We want to thank both reviewers for a thorough review and the valuable comments and suggestions on improving the manuscript's quality. As a result, notable improvements were made to the manuscript.

Below, we provide replies (blue colored text) to the specific points raised by the reviewers (black text).

Reviewer 1

Summary:

The Universal Temperature Index is a measure of thermal comfort or discomfort perceived by humans, and is estimated by a environmental model from the measured values of air temperature, radiation, humidity etc. The model is complex to run, and so polynomial approximations for a quick, albeit not totally accurate, estimations have been developed. The standard polynomial approximation incurs in errors that are deemed too large. The present study presents another approximation method, based on orthogonal polynomial regression that seems to provide more accurate results.

Recommendation: The manuscript is well written and the study seems to have not technical flaws. For that standpoint I have very few comments. However, I do have a more general question on the motivation of the study, which I think the authors should address or justify more thoroughly

We thank the reviewer for the careful reading of our manuscript and for the positive assessment of its clarity and technical soundness. We also appreciate the insightful comment regarding the broader motivation of the study, which we agree is important to clarify and strengthen.

Main point

1) The manuscript mentions another alternative method, namely interpolation from an available look-up table that contains about 100 thousand values. This is also the approach recommended by Bröde (2021a). The manuscript argues that the storage of 100 thousand values makes the calculation cumbersome, but I clearly disagree. This storage would amount to roughly 1 MB of data, which is a very small space. Intuitively, I would argue that an interpolation of that table can produce very accurate values with a simple spline or linear algorithm. So the question arises as what would be the advantages of the algorithm presented in this manuscript relative to the look-up table interpolation.

We thank the reviewer for this important and thoughtful comment. We agree that the memory requirements of the look-up table are indeed modest, and we acknowledge that storage alone is not a limiting factor in modern computational environments. However, the polynomial approximation offers several practical advantages over the look-up-table-based interpolation approach:

(i) **Licensing and reproducibility**. The precomputed UTCI look-up table is not freely reusable or redistributable in all contexts. Namely, the table is published as Electronic Supplementary Material to the Bröde et al. (2012) paper, for which the copyright owner is ISB and the license does not allow free reuse or redistribution of this data. This creates limitations on directly incorporating such data into open-source software packages or operational tools. In contrast, polynomial approximation is fully self-contained and can be implemented and redistributed without restrictions, which is particularly important for reproducibility and broad adoption.

(ii) **Computational efficiency**. While interpolation methods can be accurate, they are computationally more demanding. As noted in Bröde et al. (2012), the polynomial approximation is on the order of three orders of magnitude faster than the look-up-table-based interpolation. This difference becomes especially relevant in large-scale applications, such as numerical weather prediction or climate reanalysis, where the index must be evaluated millions of times.

(iii) **Simplicity and portability of implementation**. A polynomial approximation requires only basic arithmetic operations and can be easily translated into virtually any programming language (e.g., Fortran, C/C++, embedded systems). In contrast, implementing multi-dimensional interpolation requires additional logic for data handling, indexing, and neighborhood search, which increases code complexity and reduces portability—particularly in constrained or legacy environments.

(iv) **Interpolation is still required**. Even with the lookup table, values are not available for all possible combinations of input variables. In particular, there are gaps at intermediate values (e.g., humidity), so interpolation (and, in some cases, extrapolation) is unavoidable. Therefore, the lookup-table approach does not eliminate approximation error; rather, it shifts it to the interpolation scheme. In this sense, both approaches rely on approximation, but the polynomial form provides a direct, continuous, and analytically defined mapping.

(v) **Generalization**. An additional advantage of the proposed approach is that its behavior on unseen data can be assessed directly. In our case, the model was trained on only 20% of the dataset and tested on the remaining 80%, while maintaining very good predictive performance. This provides strong evidence of generalization. Such validation is less straightforward for local interpolation schemes, whose reliability depends more directly on the local density and coverage of tabulated points, especially in regions where extrapolation is necessary.

For these reasons, we view the polynomial approximation not as a replacement for the lookup table in all contexts, but as a complementary approach that is particularly advantageous in settings that require speed, portability, and ease of deployment. We have clarified this positioning and expanded the discussion of these trade-offs in the revised manuscript; see Section 1, Introduction, pages 2-3, lines 70-101.

I am not arguing that the study is not valuable, as it presents a possible way of producing more accurate estimation of the index, but the reader would ask themselves if it really worth the effort. Bröde (2021a) argues that " This chapter provides hints and guidelines on how to handle these issues, and especially encourages the application of the hardly used look-up table approach, which will help avoiding many, if not all concerns related to UTCI calculation via the regression polynomial"

We agree that the lookup-table approach is valuable and, in some settings, may be preferred. Our main point is that the proposed approximation is not a complex or highly technical alternative, but rather a simple and general improvement of the polynomial approach already widely used in practice. With only minimal code changes, it yields substantially better accuracy and results close to the theoretical optimum in the least-squares sense.

We also believe it is worthwhile to highlight and popularize this approach because it is not specific only to the UTCI dataset, but is a general strategy for building accurate, interpretable, and computationally efficient approximations.

