or goal of the figure.

    Response: We have rephrased the figure title; however, section 5.3 (Figure 10) has been removed in the revised manuscript.

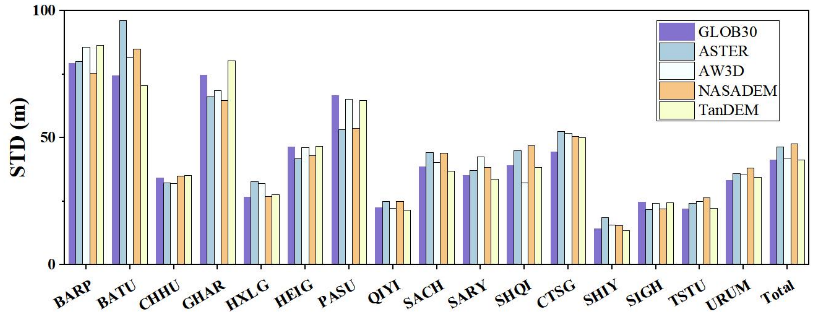


Figure 10: Accuracy of ice thickness inversion using GLOB30, ASTER, AW3D, NASADEM, and TanDEM-30 as input DEMs

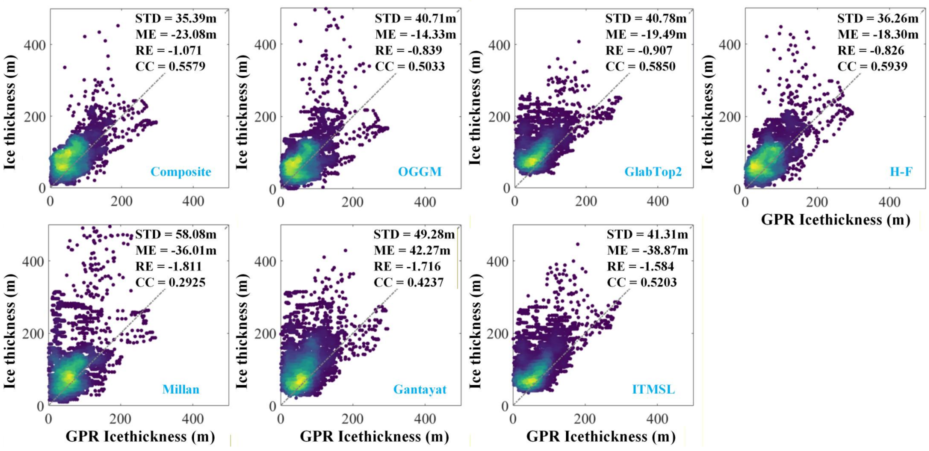


Figure 12: Evaluation of model-inverted ice thickness accuracy using GPR-measured ice thickness.

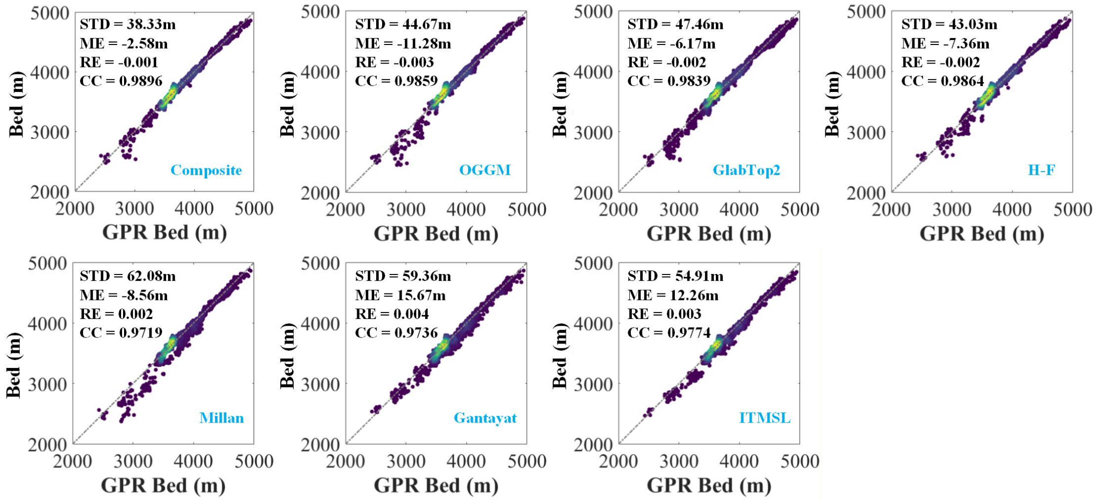


Figure 13: Evaluation of model-inverted subglacial topography accuracy using ice-bed measurements.

