

# GSV-SRTS: A Heterogeneous Landscape Soil-Canopy Reflectance Model Over Sloping Terrain with an Extended GSV and Stochastic Radiative Transfer Theory

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**Abstract.** Accurately modelling radiation interactions within canopy layers and soil backgrounds is crucial for biophysical variables retrieval across regional or global scales. However, terrain relief introduces significant uncertainties in modeling CR by modulating solar and viewing geometry and exacerbating landscape heterogeneity. Conventional CR models, designed primarily for flat and homogeneous landscapes, often inadequately represent these complex interactions. In this study, we present a canopy reflectance model suitable for heterogeneous structures on sloping terrain. By extending the stochastic radiative transfer theory from flat terrain to sloping terrain and integrating the soil General Spectral Vector, the GSV-SRTS model was introduced. This enables accurate prediction of soil-canopy radiative transfer within subpixel 3-D heterogeneous mountain landscapes. The proposed GSV-SRTS model was evaluated against the Discrete Anisotropic Radiative Transfer (DART) model, compared with typical mountain canopy reflectance models, and validated against remote sensing observations at varying spatial resolutions. The results showed that the GSV-SRTS model achieves good accuracy in the comparisons with DART ( $R^2 = 0.9136$  (0.9052) and root-mean-square errors (RMSE) = 0.0246 (0.0216) in the red (NIR (Near-Infrared)) band) and performs well in real mountainous areas, particularly with high spatial resolution remote sensing observations ( $R^2 = 0.9078$  (0.9143) and RMSE = 0.0201 (0.0212)). Furthermore, the GSV-SRTS model effectively captures the impacts of canopy structure and terrain factors on bidirectional reflectance. This underscores the GSV-SRTS model as a reliable physical tool for simulating radiation regimes over sloping terrain, with the potential to enhance the accuracy of biophysical variable retrieval from remote sensing observations.

## 31 **1 Introduction**

32 Canopy reflectance (CR) models simulate radiative transfer within vegetation-soil systems to link  
33 biophysical parameters with remote sensing data (Widlowski et al., 2013). Accurate simulation of these  
34 processes is fundamental for interpreting remote sensing observations and retrieving biophysical  
35 variables across scales (Verrelst et al., 2015). However, to manage the inherent heterogeneity of reality,  
36 CR models necessarily employ abstractions and simplifications (Yin et al., 2017), which consequently  
37 limits their ability to characterize fine-scale landscape details.

38 In addition, topography presents challenges in CR modeling, as terrain relief modifies illumination  
39 and viewing geometry, leading to variations in reflected radiance through surface anisotropic reflectance  
40 (S. Sandmeier, 1997), and significantly affecting the gap probability of canopy layers, which alters  
41 radiation contributions from both the canopy and soil background in solar and viewing directions (Wen  
42 et al., 2018). Therefore, the heterogeneous landscapes and sloping terrain are the primary sources of  
43 uncertainty in CR modeling (Yin et al., 2017).

44 In recent decades, a variety of models for canopy reflectance on sloping terrain have been developed.  
45 Schaaf et al. (1994) introduced the geometric-optical (GO) and mutual shadowing models to sloping  
46 terrain (GOMST) by applying the coordinate rotation (Schaaf et al., 1994). Combal et al. (2000)  
47 expanded the Ross model (Ross, 2012) to account for sloping terrain while maintaining geotropic  
48 vegetation growth (Bruno Combal, 2000). Verhoef and Bach (2007, 2012) developed a framework that  
49 integrated radiative transfer (RT) processes within within a soil-leaf-canopy (SLC) medium and later  
50 extended this model to sloping terrain (SLCT) (Verhoef and Bach, 2007, 2012). However, these CR  
51 models overlooked the impact of the heterogeneity of canopy structure and soil properties and assumes  
52 that they are uniformly distributed in pixels.

53 The characterization of canopy heterogeneity structure is a crucial aspect in remote sensing studies.  
54 The geometric-optical model over sloping terrain (GOST) has been utilized to represent the realistic 3-  
55 D structure of canopy (Fan et al., 2014). However, the computational complexity of GOST has limited  
56 its performance in parameter retrieval. The canopy reflectance model suitable for both continuous and  
57 discontinuous canopies over sloping terrain (BOST) was developed (Hu and Li, 2022). However, the  
58 internal canopy structure was based on the assumption of turbid media which inherited from the SLC  
59 model. To overcome these limitations, the Stochastic Radiative Transfer (SRT) theory was drawn into

60 canopy RT modeling, which simulates the random scattering and absorption of photons as they interact  
61 with various canopy elements, allowing for a more realistic representation of the three-dimensional (3D)  
62 structure of vegetation (Shabanov et al., 2000). This makes SRT a suitable approach for complex and  
63 heterogeneous scenarios (Shabanov et al., 2007). Zeng et al. developed a RT model specifically designed  
64 for patchy landscapes based on SRT theory (Zeng et al., 2020), which considered landscape heterogeneity  
65 by integrating stochastic processes to simulate radiation interactions within fragmented vegetation  
66 canopies, addressing variability in canopy structure and spatial distribution. Yan et al. applied the SRT  
67 model to heterogeneous discrete canopies (Yan et al., 2021), achieving an optimal balance between  
68 accuracy and efficiency in modeling radiation regimes of discontinuous canopies. Li et al. (2020)  
69 expanded SRT to simulate BRF in forests with within-crown heterogeneity, parameterizing vertical and  
70 horizontal foliage distributions to enable simulations of complex canopy structures while maintaining  
71 theoretical consistency with 3D radiative transfer (Li et al., 2020). While SRT models have proven  
72 effective in characterizing RT within heterogeneous canopies, their performance in mountainous regions  
73 is limited by the neglect of terrain effects. Extending the SRT theory to sloping terrain could enhance the  
74 characterization of RT within mountain vegetation, providing a more comprehensive understanding of  
75 canopy heterogeneity in such challenging environments.

76 In terms of quantitative impacts on CR modelling, soil reflectance is a key factor in the accurate  
77 modeling of canopy reflectance, especially in areas with sparse vegetation. Under canopies with a low  
78 leaf area index ( $LAI < 2$ ), soil reflectance can contribute significantly to the total canopy reflectance,  
79 particularly in the near-infrared (NIR) region (Jacquemoud et al., 2009). Neglecting soil reflectance in  
80 canopy reflectance (CR) models can lead to errors of 10–30% in reflectance estimates (Baret et al., 1993).  
81 Research has shown that incorporating precise soil reflectance data into radiative transfer (RT) models  
82 can improve simulation accuracy by 5–15% (Jacquemoud et al., 2009). To better capture the  
83 heterogeneity of soil properties across landscapes, empirical methods are commonly used to model soil  
84 spectral reflectance. Jiang and Fang (2019) developed the General Spectral Vector (GSV) model, which  
85 combines site-specific observational data with soil databases to accurately simulate soil reflectance  
86 spectra (Jiang and Fang, 2019). The GSV model has been found to outperform traditional wet chemical  
87 methods in soil monitoring (Nocita et al., 2015) and has demonstrated effectiveness in predicting soil  
88 properties even in vegetated areas (Pinheiro et al., 2017). -

89 To address the terrain effects and consider both the heterogeneity of canopy structure and soil  
90 properties, a new soil-canopy reflectance model known as GSV-SRTS was developed in this study  
91 specifically for sloping heterogeneous landscapes by expanding on the GSV and SRT theory. The canopy  
92 gap probability modulated by terrain factors, geotropic growth of vegetation, and terrain-induced  
93 variations in illumination and viewing geometry were taken into account. GSV-SRTS can offer a realistic  
94 representation of radiation propagation within soil-canopy objects over sloping terrains, serving as a  
95 theoretical tool for forward canopy reflectance modeling over sloping terrains with heterogeneous  
96 vegetation structures. This model has the potential to be used to accurately retrieve biophysical  
97 parameters related to vegetation. To further explore how terrain and vegetation characteristics impact  
98 canopy and soil reflectance, the GSV-SRTS model was used to analyse the effects of different terrain  
99 factors and canopy gap probabilities on model performance.

100 In this study, ‘discontinuous canopies’ refer to sub-pixel to pixel-scale discrete tree patches; ‘patchy  
101 landscapes’ denote 10–100 m scale mixtures of vegetation and non-vegetation covers; ‘heterogeneous  
102 landscapes’ collectively describe mountainous terrain with varying canopy structure, soil properties, and  
103 topography.

104 The following sections will provide a detailed overview of the article. In Section 2, we will delve  
105 into the development of the GSV-SRTS model. Section 3 will outline the evaluation strategy, including  
106 the DART configuration and validation using actual remote sensing images. Moving on to Section 4, we  
107 will present the results of the validation and performance assessment of the model. Sections 5 and 6 will  
108 then offer a discussion of the findings and conclude with final remarks.

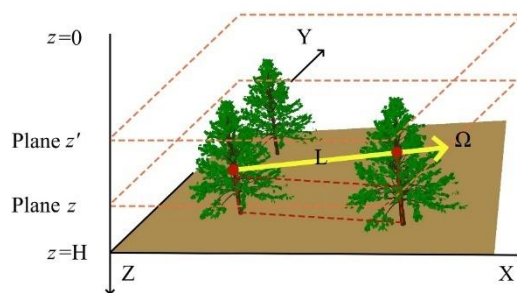
## 109 **2 Methods**

110 The accurate simulation of canopy reflectance (CR) over sloping terrain requires a physically consistent  
111 coupling of the soil background contribution with the radiative transfer within the vegetation canopy.  
112 The soil background is not merely a static lower boundary but a spectrally variable source of reflected  
113 radiation that interacts multiple times with the canopy (e.g., through ground-canopy-ground scattering).  
114 In heterogeneous mountainous landscapes, both the soil spectral properties and the canopy architecture  
115 vary spatially. Therefore, an integrated framework is necessary. In this study, we couple a Generalized  
116 Spectral Vector (GSV) model for the soil background with a Stochastic Radiative Transfer (SRT) model

117 for the canopy. The GSV model efficiently represents the high-dimensional spectral variability of soils,  
 118 providing the lower boundary condition. The SRTS model, operating within a local slope coordinate  
 119 system, simulates the absorption and scattering of radiation within a 3D heterogeneous canopy. The  
 120 coupling occurs physically at the soil-canopy interface: the anisotropic reflectance field from the GSV  
 121 soil model serves as the upwelling boundary condition for the SRT equation, while the SRT-computed  
 122 transmission and multiple scattering determine the irradiance incident upon the soil. This bidirectional  
 123 coupling ensures that the combined GSV-SRTS model captures the integrated effect of variable soils and  
 124 complex canopies under terrain-modified illumination.