Finally, we interpret the statement in Bröde (2021a) as a useful recommendation under favorable conditions, but also as somewhat optimistic and prescriptive, since it does not cover all practical situations. In many applications, issues such as licensing, portability, ease of implementation, and computational speed still make a polynomial approximation an attractive option.

We have revised the manuscript to clarify this motivation more explicitly; please see Section 1, Introduction, pages 2-3, lines 70-101 and the Conclusion, page 15, next-to-last paragraph in the paper, lines 462-474.

Minor points

2) The labels in Figure 1 are too small. This also the case to a lesser degree in other Figures. Figure 3 is ok, so I would recommend to homogenize the font size in all figures.

We have increased the font size and made it uniform across all the figures.

3) In table 2, the reader has to infer which is the train loss and the test loss. It seems that the train loss is the upper number, but this could be indicated more explicitly. It seems that the train loss numbers require to be wrapped by a []

We have highlighted the train/test distinction in Table 2 by labeling the train and test entries clearly.

Reviewer 2

The article presents a new approximation for the Universal Thermal Climate Index by applying sparse regression with orthogonal polynomials to the Fiala thermo-physiological model. The proposed approach improves predictive accuracy and numerical stability over the existing standards (particularly in extrapolation), while maintaining comparable computational efficiency.

The manuscript is very well written and well illustrated .

We thank the reviewer for the positive evaluation of our work and for the clear summary of its contributions. We appreciate the recognition of the improvements in predictive accuracy, numerical stability, and computational efficiency. We are also grateful for the kind remarks regarding the clarity of the presentation and the quality of the illustrations.

I understand that the techniques (sparse model discovery) used for the regression are standard and no novelty is presented in that front. But I would add at least the kind of equations that these methods are aiming at. This will help interpreting the results (e.g. Table 1 or Figure 3, Why for a given polynomial degree the number of parameters change?).

We thank the reviewer for this helpful suggestion. We agree that the manuscript should describe the fitted functional form more explicitly to make the results easier to interpret.

While sparse regression itself is a well-established technique, the use of orthogonal polynomial bases in this context is much less common and, to the best of our knowledge, has been applied only in a more limited and less systematic way. Our contribution is therefore not methodological novelty in sparse regression itself, but rather the use of sparse orthogonal regression as a simple and unified framework for constructing accurate polynomial approximations with good numerical properties. In particular, the orthogonal basis makes the approximation order-by-order consistent, so that low-order truncations provide the best approximation for a given small model complexity in the least-squares sense; please see the Conclusion, page 15, next-to-last paragraph in the paper, lines 462-474.

Is the proposed function a linear combination of Legendre polynomials? Are Ta, va, Tr−Ta and rH its input variables? Please present the shape of the polynomial basis expansions that you are fitting.

Yes. The proposed approximation is a linear combination of Legendre polynomial basis functions. The input variables are the four environmental variables Ta, va, Tr − Ta, and rH, each normalized to the interval [-1, 1].

More specifically, the approximation is built as a sum of products of Legendre polynomials in these four normalized input variables, up to a chosen maximum polynomial degree. Sparse regression is then used to select only a subset of all candidate terms, so the final model retains only the coefficients chosen by the Lasso regularization.

In this sense, the model can be viewed as a Fourier-like decomposition in an orthogonal polynomial basis. The number of parameters changes because, for each maximum degree, the pool of candidate basis terms changes, and the Lasso penalty selects a sparse subset from that pool. Higher polynomial degree provides a larger set of possible terms, while the regularization strength determines how many of them are retained in the final approximation.

Our main goal here is not to assign a direct physical interpretation to each retained term, but to obtain an approximation that is more accurate, numerically stable, and still easy to implement. We have included in the text the general form of the fitted approximation and discussed how the polynomial degree and regularization impact the fit; see Section 3, Results and discussion, pages 8-9, lines 253-282.

Minor comments

- In the introduction, when it is explained that the water vapor is not included, I would add that the relative humidity is included to account for its effect.

We agree and have clarified that relative humidity is included to account for the effect of water vapor; see Table 1, caption.

- "Training is conducted on only 20% of the available data, while performance is assessed on the remaining 80%"

Are these sets taken randomly?

Is this needed for a better fitting?

Is the number of points a limitation of the regression method?

We thank the reviewer for these questions.

Yes, the training and test sets are selected randomly. We also verified that the results are robust under different random splits through bootstrapping.

Using only 20% of the data for training is not required for better fitting. On the contrary, it makes the task more challenging. The motivation is to demonstrate the generalization capability of the method. Despite being trained on a relatively small subset, the model achieves similar performance on the remaining 80% of the data, which indicates strong generalization.

This setting is more demanding than typical machine learning practice, where a larger fraction of the data is used for training. Our results show that increasing the amount of evaluated data by a factor of five does not degrade performance, suggesting that the method remains reliable and would likely achieve an equally good approximation on unseen data.

Finally, the number of data points is not a limiting factor of the regression method. If anything, the fact that accurate approximations are obtained from a relatively small training set highlights the robustness and efficiency of the approach; see Conclusion, page 15, next-to-last paragraph in the paper, lines 462-474.