11. Line 391-392: Your sentence highlights that the best performing case when looking at ME is the composite while figure 12 shows that HF has the smallest ME. Ho do you compute the percentage given in this sentence?

Response: ME reflects the overall overestimation of ice thickness by the model relative to the measured values; the percentage mentioned here refers to the change in RMSE of other models relative to that of the Composite model. Our previous expression was not clear enough, and we have now rewritten the sentence **(P.24, L484-488)**.

"The ME quantification confirms a systematic positive bias. Farinotti et al. (2021) emphasized that no single model clearly outperforms all others, highlighting the advantage of multi-model ensembles over individual approaches. Compared to OGGM, the Composite model reduces thickness errors (RMSE) by 13.1%; by 13.2% relative to GlabTop2; by 2.4% relative to H-F; by 39.1% relative to Millan; by 28.2% relative to Gantayat; and by 14.3% relative to ITMSL."

12. Fig12 and 13: Name of the model as title of each panel and only "ice thickness" or "bed" as y-label. Thickbness --> thickness in the xlabel.

Response: We have revised the figure and corrected the errors **(P.24, L492)**.

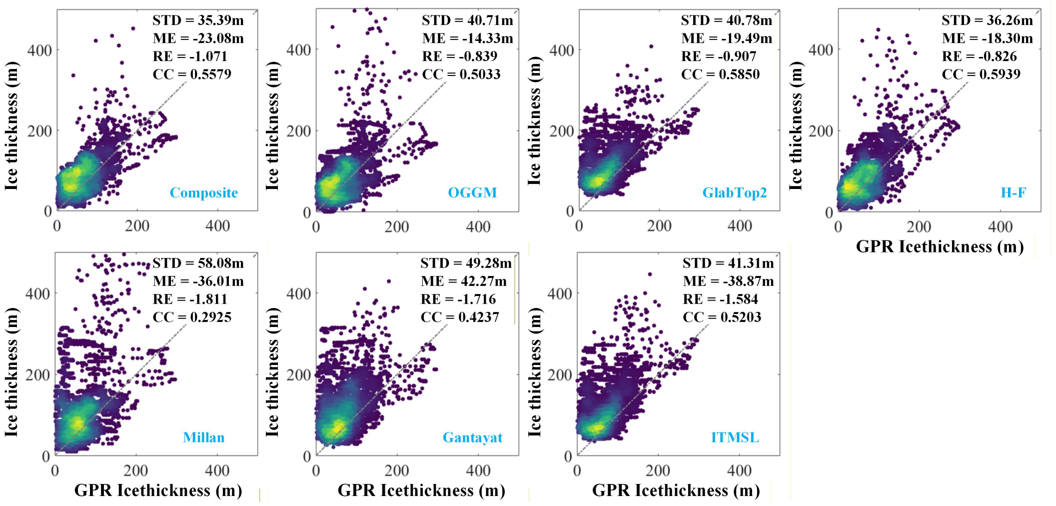


Figure 12: Evaluation of model-inverted ice thickness accuracy using GPR-measured ice thickness.

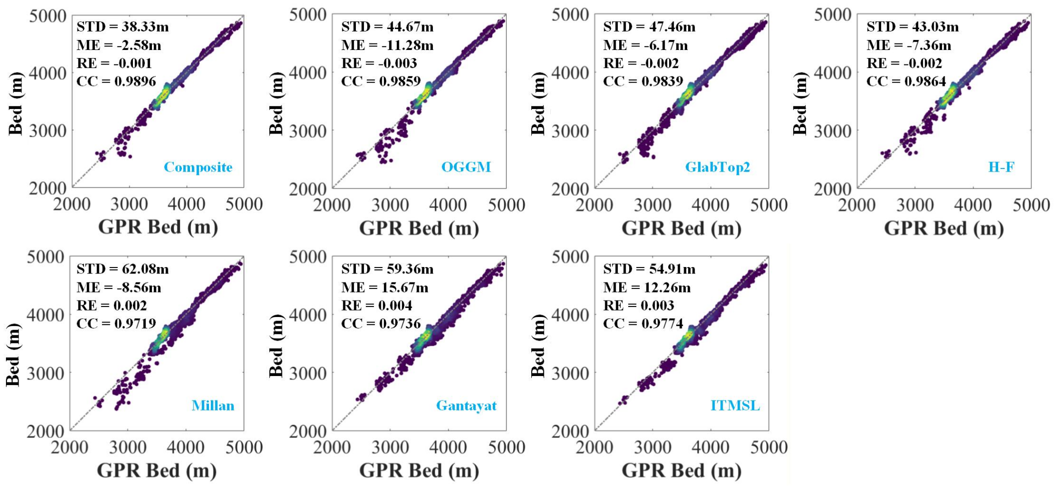


Figure 13: Evaluation of model-inverted subglacial topography accuracy using ice-bed measurements.