## 125 2.1 General Framework of the GSV-SRTS Model

126 The GSV-SRTS model is a sophisticated theoretical framework designed to address the intricate nature  
 127 of radiation transfer processes within the layers of a canopy. By expanding upon the traditional radiation  
 128 transfer equation to include the soil GSV and integrating topographic elements, this model offers a more  
 129 precise depiction of the inherent variations present in canopy-soil systems. One key feature of the GSV-  
 130 SRTS model is its utilization of a local slope coordinate system for all internal calculations, ensuring a  
 131 geometrically accurate representation of how light interacts within a rugged mountainous terrain. To  
 132 illustrate the application of the GSV-SRTS model, this study presents a 3D discontinuous canopy made  
 133 up of identical trees situated within the layer defined as  $0 < z < H$  in the coordinate system (Fig. 1) The  
 134 number of trees housed within a pixel is determined randomly using the Poisson distribution (Huang et  
 135 al., 2008). From a stochastic view, the 3D canopy structure is conceptualized as a spatial stochastic  
 136 process, symbolized by an indicator function denoted as  $\chi(x, y, z)$ . This function yields a value of 1 if the  
 137 specific point  $(x, y, z)$  falls under vegetation cover, and 0 if it does not.



138  
 139 **Figure 1: A typical discontinuous vegetation canopy and the coordinate system used for the stochastic RT equation. The**  
 140 **planes  $z = 0$  and  $z = H$  represent the top and bottom boundaries of the vegetation canopy, respectively. A specific point**  
 141 **within the vegetation canopy, located on the plane  $z$ , moves upward along the unit vector  $\Omega$ , covering a distance  $L$  until it**  
 142 **reaches the plane  $z'$ .**

143 **2.2 Stochastic radiative transfer equation in sloping terrains**

144 The adoption of a local slope coordinate system ( $x', y', z'$ ), aligned with the inclined ground surface, is  
 145 fundamentally necessary to render the radiative transfer problem tractable over slopes. In the  
 146 conventional horizontal coordinate system, a sloping ground surface presents a moving, tilted boundary  
 147 condition that vastly complicates the formulation of the extinction coefficient and the boundary  
 148 conditions for the radiative transfer equation (RTE). By transforming to a coordinate system where the  
 149  $z'$ -axis is normal to the local slope, the ground surface is redefined as a flat plane ( $z' = 0$ ). This crucial  
 150 simplification allows the use of a standard volumetric formulation for canopy extinction (where the  
 151 extinction coefficient is defined relative to the direction of propagation) and enables the application of  
 152 the stochastic approach for gap probability calculation on this effectively ‘level’ domain. Consequently,  
 153 all canopy structural statistics and radiative transfer calculations are performed relative to this local frame,  
 154 and the effects of terrain are encapsulated in the modified solar and viewing direction vectors within this  
 155 frame, as defined in Equations (1)-(3). For any given global direction  $\Omega$ , its corresponding local sun  
 156 zenith angle  $\cos\theta_s$  and local view zenith angle  $\cos\theta_v$  are calculated as (Gu and Gillespie, 1998):

$$157 \quad \cos\theta_{s(v)} = \cos\theta_g \cos\theta'_{s(v)} + \sin\theta_g \sin\theta'_{s(v)} \cos\varphi_{sg(vg)} \quad (1)$$

158 where  $\theta_g$  and  $\varphi_g$  represent the slope and aspect, respectively.  $\varphi_{sg}$  is the relative azimuth angle  
 159 between the sun and the sloping background; and  $\varphi_{vg}$  is the relative azimuth angle between the viewer  
 160 and the sloping background. All subsequent RT computations inside the canopy are performed using  
 161 these local angles, ensuring that all physical interactions are relative to the actual inclined surface.  
 162

163 The Stochastic Radiative Transfer Equations (SRTE) are obtained through horizontal averaging of  
 164 the 3D RT process, thereby integrating the influence of canopy structure. Solving these equations (Eqs.  
 165 (2)-(3)) necessitates determining the mean radiation intensities for both the entire scene,  $\bar{I}(z, \Omega)$ , and the  
 166 vegetation-covered area within a pixel,  $U(z, \Omega)$ , at depth  $z$  in direction  $\Omega$ , which are defined as (Vainikko,  
 167 1973; Shabanov et al., 2000):

$$168 \quad \mu \frac{\partial \bar{I}(z, \Omega)}{\partial z} = -\sigma'(\Omega) \bar{I}(z, \Omega) + \frac{\sigma_s}{4\pi} \int_{4\pi} \bar{I}(z, \Omega') \sigma'_s(\Omega', \Omega) d\Omega' + F_0(z, \Omega) +$$

$$169 \quad F_H(z, \Omega) \quad (2)$$

$$171 \quad \mu \frac{\partial U(z, \Omega)}{\partial z} = -\sigma'(\Omega) U(z, \Omega) + \frac{\sigma_s}{4\pi} \int_{4\pi} U(z, \Omega') \sigma'_s(\Omega', \Omega) d\Omega' +$$

$$172 \quad Q_0(z, \Omega) + Q_H(z, \Omega) \quad (3)$$

174 where  $\mu = \cos\theta_v$ , and  $\theta$  is the local view zenith angle.  $\Omega$  indicates the direction of incoming direct light,  
 175 and  $\Omega'$  indicates the viewing direction.  $F_0(z, \Omega)$  and  $F_H(z, \Omega)$  in Eq.(2) represent the diffuse and direct  
 176 radiation entering through the upper and lower boundaries.  $Q_0(z, \Omega)$  and  $Q_H(z, \Omega)$  in Eq. (3) refer to  
 177 the mean diffuse and direct radiation that penetrate the upper and lower canopy boundaries and  
 178 subsequently experience scattering in the heterogeneous medium.  $\sigma'(\Omega)$  is the extinction coefficient. The  
 179 extinction coefficient is geometrically corrected to account for local slope-induced projection effects.  
 180 This correction reduces effective extinction when illumination or viewing directions are highly oblique  
 181 relative to the local normal, ensuring the local radiative balance remains consistent with the actual slope  
 182 geometry, which is shown as follows (Mousivand et al., 2015):

$$183 \quad \sigma'(\Omega) = \sigma(\Omega) \sqrt{\max(0, \mu'_s) \max(0, \mu'_v)}$$

$$184 \quad (4)$$

185 The terrain-induced azimuthal dependence of scattering is characterized by a topographic  
 186 modulation function (S. Sandmeier, 1997). The modulation function, which depends on the local slope  
 187 angle and the directions relative to the slope normal, physically approximates the effect of this truncated  
 188 geometry on the scattering process. It adjusts the standard volumetric scattering phase function to better  
 189 represent the anisotropic scattering behavior in the vicinity of the sloping ground boundary, where the  
 190 hemispherical distribution of scatterers is asymmetrically constrained. It is described as follows:

$$191 \quad C_{\text{topo}}(\phi_v) = \sin\theta_g \cos\phi_{vg} \mu'_s \mu'_v \quad (5)$$

192 which enhances scattering in the downslope direction and weakens it in the upslope direction. The  
 193 scattering kernel  $\sigma_s(\Omega', \Omega)$  in Eq.(2)(3) is adjusted by the slope-induced azimuthal modulation function:

$$194 \quad \sigma'_s(\Omega', \Omega) = \sigma_s(\Omega', \Omega) [1 + C_{\text{topo}}(\phi_v)] \quad (6)$$

195 where  $\sigma_s(\Omega', \Omega)$  is the original differential scattering coefficient (using a Henyey–Greenstein phase  
 196 function).

197 The average horizontal intensity across over vegetated area  $U(z, \Omega)$  or the entire horizontal plane  
 198  $\bar{I}(z, \Omega)$  comprise two components: the direct and the diffuse parts of the incoming irradiance, which can  
 199 be given as follows (Shabanov et al., 2000):

$$200 \quad U(z, \Omega) = U_{\text{dir}} \delta(\Omega' - \Omega) + U_{\text{dif}}(z, \Omega)$$

$$201 \quad \bar{I}(z, \Omega) = f_{\text{dir}} I_{\text{dir}} \delta(\Omega' - \Omega) + I_{\text{dif}}(z, \Omega) \quad (7)$$

202 where  $f_{dir}$  is the fraction of incident direct radiation on the upper boundary;  $\delta(\Omega'-\Omega)$  is Dirac's delta  
 203 function (Dirac, 1981) , which approaches infinity when the polar angle in direction  $\Omega$  matches the  
 204 direction of direct solar radiation  $\Omega$ , and zero otherwise;  $U_{dir}(z)$  ( $I_{dir}(z)$ ) and  $U_{dif}(z,\Omega)$  ( $I_{dif}(z,\Omega)$ ) represent  
 205 the direct component and the diffuse component over the vegetation-occupied area (or the entire scene  
 206 area), respectively, which is described in Eq.(7). The boundary conditions for the entire scene are  
 207 simplified as follows (Shabanov et al., 2000; Bird and Hulstrom, 1981).

