Response to RC1

Comments from referee

The description of the NICAS method outlined in this manuscript is clear, well-written, and thorough. It provides an excellent outline of the method including the motivation behind developing the process, the flexibility and extra features (such as the non-isotropic extension, in-homogeneous sub-grid generation, and modifying interpolation methods), descriptions of the best-use cases for the method, and a study of the computational performance and scalability of the method. The examples and demonstrations showcasing the method, and associated figures, tie together very well.

I have a few very minor suggestions that may improve this manuscript. First, adding a figure in section 2.1 "Motivations" that shows a full-resolution grid side-by-side with an associated sub-grid generated from the full grid would help the reader follow the argument on computational affordability and prepare them for the illustration of the application of the method in Figures 8 & 9. Second, Section 3.5 "Steps illustration" should add a few-sentence written description of the NICAS 'setup' step with some more details what happens during the setup to help give more context for the results in Figures 10, 11, and 12. Finally, adding a figure around section 3.1 "Splitting horizontal and vertical directions" to illustrate the two-step vertical and horizontal sub-sampling would be a nice, though not-necessary, addition.

Author's response

Thank you for your positive and constructive feedback. As you suggested, I have added:

- a first figure in section 2.1 illustrating the full grid and subgrid parameters,
- a second figure in section 3.1 illustrating the subsampling in vertical and horizontal directions,
- a paragraph in section 3.5 to clarify the distinction between the setup step and the application step.

Author's changes in manuscript

- For the first figure, subsection 2.1 has been slightly rephrased to make it clearer and reference the new Figure 1.
- For the second figure, subsection 3.1 has been modified to introduce and explain the new Figure 7.
- Subsection 3.5 has been extended to provide a full description of the "setup" and "application" step of the NICAS method:

However, it is possible to split the NICAS usage into two successive steps:

- A *setup* step, where i) the subgrid is generated, ii) the interpolation from the subgrid to the full grid is defined, iii) the convolution on the subgrid is computed, and iv) the normalization of the whole operator is estimated. Most of the computational cost occurs during this phase, including the additional cost associated with advanced features such as inhomogenous, anisotropic, or boundary-aware correlation functions. The resulting operators are generally stored into dedicated files (NetCDF format), either one file per MPI task or in a common global file, for a future use. If a common global file is read in a subsequent execution, a data redistribution is needed, which comes with a extra communication cost.
- An *application* step, where the NICAS operator is applied on fields. This application includes computation and communication sub-steps for both the interpolation and the convolution applications, and the multiplication with the normalization field.

Response to RC2

Comments from referee

This interesting paper is clearly and carefully written and provides a good general description of the extremely versatile NICAS correlation and covariance operator algorithm. This will become an important general reference to the NICAS work. My only serious comment is to point out that Section 4, supposed to describe the performance of the method, seems to be missing, or at least to be very thin. I hope the author can fill out this Section 4 into a more substantial overview of the scheme's performance under different conditions when the manuscript is revised. Alternatively, if the author feels that this discussion is already essentially present in his Section 5.3, it might be better to remove the redundant section 4 altogether.

Has any thought been given to extending the scope of the scheme to handle cases where it might be desirable to incorporate nontrivial correlations between variables in an adaptive way? Although these kinds of correlations are typically reserved for the ensemble approaches to sort out, it would nevertheless be a valuable attribute of a parameterized correlation scheme, such as the NICAS, since it would presumably partially reduce the reliance of the data assimilation schemes on large and costly ensembles.

I look forward to the formal acceptance of the revised manuscript.

Author's response

Thank you for your positive and constructive feedback.

Regarding the very short section 4, it was a Latex typo: I defined subsection 4.1 as a "section" instead of a "subsection". As a consequence, section 5 should have been section 4.1, section 4.1 should have been 4.2, etc. In the revised manuscript, the whole section 4 is about performances, while section 5 is a discussion about the different similar methods and section 6 contains the conclusions. I am sorry for this mistake that I should have noticed earlier.

For the relations between variables, it is a very interesting question but I think it is outside of the scope of this article. However, it should be mentioned more clearly in the article, so I have added a paragraph in the introduction. It points to references for both correlation operators and localization operators.

Author's changes in manuscript

The subsections numbering issue in section 4 has been fixed, and a paragraph has been added in the Introduction regarding multivariate aspects:

> The scope of this article is limited to univariate aspects of correlation operators: the NICAS method does not handle the correlations between variables in multivariate cases. In most variational data assimilation systems, univariate and multivariate aspects are separated. For correlation operators, so called "balance operators" can introduce complex relationships between variables, as described for the atmosphere in Bannister (2008a, b) for large scales and Bannister (2021) for convective scales, or as in Weaver et al. (2005) for the ocean. For localization operators, the multivariate treatment is often simpler, as described in Ménétrier (2023) and Lee et al. (2024).

The new references have been added to the list.