13. What is the difference between the GPR ice thickness measurement subtracted to the DEM and the GPR derived bed measurement ? This should be explained to understand the difference between fig 12 and 13.

Response: The comparison we made is between the subglacial topography derived from DEM minus the inverted ice thickness and the ice bed elevation measured by GPR. There is a difference in accuracy between the two: the former is an indirect result based on inverted ice thickness, while the latter is a direct measurement. It is precisely because of this difference in accuracy that we conducted the comparative analysis **(P.24, L495-497)**.

"There is a difference between the two: the former is an indirect result derived from inverted ice thickness, while the latter is a direct measurement."

6) Conclusion

1. **This** comment hold for the discussion or the conclusion : what are the advantage of ITMSL with respect to the other inversion model analyzed in the discussion ? They all perform slightly better than laminar flow estimation. Why should we keep using a laminar flow estimation ? Especially knowing that ITMSL require a calibration of parameters B, which is glacier sensitive. How do you reproduce it a large scale, with no thickness measurement ? And when compared to the other ITMIX model or newer method such as IGM ?

   Response: This study aims to improve the simulation accuracy of basal sliding within the laminar flow equation by explicitly incorporating a sliding law. In the Discussion, we compare the ice thicknesses inverted by ITMSL with those derived from other models used by Farinotti et al., (2019) to verify the reliability and effectiveness of our proposed method. Although the overall accuracy of ITMSL on the 16 glaciers is not significantly higher than that of H-F, OGGM, and GlabTop2, the ITMIX/ITMIX2 have shown that no single model performs best for all glaciers, and that weighted multi-model ensembles can integrate the strengths of individual models to yield more reliable results. Under this premise, improving the accuracy of a single ice thickness model based on a specific method is a fundamental condition for ensemble averaging. Therefore, the method proposed in this study still plays a positive role in research on glacier thickness and volume.

   (a) What are the advantage of ITMSL with respect to the other inversion model analyzed in the discussion ?

   Compared to other simplified models based on the laminar flow equation (e.g., GV14, GV22), the core advantage of ITMSL is the introduction of a physically-based basal sliding parameterization rather than empirical coefficients. This allows ITMSL to more realistically capture glacier dynamics, especially in fast-flowing regions where sliding dominates. In addition, ITMSL provides an interpretable physical mechanism that can separate the contributions of ice deformation and basal sliding, facilitating the understanding of dynamic differences among glaciers. Moreover, compared to GV14 and GV22—which are also based on the laminar flow equation—ITMSL reduces the standard deviation by 16.2% and 28.9%, respectively.

   (b) They all perform slightly better than laminar flow estimation. Why should we keep using a laminar flow estimation ?

   The laminar flow equation is a fundamental physical basis of glacier dynamics. It requires few input parameters (surface velocity, DEM, and glacier outline) and is easy to apply at large scales. Compared to full-physics models that require iterative solution of the ice flow equations (e.g., IGM), the laminar flow model has a low computational cost, making it suitable for rapid global-scale assessments. Farinotti et al. (2021) pointed out that even the ITMIX multi-model ensemble has considerable uncertainty. Therefore, ITMSL can serve as one member of such an ensemble, contributing a sliding-sensitive estimate under physical constraints.

(c) Especially knowing that ITMSL require a calibration of parameters B, which is glacier sensitive. How do you reproduce it a large scale, with no thickness measurement ?

This is a very practical issue. Based on the inverted B values from glaciers with available data, we can establish regression relationships between B and glacier characteristics such as area, slope, velocity, and surface temperature, and then recommend regional empirical B values for unmeasured glaciers.

(d) And when compared to the other ITMIX model or newer method such as IGM ?

Compared to full-ice-flow models like IGM (Jouvet, 2023), ITMSL has a lower computational cost and does not rely on an initial field that evolves over time. However, IGM is capable of simulating future glacier changes, which ITMSL cannot do. In summary, ITMSL is suitable for rapid static thickness estimation in data-scarce regions, whereas IGM is appropriate for dynamic simulations where sufficient meteorological forcing data are available.

References

Farinotti D. et al, (2017) How accurate are estimates of glacier ice thickness? Results from ITMIX, the Ice Thickness Models Intercomparison eXperiment. The Cryosphere, 11(2), 949–970 (doi:10.5194/tc-11-949-2017)

Farinotti D. et al, (2021) Results from the Ice Thickness Models Intercomparison eXperiment Phase 2 (ITMIX2). Frontiers in Earth Science, 8 (doi: 10.3389/feart.2020.571923

Jouvet G., (2023) Inversion of a Stokes glacier flow model emulated by deep learning. Journal of Glaciology, 69(273), 13–26 (doi: 10.1017/jog.2022.41)