$$208 \quad \begin{aligned} \bar{I}_0(z, \Omega) &= F_{dir}(\Omega_0)\delta(\Omega - \Omega_0) + d_0(z, \Omega), u < 0 \\ I_H(z, \Omega) &= \frac{R(\Omega, \Omega')}{\pi \int_{2\pi^+} R(\Omega, \Omega') \cos\theta_v d\Omega} + d_H(z, \Omega), u > 0 \end{aligned} \quad (8)$$

$$209 \quad F_{dir} = S_0 \cos\theta_s T \quad (9)$$

$$210 \quad \rho_{soil} = \frac{1}{\pi} \int_{2\pi^+} R(\Omega, \Omega') \cos\theta_v d\Omega \quad (10)$$

211 where  $\delta(\Omega-\Omega_0)$  is the Dirac's delta function, which enables precise estimation of the solar radiation.  
 212  $F_{dir}(\Omega_0)$  represents the intensity of direct radiation incident on the upper boundary.  $S_0$  is the solar  
 213 constant, and  $T$  is the atmospheric transmittance.  $d_0(z, \Omega)$  denotes the diffuse radiation intensity from  
 214 the upper boundary, while  $d_H(z, \Omega)$  represents the intensity entering the canopy through the lower  
 215 boundary. These two parameters can be calculated using the Second Simulation of the Satellite Signal in  
 216 the Solar Spectrum (6S) model (Roujean et al., 1992). The 6S model calculates  $d_0(z, \Omega)$  as atmospheric  
 217 diffuse skylight via RTE accounting for Rayleigh and aerosol scattering and  $d_H(z, \Omega)$  by integrating  
 218 surface/canopy scattering effects on transmitted radiation.  $R(\Omega, \Omega')$  is the bidirectional soil reflectance  
 219 factor, the same as  $R_{1 \times 1}$  in Eq (20) and  $\rho_{soil}$  is the soil hemispherical reflectance. The incoming radiation  
 220 can be parameterized by two values: the total flux  $F_{dir+dif}(\Omega_0)$ , defined as follows (Shabanov et al., 2000):

$$221 \quad F_{dir+dif}(\Omega_0) = \int_{2\pi^-} \bar{I}_0(z, \Omega) \cos\theta_v d\Omega = F_{dir} \cos\theta_v + \int_{2\pi^-} d_0 \cos\theta_v d\Omega \quad (11)$$

222 The horizontal average intensity  $U(z, \Omega)$  over the vegetation-covered area can be decomposed into  
 223 the direct ( $U_{dir}(z)$ ) and diffuse ( $U_{dif}(z)$ ) components of the incoming solar radiation, defined as follows  
 224 (Shabanov et al., 2000):

$$\begin{cases}
U_{dir}(z) + \frac{\sigma(\Omega)}{|\cos\theta_v|} \int_0^z K(z, z', \Omega) U_{dir}(z', \Omega) dz' = 1 \\
U_{dif}(z, \Omega) + \frac{\sigma(\Omega)}{|\cos\theta_v|} \int_0^z K(z, z', \Omega) U_{dif}(z', \Omega) dz' = \frac{1}{|\cos\theta_v|} \int_0^z K(z, z', \Omega) S_{dif}(z', \Omega) dz' + F_0(z, \Omega), \mu < 0 \\
U_{dif}(z, \Omega) + \frac{\sigma(\Omega)}{|\cos\theta_v|} \int_z^H K(z, z', \Omega) U_{dif}(z', \Omega) dz' = \frac{1}{|\cos\theta_v|} \int_z^H K(z, z', \Omega) S_{dif}(z', \Omega) dz' + F_H(z, \Omega), \mu > 0
\end{cases}
\quad (12)$$

$$S_{dif}(z', \Omega) = \int_{4\pi} \sigma_s(\Omega \rightarrow \Omega') U_{dif}(z', \Omega') d\Omega' \quad (13)$$

where  $S_{dif}(z', \Omega)$  denotes the spherical integration of scattering.  $K(z, z', \Omega)$  is the Conditional Probability Correlation Function (PCF). This core function quantifies the probability of spatial correlation between vegetation elements at depths  $z$  and  $z'$  along direction  $\Omega$ , thereby statistically capturing the effect of canopy 3D structure and heterogeneity on the radiation field. The stochastic canopy structure is embedded in the SRTE solution via the PCF, which encodes spatial correlations of vegetation presence derived from  $\chi(x, y, z)$ . This function directly modulates the integral terms in Eq. (12), ensuring canopy heterogeneity and terrain-slope effects are propagated through radiation intensities. Its mathematical form is shown as follows (Shabanov et al., 2007):

$$q(z, z', \Omega) = \langle \frac{1}{S} \iint_S \chi(x, y, z) \chi(x', y', z') \cdot f(\theta_g, \Delta h) dx dy \rangle \quad (14)$$

$$f(\theta_g, \Delta h) = \exp\left(-\frac{\Delta h}{H \cdot \tan\theta_g}\right) \quad (15)$$

$$p(z) = \langle \frac{1}{S} \iint_S \chi(x, y, z) dx dy \rangle \quad (16)$$

$$K(z, z', \Omega) = \frac{q(z, z', \Omega)}{p(z)} = \frac{\langle \int_S \chi(x, y, z) \chi(x', y', z') dx dy \rangle}{\iint_S \chi(x, y, z) dx dy} \quad (17)$$

where  $p(z)$  is the canopy density at canopy depth  $z$ ,  $\chi(x, y, z)$  is the indicator function (1 for vegetation, 0 otherwise). The point  $(x, y, z)$  moves travels a distance  $L$  in the direction  $\Omega$  to reach  $(x', y', z')$ . The bracket  $\langle \cdot \rangle$  denotes the ensemble average over all realizations of  $\chi(x, y, z)$  within a finite pixel.  $\Delta h$  is the vertical elevation difference between two points,  $H$  is the total vertical height of the vegetation canopy.

### 253 2.3 Reflectance factors of the soil background

254 Soil reflectance properties are important elements that should be carefully formulated in canopy radiative  
255 transport process modelling (Knyazikhin et al., 1998b; Verhoef and Bach, 2007). However, modelling  
256 soil reflectance is challenging due to the complex optical properties of soil in shortwave bands. The GSV  
257 soil reflectance model addresses this by providing an effective method for approximating the reflectance  
258 spectra of both dry and wet soils (Jiang and Fang, 2019). Coupling GSV with SRT is critical because: (1)  
259 soil drives reflectance at low LAI (<2), requiring precise spectral representation; (2) SRT resolves canopy  
260 heterogeneity unaddressed by homogeneous models; (3) slope effects jointly modulate soil exposure and  
261 canopy gap probability. GSV-SRTS integrates these via local slope coordinates and coupled soil-canopy  
262 radiation.

263 The GSV refers to the soil spectral vector, which can be obtained by integrating dry and wet soil  
264 data. The global soil reflectance matrix  $R_{m \times l}$  can be expressed as follows (Jiang and Fang, 2019):

$$265 R_{m \times l} \approx U_{m \times m} \Sigma_{m \times l} V_{l \times l} \quad (18)$$

266 where  $m$  represents the number of dry soil samples;  $l$  represents the number of bands; and  $R_{m \times l}$   
267 represents the dry soil spectral matrix.  $U_{m \times m}$ ,  $\Sigma_{m \times l}$ , and  $V_{l \times l}$  can be calculated by the Python callable  
268 function *svd* (singular value decomposition).

269 Principal component analysis is used to obtain  $\Sigma_{m \times n}$  and  $V_{n \times l}$ , where  $V_{n \times l}$  takes the first few  
270 vectors.  $U_{m \times m} \Sigma_{m \times n}$  can be written as  $C_{m \times n}$ . For our GSV model, instead of  $m$  row vectors,  $C_{1 \times (n+1)}$  is  
271 needed, which can be expressed as follows (Jiang and Fang, 2019):

$$272 C_{1 \times (n+1)} = R_{1 \times d} V_{(n+1) \times d}^{-1} \quad (19)$$

273 where  $R_{1 \times d}$  represents the soil spectral data from the region of interest. The first several rows of  $V_{(n+1) \times d}$   
274 can account for the first  $n$  rows of  $V_{(n+1) \times d}$  because we take the first several vectors through principal  
275 component analysis. The  $n+1$  row of  $V_{(n+1) \times d}$  represents the vector of the  $n$  rows of wet soil and one soil  
276 spectral data of the study area. Finally, the reflectance spectrum of the soil background can be calculated  
277 as follows (Jiang and Fang, 2019):

$$278 R_{1 \times l} = C_{1 \times (n+1)} V_{(n+1) \times l} \quad (20)$$

281 where  $R_{1 \times l}$  is the final soil spectral vector.

## 283 2.4 BRF Calculation

284 We apply the discrete ordinates method with  $N_\mu$ =polar and  $N_\phi$ =azimuthal angles to discretize scattering  
285 integrals. The coupled SRTE system is solved iteratively with Gauss-Seidel updates until intensity  
286 residuals fall below  $10^{-4}$ , ensuring stability and energy conservation under slope-modified coefficients.  
287 The discrete ordinates method, a standard approach for solving radiative transfer problems, was used to  
288 numerically solve the coupled SRTS system with its boundary conditions. The incoming radiation is  
289 parameterized by the total flux  $F_{dir+diff}(\Omega_0)$  (Shabanov et al., 2000). The horizontal average intensities  
290  $\bar{I}(z, \Omega)$  and  $U(z, \Omega)$  are divided into direct and diffuse components for computational solution. The final  
291 simulated BRF is calculated from the resulting upwelling radiation intensity at the canopy top ( $z = 0$ ) for  
292 the viewing direction  $\Omega_v$ :

$$293 \text{BRF}(\Omega_v, \Omega_s) = \frac{\pi \cdot \bar{I}(0, \Omega_v)}{F_{dir}(\Omega_s) + E_{diff}} \quad (21)$$

294 where  $E_{diff} = \int_{2\pi} d_0(\Omega) |\cos\theta_v| d\Omega$  is the total diffuse irradiance on the horizontal plane.

## 296 3 Dataset

### 297 3.1 Strategy for Evaluating the Performance of the SRTS Model

298 To comprehensively evaluate the proposed GSV-SRTS model, the simulations were conducted as  
299 follows: First, the overall accuracy of GSV-SRTS was evaluated against DART using scatterplots to  
300 validate the model feasibility. Next, GSV-SRTS was compared with the typical mountain CR models to  
301 validate its advantage in various terrain and canopy conditions after incorporating the SRT theory and  
302 soil spectral vector. The angular distribution of GSV-SRTS was analyzed to reveal the response  
303 mechanism of terrain and canopy conditions to multi-angle observations. Finally, multi-resolution  
304 satellite imagery was used to evaluate model suitability in real mountainous areas.

### 305 3.2 Comparison of DART-based Simulation Models

306 The DART model is a highly regarded analysis tool utilized for modeling the transmission of radiation  
307 signals within soil-leaf-canopy systems (Gastellu-Etchegorry et al., 2012). It has the capability to  
308 accurately simulate the canopy reflectance in various types of terrain and canopy structures. During the  
309 validation of the model, DART simulations were used as the standard to assess the reliability of the GSV-

310 SRTS model as well as other comparable models. In the virtual scenarios created by DART, specific  
311 parameters such as the solar zenith angle of  $30^\circ$  and solar azimuth angle of  $0^\circ$  were set. Additionally, to  
312 evaluate the performance of the model in multi-angle observations, the view zenith angle was varied  
313 from  $0^\circ$  to  $60^\circ$ , while the view azimuth angle ranged from  $0^\circ$  to  $360^\circ$ , encompassing the entire  
314 observation hemisphere. To investigate the accuracy of the model across different sloping surfaces, the  
315 slope angle was adjusted from  $0^\circ$  to  $50^\circ$  in increments of  $10^\circ$ , covering gentle, moderate, and steep slopes.  
316 The aspect was fixed at  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$  and  $270^\circ$ , corresponding to surfaces facing north, east south and  
317 west respectively. Various values of Leaf Area Index (LAI) ranging from 0.5 to 4, canopy height from 2  
318 m to 30 m, and canopy density from 0.1 to 0.8 were set to represent sparse, moderate, and dense  
319 vegetation coverage scenarios. The virtual vegetation scenes were standardized at  $100 \times 100$  m with  
320 diverse canopy structures, terrains, and viewing conditions. The specifics of the input parameters for  
321 generating scenes in DART are outlined in Table 1. In comparison to traditional mountain canopy  
322 reflectance models, the SLCT and GOST2 models were utilized, which are based on radiative transfer  
323 and geometrical optical theories for sloping terrains. All results from the compared models were  
324 juxtaposed with the DART simulations in the same virtual scenes as the benchmark for accuracy  
325 assessment.

326 **Table 1.** Specification of the input parameters of the scenes generated by DART

Model Parameters		Units	Range
Leaf parameters			
$S_L$	Leaf linear characteristic dimension	[m]	0.05-0.2
Canopy structure parameters			
$LAI$	Leaf area index	[m <sup>2</sup> / m <sup>2</sup> ]	0.5-4
$ALA$	Average leaf angle	[°]	30
$p$	Canopy density	unitless	0.1-0.8
$H$	Canopy height	[m]	2-30
$\chi(x, y, z)$	Indicator function	unitless	0 or 1
Illumination view geometry			
$\theta_s$	Sun zenith angle	[°]	0
$\theta_v$	View zenith angle	[°]	0-360
$\varphi_s$	Sun azimuth angle	[°]	30
$\varphi_v$	View azimuth angle	[°]	0-60
Atmospheric condition			
$f_{dir}$	Diffuse radiation fraction	unitless	0.2
$T$	Atmospheric transmittance	unitless	0.7
Topographic factors			
$\theta_g$	Slope	[°]	0-60

### 327 **3.3 Assessment of Canopy Structural Effects on Multi-angle Reflectance**

328 To discover the influence of canopy structure, multi-angle CR simulations were conducted. The virtual  
329 vegetation scenes were set with stable terrain and view conditions. The sun zenith angle was set to 30°  
330 and the sun azimuth angle was set to 0°. The view zenith angle was set to range from 0° to 60°, and the  
331 view azimuth angle ranged from 0° to 360° to cover the hemispheric viewing space. And the canopy  
332 densities were set to 0.2, 0.5 and 0.8 to demonstrate sparse, medium and dense vegetation coverage. On  
333 multi-angle CR simulations, the terrain and canopy condition effects were assessed explicitly.

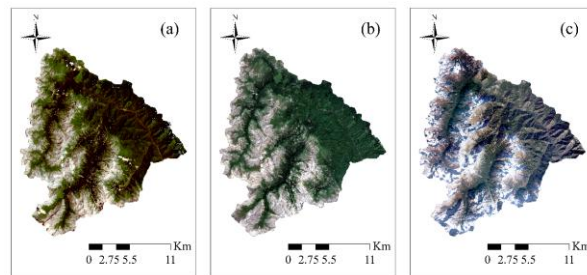
### 334 **3.4 Assessment of the Terrain and Canopy Conditions Effects on Multi-angle CR Simulations**

335 To assess the effectiveness of the proposed model, it is imperative to simulate canopy reflectance in a  
336 real mountainous setting. The chosen location for this study is the Wanglang Field Observation Station,  
337 situated in the northeastern region of the Tibetan Plateau at coordinates approximately 32.94°N and  
338 104.07°E. This area boasts a diverse landscape characterized by towering mountain peaks and deep  
339 gorges, with elevations ranging from 2000 to 4200 meters and slopes that can vary from gentle inclines  
340 to steep gradients of up to 76°. The various vegetation types that thrive in this unique environment include  
341 deciduous forests, coniferous forests, mixed forests combining both coniferous and deciduous species,  
342 as well as scrublands. Field measurements were stratified by elevation, slope, and aspect to cover  
343 dominant vegetation types. Canopy parameters (LAI, height) were measured via hemispherical  
344 photography and UAV LiDAR, with soil spectra sampled from exposed sites aligned with target pixels.

345 Satellite products from Landsat 9-OLI, Sentinel-2, and Jilin-1 with spatial resolutions of 30 m, 10  
346 m, and 0.5 m, respectively, are presented in Fig. 2. Cloud-free remote sensing images from these sensors,  
347 covering the experimental area, served as reference data for model performance assessment. The Landsat  
348 9-OLI image was acquired on July 30, 2024, during the vegetation growing season, with solar zenith and  
349 azimuth angles of 25.5° and 119.6°, respectively. It was obtained from the Earth Resources Observation  
350 and Science (EROS) Center Science Processing Architecture (ESPA) on-demand interface  
351 (<https://espa.cr.usgs.gov/>), which provides surface reflectance products corrected using the Second  
352 Simulation of the Satellite Signal in the Solar Spectrum Vector (6SV) model (Vermote et al., 2016). The  
353 Sentinel-2 image was captured on July 8, 2022, with corresponding solar zenith and azimuth angles of

354 18.73° and 119.2°, and was downloaded from the Copernicus Open Access Hub  
 355 (<https://dataspace.copernicus.eu/>). The Jilin-1 remote sensing images were acquired and mosaicked from  
 356 the Jilin-1 Satellite Network (<https://www.jl1mall.com/>), with data collected between June and August  
 357 from 2022 to 2024. These three remote sensing images are accompanied by corresponding digital  
 358 elevation models (DEMs) for topographic analysis. Topographic factors for the study area were derived  
 359 from the following sources: the ASTER Global Digital Elevation Model (GDEM) version 2 (spatial  
 360 resolution of 1 arc-second, approximately 30 m) was used for the Landsat 9-OLI image; the Copernicus  
 361 DEM GLO-10 product (10 m resolution) was acquired from the Copernicus Open Access Hub to match  
 362 the Sentinel-2 image; and high-resolution DEM data were obtained via unmanned aerial vehicle (UAV)  
 363 laser scanning and resampled to a 0.5 m resolution to align with the Jilin-1 imagery.

364



365

366 **Figure 2: Study area Wanglang National Nature Reserve, Pingwu County, Mianyang with three different sources of remote**  
 367 **sensing images, from which (a) Landsat9-OLI, (b) Sentinel2, (c) Jilin-1.**

368 The input parameters for driving the SRTS model are summarized in Table 2. These parameters  
 369 have been carefully defined using a combination of field measurements, remote sensing products, and  
 370 relevant literature. For example, the leaf linear characteristic dimension ( $S_L$ ) has been designated as 0.05-  
 371 02 m based on typical values for vegetation (Jacquemoud et al., 2009). Canopy structure parameters  
 372 included in the model consist of a leaf area index (LAI) ranging from 2-4  $m^2/m^2$ , canopy height ( $C_H$ )  
 373 varying from 5-30 m to encompass the range of measurements, and an average leaf angle (ALA) of 30°  
 374 which follows an ellipsoidal distribution assumption (Campbell, 1990). In order to accurately capture the  
 375 topographic variability of the study area, parameters such as slopes ( $\theta_g$ ) ranging from 0-60° and aspects  
 376 ( $\phi_g$ ) from 0-360° have been incorporated into the model. These parameters ensure that the full terrain  
 377 characteristics are taken into consideration during the simulations. Furthermore, atmospheric parameters  
 378 for the study area have been derived using the MODTRAN atmospheric radiative transfer model (Berk  
 379 et al., 2004). This ensures that the atmospheric conditions are accurately represented in the model

380 simulations. The illumination geometry has been fixed based on remote sensing observations, providing  
 381 a consistent and reliable input for the model. By incorporating all of these parameters into the SRTS  
 382 model, we are able to accurately simulate and analyze the surface reflectance properties of the study area.

383 The input parameters for the model are outlined in Table 2, with a summary provided for reference.  
 384 A dataset based on these parameters was used to generate 10000 combinations of input values. Each  
 385 combination was then used to compare the remote sensing reflectance with the simulated reflectance  
 386 stored in a lookup table (LUT) on a pixel-by-pixel basis. In order to mitigate the challenges posed by the  
 387 ill-posed nature of the model inversions (Vermote et al., 1997) the final outcome was determined by  
 388 averaging the results from the 100 best-matching simulations. The degree of agreement between the  
 389 observed and simulated reflectance was evaluated using the following cost function:

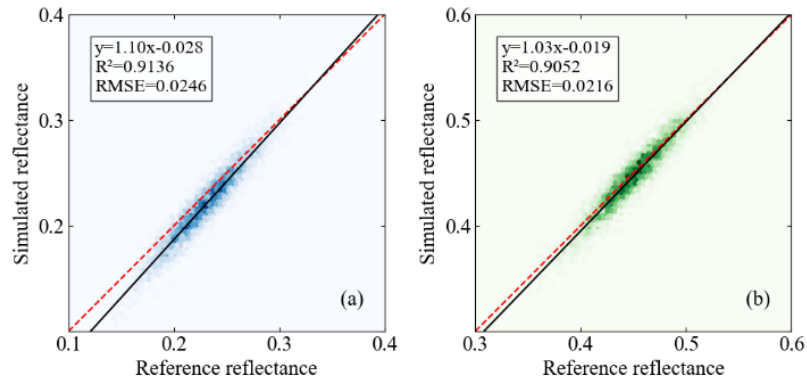
$$390 \quad \text{Cost} = \sqrt{\left(\frac{\text{Sim}_r - \text{Obs}_r}{\text{Obs}_r}\right)^2 + \left(\frac{\text{Sim}_n - \text{Obs}_n}{\text{Obs}_n}\right)^2}$$

391 (22)

392 where  $\text{Sim}_r$  and  $\text{Sim}_n$  are the simulated reflectances in the red and NIR bands, respectively, and  $\text{Obs}_r$  and  
 393  $\text{Obs}_n$  are the observed reflectances in the red and NIR bands, respectively.

394 **Table 2.** Specification of the input parameters to generate the LUT

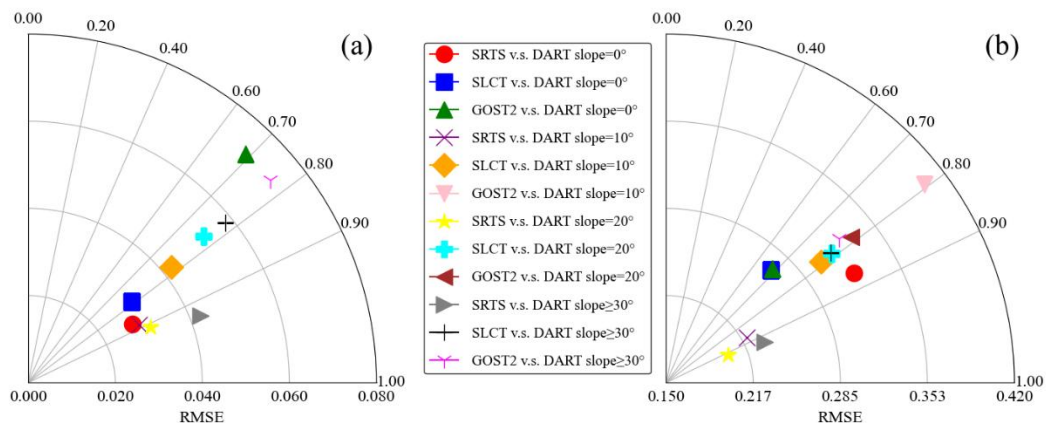
Model Parameters		Units	Range
Leaf parameters			
$S_L$	Leaf linear characteristic dimension	[m]	0.05-0.2
Canopy structure parameters			
$LAI$	Leaf area index	[m <sup>2</sup> / m <sup>2</sup> ]	2-4
$ALA$	Average leaf angle	[°]	30
$p$	Canopy density	unitless	0.2-0.8
$C_H$	Canopy height	[m]	5-30
$\chi(x, y, z)$	Indicator function	unitless	0 or 1
Illumination view geometry			
$\theta_s$	Sun zenith angle	[°]	25.5
$\theta_v$	View zenith angle	[°]	0-360
$\varphi_s$	Sun azimuth angle	[°]	119.6
$\varphi_v$	View azimuth angle	[°]	0-60
Atmospheric condition			
$f_{dir}$	Diffuse radiation fraction	unitless	0.2
$T$	Atmospheric transmittance	unitless	0.7
Topographic factors			
$\theta_g$	Slope	[°]	0-60

395 **4 Results and analysis**396 **4.1 Model Evaluation by DART and Typical CR Models**

397

398 **Figure 3: Density scatterplots between the canopy reflectance simulated by DART and the GSV-SRTS models. (a) and (b)**  
 399 **represent the DART simulations compared with the SRTS model in the red and NIR bands.**

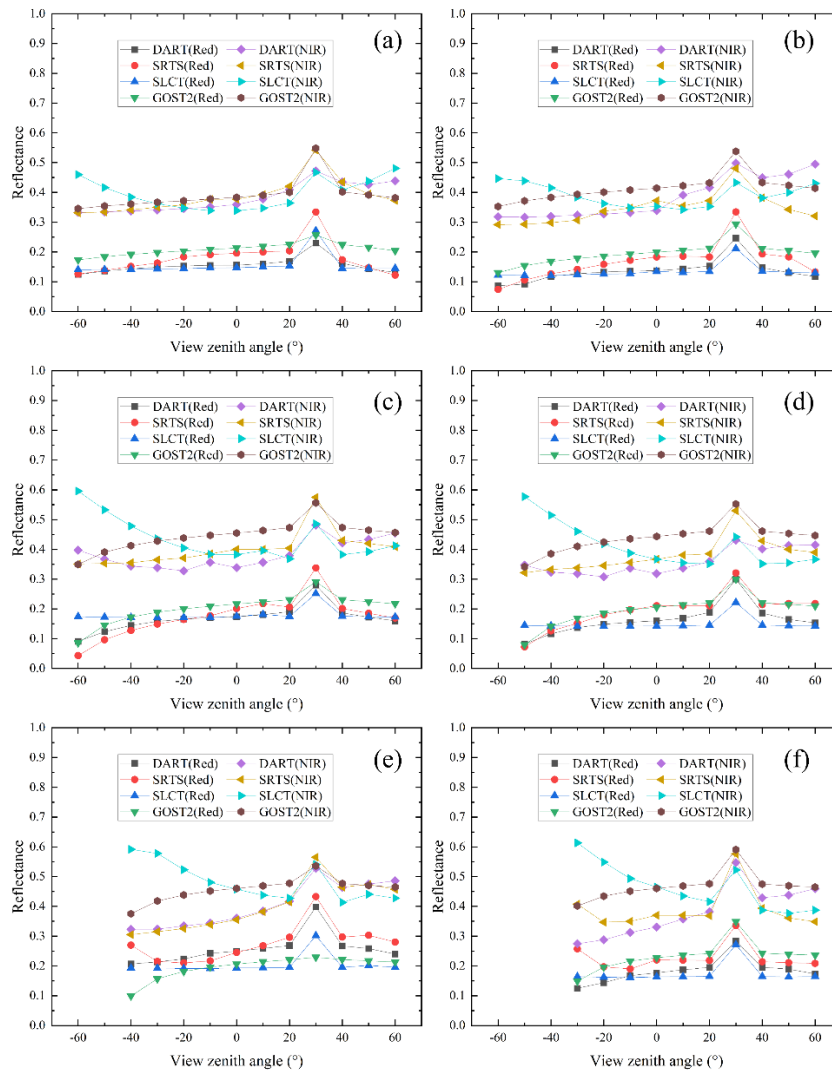
400 To validate the overall simulation accuracy of the GSV-SRTS model, the GSV-SRTS simulations were  
 401 compared against the DART simulations. As shown in Fig. 3 and Fig. 4, the scatter plots were generated  
 402 using comprehensive input parameter combinations, including varying slope, aspect, canopy density, and  
 403 observation direction. The results showed that GSV-SRTS achieved the highest  $R^2$  value of 0.9136  
 404 (0.9052) and the lowest RMSE of 0.0146 (0.0106) in the red (NIR) band, outperforming other compared  
 405 models.



406

407 **Figure 4: The comparison between GSV-SRTS, SLCT, GOST2, and DART over different sloping terrains. (a) and (b)**  
 408 **represent the results in the red and NIR bands, respectively.**

409 As shown in Figure 4, the model comparison across different slopes shown that the GSV-SRTS and  
 410 DART simulations have good consistency over different sloping terrains. In comparisons between DART  
 411 and other models, the performance of GOST2 and SLCT deteriorates as slope increases. The SLCT  
 412 model, which assumes trees grow perpendicular to the ground, exhibits distortions in simulating crown  
 413 gap fraction on steeper slopes. This in turn affects the accurate characterization of radiation contributions  
 414 over sloping terrain. GOST2 assumes that trees follow the Neyman type-A distribution pattern (Fan et  
 415 al., 2015). Although the GOST2 model can accurately describe the spatial distribution of the trees, the  
 416 scenes generated by DART where trees are randomly distributed is more suitable for SRTS. Specifically,  
 417 in the red (NIR) band, SRTS precision increases from 0.8702 (0.8635) at 0° to 0.9318 (0.9234) at slopes  
 418  $\geq 30^\circ$ . This comparison demonstrates enhanced capability of SRTS in complex terrain conditions.



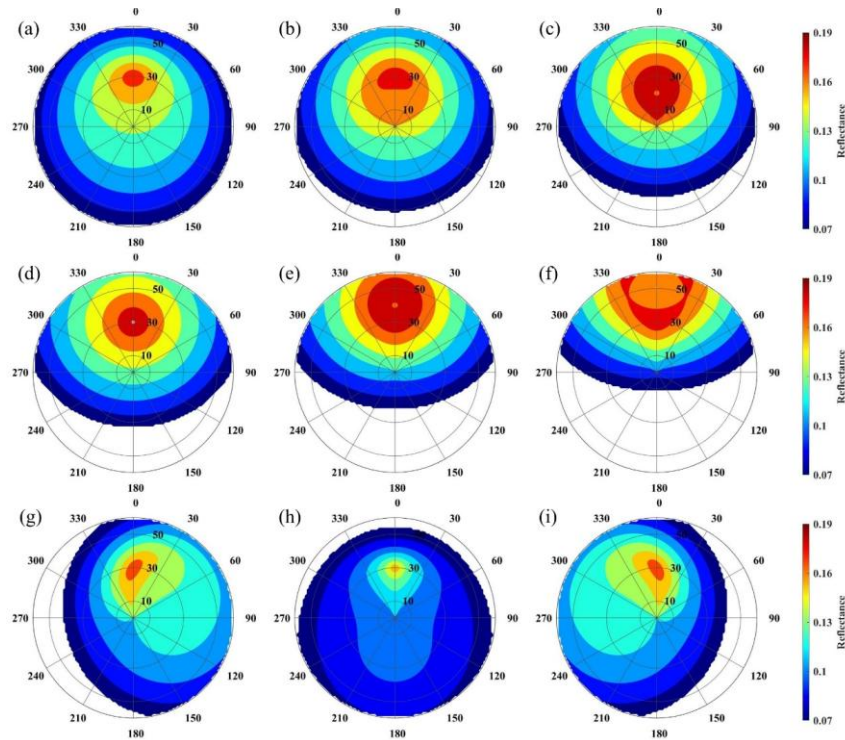
419

420 **Figure 5: Comparison of canopy reflectance simulated by SRTS, SLCT, and GOST2 models in the PP (principal plane) for**  
421 **slopes of (a) 0°, (b) 10°, (c) 20°, (d) 30°, (e) 40°, and (f) 50°. A view azimuth angle of 0° corresponds to the positive X-axis**  
422 **direction, while 180° corresponds to the negative X-axis direction.**

423 To specifically compare the reflectance under different view zenith angles, the different slopes were  
424 set from 0° to 50° with an interval of 10°, as illustrated in Fig. 5. As the terrain slope increases, the GSV-  
425 SRTS model achieves the best match with DART simulations in the red and NIR bands, despite SLCT  
426 performing better on flat terrain. As the slope steepens, the fundamental differences in physical approach  
427 of each model become increasingly apparent. The SLCT model relies on the simplified plane-parallel  
428 RT theory, assuming homogeneous leaf distribution and ignoring slope-induced shadowing effects. This  
429 leads to systematic underestimation of multiple scattering contributions in steeper terrain. The GOST2  
430 assumes that the trees follow the Neyman type-A distribution, so when view zenith angle is near nadir,  
431 the probability of observing the background is larger than that of the other compared models. The GSV-  
432 SRTS model accounts for terrain conditions by coupling a voxel-based canopy representation with a  
433 Monte Carlo photon-tracking approach, enabling it to resolve anisotropic scattering patterns caused by  
434 tilted surfaces. This ability to adjust and remain accurate even in more complex conditions makes GSV-  
435 SRTS especially well-suited for sloping terrains.

#### 436 **4.2 Terrain Effects on CR in Hemispheric Space**

437 To reveal the effects of terrain factors and viewing conditions to canopy reflectance, the angular  
438 distributions of reflectance under various scenarios were simulated by the GSV-SRTS model. The results  
439 are presented in the form of polar coordinate maps. In these maps, the radial distance corresponds to the  
440 zenith angle, while the angular position represents the viewing azimuth angle. The polar contour maps  
441 for the red and NIR bands are displayed in Fig. 6 and Fig. 7, respectively.



442

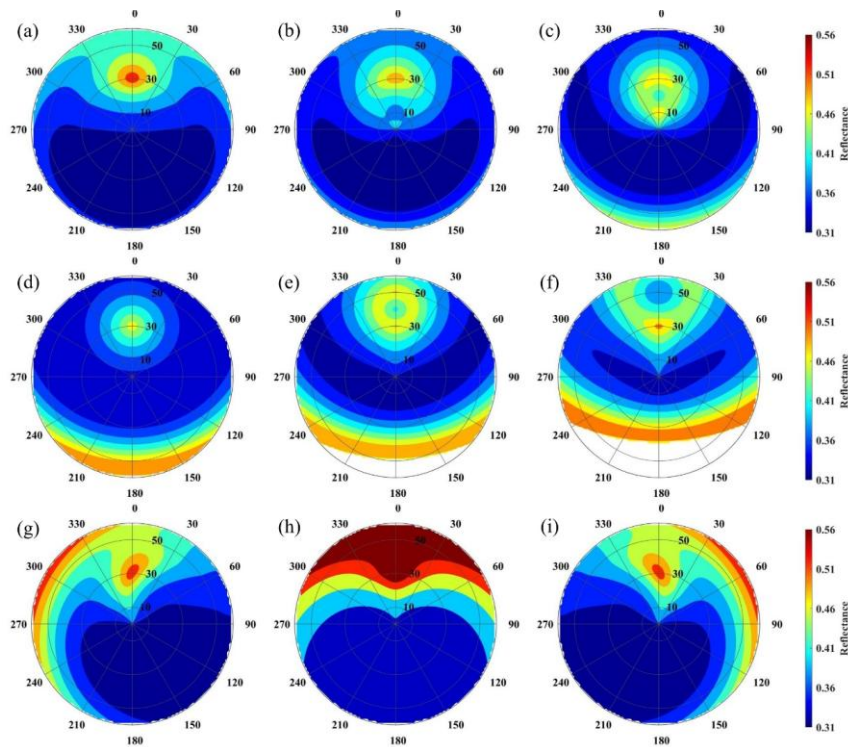
443 **Figure 6: Angular distribution of canopy reflectance in the red band: (a-f) for an aspect of 0° and slopes of 0°, 10°, 20°, 30°, 40°, and 50°; (g-i) for a slope of 20° and aspects of 90°, 180°, and 270°.**

445 As shown in Fig. 6, reflectance in the red band ranges primarily between 0.07 and 0.19. The peak  
 446 of reflectance occurs at the hotspot, where the view zenith angle is 30° and the azimuth angle is 0°. At  
 447 the same azimuth angle, reflectance decreases as the zenith angle increases. This angular dependence  
 448 occurs due to geometric shadowing, path length effects and volume scattering reduction. Photons  
 449 penetrating the canopy at off-nadir angles traverse longer effective path lengths through the vegetation  
 450 medium, enhancing absorption by the canopy.

451 When the aspect is 0° (facing the illumination direction), increasing the slope expands the high-  
 452 reflectance zone and intensifies maximum reflectance at the azimuth angle of 0°. This amplification  
 453 occurs because steeper slopes facing the light source create optimal conditions for enhanced canopy  
 454 reflectance. The tilted surface exposes a larger area to direct solar radiation, enhancing the sunlight  
 455 received by slope-facing vegetation by a factor of  $1/\cos\theta$  (where  $\theta$  is slope angle) (Dubayah and Rich,  
 456 1995). On the slope of 20°, solar radiation increases by 64% compared to that in flat terrain.

457 However, when the slope reaches 20° and the aspect shifts to 90° (panel (g)) or 270° (panel (i)),  
 458 high reflectance concentrates in specific sections of the upper hemisphere. Reflectance is higher between  
 459 azimuth angles of 0°–90° in panel (g) and 270°–360° in panel (i) because the probability of sunlight  
 460 reaching the ground becomes larger. For slopes with an aspect of 90°, maximum reflectance occurs when

461 the solar azimuth angle is approximately  $90^\circ$ . The PP (principal plane) of incidence is the same with the  
 462 normal vector of the slope, minimizing the shadowing effects. This geometry enhances the backscatter  
 463 direction observed by the sensor. It also creates minimal cast shadows and maximum visible illuminated  
 464 areas, increasing the apparent reflectance. Similarly, slopes with a  $270^\circ$  aspect exhibit peak reflectance  
 465 near a solar azimuth angle of  $270^\circ$ , following the same geometric principle. Notably, polar contour maps  
 466 of a  $20^\circ$  slope with aspects of  $90^\circ$  and  $270^\circ$  exhibit symmetry about the vertical axis. This symmetry  
 467 stems from the fact that the relative azimuth angle between the illumination and viewing directions is the  
 468 same in both instances. When the sun-sensor geometry is fixed, the reflectance remains consistent,  
 469 showing mirror-like symmetry around the azimuth of  $0^\circ$ . This confirms that light interacts with slopes in  
 470 the same way, regardless of their facing direction.



471  
 472 **Figure 7: Angular distribution of canopy reflectance in the NIR band: (a-f) for an aspect of  $0^\circ$  and slopes of  $0^\circ$ ,  $10^\circ$ ,  $20^\circ$ ,**  
 473  **$30^\circ$ ,  $40^\circ$ , and  $50^\circ$ ; (g-i) for a slope of  $20^\circ$  and aspects of  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ .**

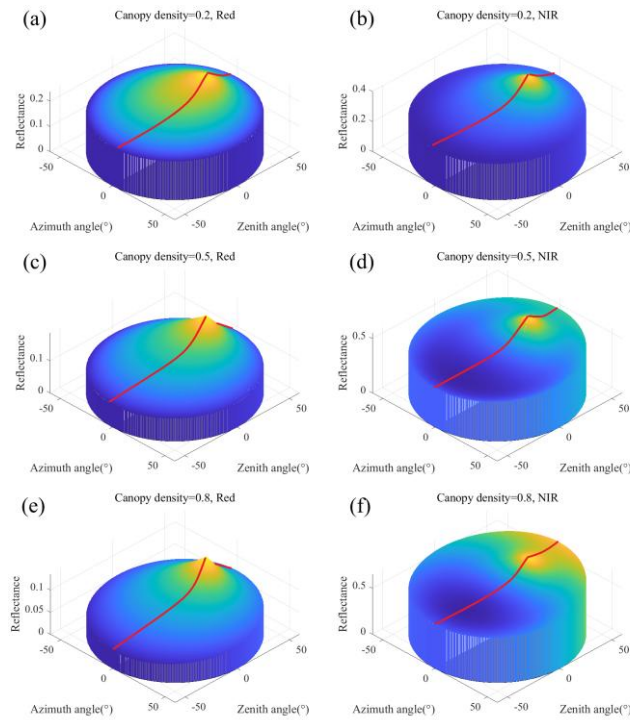
474 In the NIR band, reflectance displays similar trends yet maintains consistently higher values  
 475 relative to the red band. As shown in Fig. 7, reflectance ranges primarily from 0.31 to 0.56, with higher  
 476 values compared to the red band. This difference primarily stems from reduced chlorophyll absorption  
 477 in the NIR spectrum, enabling more effective light penetration through multiple canopy layers. These  
 478 distinct interactions lead to characteristically higher NIR reflectance and lower red reflectance in  
 479 vegetation canopies.

480           When the slope is  $20^\circ$  with the aspect of  $90^\circ$  or  $270^\circ$ , canopy reflectance increases in the  
481 hemispherical distribution at the small zenith angles. This demonstrates that steeper slopes facing the  
482 sunward direction receive more solar illumination. The consistent slope and illumination direction,  
483 combined with symmetrical aspect, result in a symmetrical distribution of gap fractions, which in turn  
484 leads to a symmetrical hemispherical reflectance distribution. For a terrain characterized by the slope of  
485  $20^\circ$  facing the aspect of  $90^\circ$ , lower reflectance is concentrated along the  $90^\circ$ - $180^\circ$  view azimuth angles.  
486 In contrast, when the aspect shifts to  $270^\circ$ , reflectance decreases along the  $180^\circ$ - $270^\circ$  view azimuth  
487 directions. When the slope faces  $90^\circ$ , sensors viewing along the azimuths of  $90^\circ$ - $180^\circ$  observe the sunlit  
488 slope but detect reduced reflectance. Because when the solar azimuth equals aspect, the illumination  
489 angle on the inclined surface is minimized, reducing casting shadows and increasing the projected sunlit  
490 area. This geometry enhances the backscattering direction observed by sensors. The same physical  
491 principles apply to the aspect of  $270^\circ$ , where reflectance maxima occur near solar azimuth of  $270^\circ$ ,  
492 demonstrating symmetrical behaviour across the PP.

493           While the hotspot fundamentally results from aligned sun-view geometry reducing mutual  
494 shadowing, sloping terrain distorts its signature: local solar/view angles (Eq. 1) shift peak reflectance  
495 toward the illuminated slope face, amplified by topographic modulation (Eq. 5). GSV-SRTS resolves  
496 this by coupling slope-corrected geometry with stochastic gap probability, capturing asymmetric hotspot  
497 broadening on inclined surfaces.

498           These results demonstrated the importance of incorporating topographic factors into GSV-SRTS in  
499 mountainous conditions, where terrain-induced factors, solar and view geometries significantly affect  
500 canopy reflectance patterns. The ability of GSV-SRTS to account for terrain-related variations ensures  
501 more accurate RT process simulations.

502 **4.3 Gap Probabilities Effects on Surface Reflectance in Hemispheric Space**



503

504 **Figure 8: Hemispheric views of GSV-SRTS-simulated reflectance over three different canopy densities (a) (b) 0.2, (c) (d)**  
505 **0.5 and (e) (f) 0.8. The left column represents the canopy reflectance in the red band, and the right column represents the**  
506 **canopy reflectance in the NIR band.**

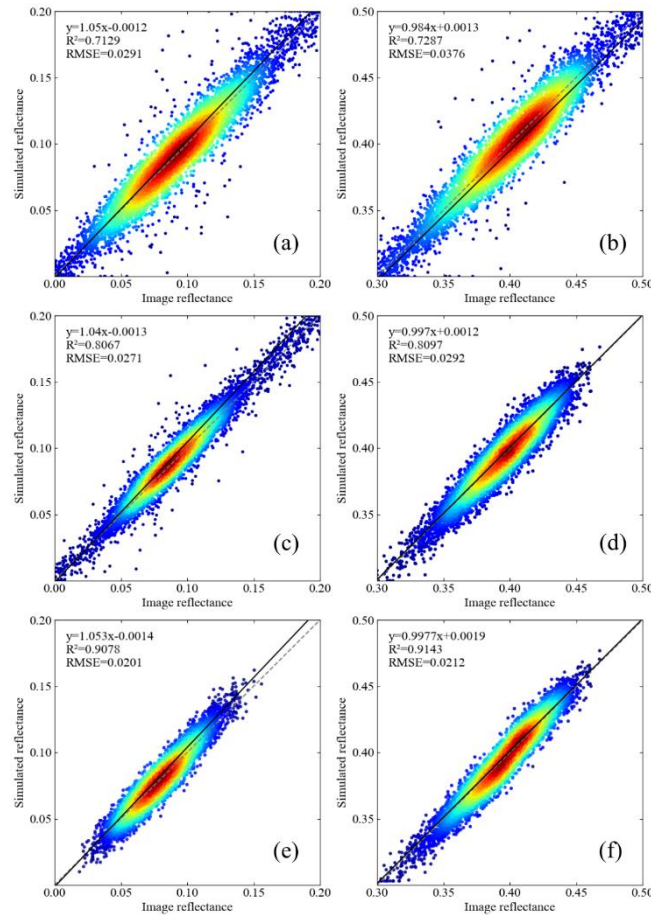
507 The gap probability, a critical factor influencing the radiative properties of vegetation and soil in remote  
508 sensing applications, was utilized in a study to investigate how canopy structure impacts surface  
509 reflectance simulations. Using the GSV-SRTS model, the hemispherical distributions of surface  
510 reflectance in the red and NIR bands were simulated under different canopy densities. Figure 8 illustrates  
511 that canopy reflectance peaks in the hotspot direction, regardless of changes in slope. This directional  
512 peak is a result of decreased canopy densities and shadowing from the ground when the sensor's viewing  
513 angle aligns with the direction of solar illumination. The GSV-SRTS model accurately captures the  
514 reduced probability of observing shaded areas by integrating terrain-modified gap probability and  
515 geometries into its simulations. Increasing canopy density leads to a noticeable increase in NIR  
516 reflectance (panels (b), (d) (f)) particularly at the nadir, due to enhanced photon interactions within the  
517 canopy and changes in gap probability affecting radiation penetration. In contrast, red band reflectance  
518 (panels (a), (c) (e)) remains low across all canopy densities, reflecting the strong absorption of red light  
519 by chlorophyll. Therefore, both spectral regions demonstrate significant bidirectional reflectance

520 distribution function (BRDF) effects, with NIR reflectance being more sensitive to changes in canopy  
521 density.

522       Moreover, reflectance in the backward-scattering direction consistently exceeds that in the forward-  
523 scattering direction. This indicates sunlit crowns become more visible at this direction and mutual  
524 shading effects are minimized in this viewing geometry, where the sunlit portions dominate the field of  
525 view of sensors. Such directional reflectance features are especially pronounced under dense canopies,  
526 where shadowing and multiple scattering further enhance the contrast between forward and backward  
527 observations. These reflectance characteristics provide critical understanding of the 3D structure and  
528 complexity of forest canopies.

#### 529 **4.4 Model Assessment in a Real Mountainous Region**

530 To assess the effectiveness of the GSV-SRTS model in a practical mountainous setting and explore how  
531 well the model works at different levels of detail, we utilized a combination of satellite imagery sources  
532 with varying resolutions. Specifically, the Landsat 9-OLI imagery with a resolution of 30 meters, the  
533 Sentinel-2 imagery with a resolution of 10 meters, and the Jilin-1 imagery with a resolution of 0.5 meters  
534 were employed to establish the reference reflectance values for our model evaluation. Additional  
535 information regarding the specific geographic area of focus for our study and the technical details of the  
536 images can be found in Section 3.4.



537

538 **Figure 9: Density scatterplots comparing the simulated reflectance from the SRTS model with the image reflectance for (a)**  
 539 **red and (b) NIR bands of Landsat 9-OLI, (c) red and (d) NIR bands of Sentinel-2, and (e) red and (f) NIR bands of Jilin-1.**

540 As illustrated in Fig. 9, the GSV-SRTS-simulated reflectance shows lower  $R^2$  values of 0.7129  
 541 (0.7287) and higher RMSE values of 0.0291 (0.0376) in the red (NIR) band when compared with the  
 542 Landsat9-OLI image reflectance. For the Sentinel-2 image, the SRTS-simulated reflectance achieves  $R^2$   
 543 values of 0.8067 (0.8097) and RMSE values of 0.0271 (0.0292) in the red (NIR) band. The SRTS-  
 544 simulated reflectance shows the highest consistency with the reflectance of the JiLin-1 image, with  $R^2$   
 545 values of 0.9078(0.9143) and RMSE values of 0.0201 (0.0212) in the red (NIR) band.

546 Even for medium-low resolution pixels, GSV-SRTS maintains advantages by statistically  
 547 aggregating sub-pixel heterogeneity into radiative parameters, avoiding homogenization biases. Its  
 548 terrain-aware structure ensures accurate BRDF shapes on slopes, where homogeneous models exhibit  
 549 systematic errors. However, the reflectance simulation accuracy of the GSV-SRTS model enhances with  
 550 higher spatial resolution of remote sensing images, owing to the ability of the underlying SRT theory  
 551 and GSV model to resolve finer details in microscale settings. This compatibility stems from the core  
 552 mechanics of the GSV-SRTS model, which simulates photon interactions within the canopy in detail.

553 The model incorporates key processes, including multiple scattering between leaves, leaf orientation,  
554 shadowing effects, and canopy structure variation. And the soil background is fully considered and  
555 accurately modelled. Consequently, the GSV-SRTS model is highly compatible with high-resolution  
556 imagery. In contrast, coarser resolution images tend to smooth out small-scale features, limiting the  
557 ability of the model to leverage its detailed photon-tracking capabilities. This limitation is clearly  
558 evidenced by the lower  $R^2$  values obtained with Sentinel-2 and Landsat 9-OLI data, which are primarily  
559 attributed to the homogenization of intra-pixel spatial details—terrain variation, canopy structure, and  
560 soil background—during the pixel integration process. Our model simulates these sub-pixel  
561 heterogeneities explicitly, but the validation data from medium-resolution sensors represents an  
562 aggregated average. This does not diminish the model's utility for coarser resolutions; rather, it highlights  
563 that the GSV-SRTS model is particularly advantageous for applications where understanding or  
564 quantifying the impact of sub-pixel heterogeneity is essential, such as in scaling studies, high-fidelity  
565 scene simulation, or informing the parameterization of larger-scale biogeophysical models. As a result,  
566 the GSV-SRTS model achieves optimal accuracy with high-resolution images, producing more realistic  
567 reflectance estimates. Therefore, the integration of high-resolution data with the photon-sensitive  
568 framework of the GSV-SRTS model enables precise capture of subtle canopy reflectance variations,  
569 optimizing model performance and achieving highly precise results.

570 As a result, the GSV-SRTS model achieves optimal accuracy with high-resolution images,  
571 producing more realistic reflectance estimates. Therefore, the integration of high-resolution data with the  
572 photon-sensitive framework of the GSV-SRTS model enables precise capture of subtle canopy  
573 reflectance variations, optimizing model performance and achieving highly precise results.

## 574 **5 Discussion**

575 Accurate modeling of soil-canopy radiation interactions is essential for retrieving biophysical variables  
576 at regional-to-global scales. However, topographic relief and heterogeneous landscapes introduce  
577 significant uncertainties in radiative transfer simulations. While numerous canopy reflectance models  
578 account for sloping terrain, most cannot take heterogeneities of canopy structure and soil properties into  
579 account, leading to distorted BRDF patterns (Xiaowen et al., 1993; Gastellu-Etchegorry et al., 1996;  
580 Chen and Leblanc, 1997). This study proposed a CR model by extending the GSV and the stochastic RT

581 model for Sloping Terrains (GSV-SRTS), improving the model applicability in real mountainous  
582 environments. The GSV-SRTS model advances vegetation and soil reflectance modeling because of its  
583 physically based BRDF framework, which intrinsically accounts for 3D canopy structure and surface  
584 interactions (Jean-Lucwidlowski et al., 2015).

585         However, the study still has limitations in the theoretical framework and experimental design,  
586 requiring further improvement. Although the GSV-SRTS model incorporates terrain effects, the  
587 influence of the surrounding terrains was not fully considered. In mountainous regions, varying slope  
588 angles significantly affect RT by altering shadow patterns, neglecting the adjacent terrain irradiance may  
589 increase inaccuracies in remote sensing modeling, especially for the surface coverage with high  
590 reflectance (Mousivand et al., 2015; Rich and M., 1995; Sandmeier and Itten, 1997). The surrounding  
591 terrains may block direct solar radiation, reducing incident energy on the target surface (Riaño et al.,  
592 2003). Surrounding terrain surfaces scatter indirect radiation, altering the total flux received by the  
593 canopy (Gu and Gillespie, 1998). Concave landforms (e.g., valleys) enhance multiple reflections, while  
594 convex features (e.g., ridges) redistribute local energy budgets (Soenen et al., 2005; Chen and Black,  
595 1992). Future studies should explore integrating surrounding terrain effects into the GSV-SRTS model  
596 to enhance accuracy under different terrains (Knyazikhin et al., 1998a).

597         The General Spectral Vector (GSV) was originally derived from the LUCAS global soil database,  
598 which comprises reflectance spectra from 19,036 dry soil samples (Jiang and Fang, 2019). While this  
599 dataset provides a robust foundation for modeling soil optical properties, its applicability to larger-scale  
600 or diverse regional studies remains limited due to geographical and pedological constraints. To enhance  
601 the generalizability and accuracy of the GSV-PROSAILT model across varied ecosystems and soil types,  
602 it is essential to expand the GSV framework by incorporating a broader and more diverse set of soil  
603 samples. This expansion would strengthen the model's utility for vegetation-soil radiative transfer  
604 modeling by enabling a more accurate representation of spatial variability in soil composition, moisture  
605 content, and surface conditions. As emphasized by Jiang and Fang (2019), extending the spectral library  
606 with additional representative samples is a critical step toward achieving higher predictive performance  
607 and broader operational applicability in remote sensing applications.

608         The topographic influence on canopy reflectance is channeled through two primary, interconnected  
609 physical mechanisms in the model: (i) the modification of the effective extinction coefficient due to  
610 changes in the relative pathlength through the canopy per unit vertical depth in the local slope coordinates,

611 and (ii) the alteration of the canopy gap probability, modeled via the PCF within the stochastic framework,  
612 which is sensitive to the terrain-modified illumination direction. The first governs the bulk attenuation  
613 of direct radiation along its path, while the second determines the probability of direct beam penetration  
614 to the soil or lower canopy layers. These two effects are not independent; the altered pathlength directly  
615 influences the spatial statistics that underpin the gap probability calculation. Our integrated approach  
616 through the local slope coordinate system ensures both effects are consistently accounted for, providing  
617 a more complete representation of terrain role in mountain canopy radiative transfer.

618 The current model uses a simplified pair-correlation function (PCF) to represent vegetation spatial  
619 structures, reducing computational demands. However, higher-order moments (e.g., three- or four-point  
620 correlation functions) in canopy structure could improve radiative characteristic predictions (Huang et  
621 al., 2008; Tang et al., 2014; Lewis and Disney, 2007). Although integrating higher-order moments would  
622 increase computational load, advancements in parallel computing and GPU technology provide potential  
623 solutions (Jonckheere et al., 2004; Lovell et al., 2003).

624 Moreover, field sampling efforts are hampered by the complex terrain and hindered by the dense  
625 vegetation cover within the study area. Increasing observation frequency and optimizing sampling  
626 schemes could enhance data representativeness (Lefsky et al., 2002). However, obtaining the canopy  
627 structure parameters in mountainous areas is challenging, which can refine the RT mechanisms (Nelson,  
628 2009). Integrating advanced sensing techniques like LiDAR provide a feasible solution for addressing  
629 these limitations (Yang et al., 2019; Bailey et al., 2020).

630

## 631 **6 Conclusion**

632 This study proposes a canopy reflectance model, the GSV-SRTS model, which is based on the extended  
633 GSV and stochastic radiative transfer theory. The model is designed to accurately analyse canopy  
634 reflectance in sloping heterogeneous landscapes. To assess the effectiveness of the GSV-SRTS model,  
635 comparisons were made with DART simulations, typical CR models, and remote sensing observations  
636 with multiple spatial resolutions. The results of the study indicate that the GSV-SRTS model adequately  
637 captures the variations in canopy reflectance induced by terrain and vegetation conditions. It exhibits  
638 strong consistency with DART benchmarks, with the DART benchmarks ( $R^2 = 0.9136$  (0.9052), RMSE

639 = 0.0246 (0.0216) in the red (NIR) band). And this study revealed the influences of various terrain and  
640 vegetation conditions on multi-angle CR simulations. By comparing the GSV-SRTS model with remote  
641 sensing observations, it was found that the model is suitable for soil-canopy radiative transfer modelling  
642 in small-scale scenes, with RMSE ( $R^2$ ) values of 0.0201 (0.9078) for the red band and 0.0212 (0.9143)  
643 for the NIR band. Therefore, the development of the GSV-SRTS model provides a feasible paradigm for  
644 CR modeling and can be used as a potential tool biophysical parameter retrieval.

645

646 *Code and data availability.* The current version of GSV-SRTS code and dataset is available from figshare  
647 website

648 <https://doi.org/10.6084/m9.figshare.30392938> (Li, 2025) (Creation date: 2025-10-19).

649

650 *Author contributions.* Conceptualization, Guyue Hu and Shaoda Li; methodology, Jinhu Bian and Guyue  
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652 Siqi Li; investigation, Siqi Li and Chenghao Liu; resources, Guyue Hu and Shaoda Li; data curation,  
653 Junxiang Tan and Ronghao Yang.; writing—original draft preparation, Siqi Li; writing—review and  
654 editing, Guyue Hu; visualization, Siqi Li; supervision, Jinhu Bian; project administration, Shaoda Li;  
655 funding acquisition, Guyue Hu, Ronghao Yang, Shaoda Li and Jinhu Bian.

656

657 *Competing interests.* The authors declare that they have no known competing financial interests or  
658 personal relationships that could have appeared to influence the work reported in this paper.

659

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665 necessarily reflect the views of the publisher.

666

667 *Acknowledgements.* This research was funded by the National Natural Science Foundation project of  
668 China, grant number 42401425, by the General Program of National Natural Science Foundation of

669 China, grant number 42465006, by the National Key Research and Development Program of China, grant  
670 number 2020YFA0608702 and by the Science and Technology Research Program of Institute of  
671 Mountain Hazards and Environment, Chinese Academy of Sciences, grant number IM-HE-CXTD-03.

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