Supplement to 'Exploring the applicability of Censored Shifted Gamma Distribution (CSGD) error model to radar based rainfall nowcasts: A UK case study'

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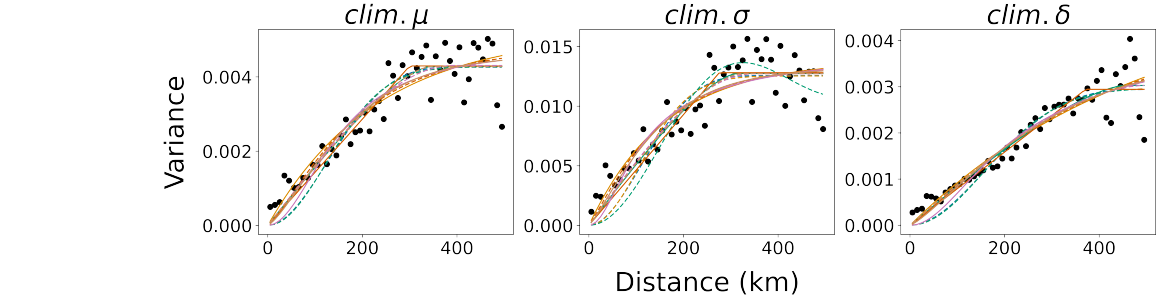
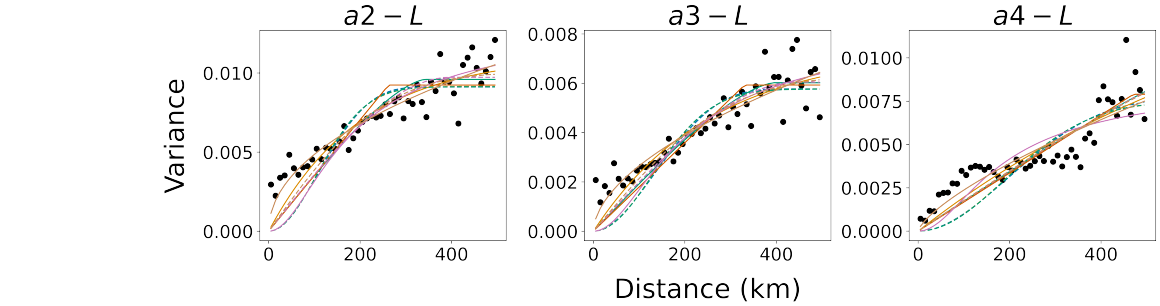
This supplement outlines the strategy for identifying optimal interpolation models to estimate CSGD (Censored Shifted Gamma Distribution) parameters at ungauged locations. Two widely-used techniques are compared: Ordinary Kriging (OK) and Radial Basis Function (RBF). The selection criteria and key intermediate results are described below.

For OK interpolation, an appropriate variogram model must first be identified, as interpolation quality depends on how well the variogram represents the spatial dependence of parameter variability with distance. Twelve commonly-used theoretical models were tested for fitting variograms of the climatological, linear, and non-linear conditional CSGD parameters. As shown in Figure S1, most fitted variograms plateau within 500 km, supporting the decision to truncate the distance threshold at 500 km to reduce outlier influence. The corresponding goodness-of-fit statistics, including R^2 and weighted least squares (WLS), are summarised in Tables S1 and S2. For each parameter, the variogram models with the highest R^2 and the lowest WLS were shortlisted as candidates for OK interpolation.

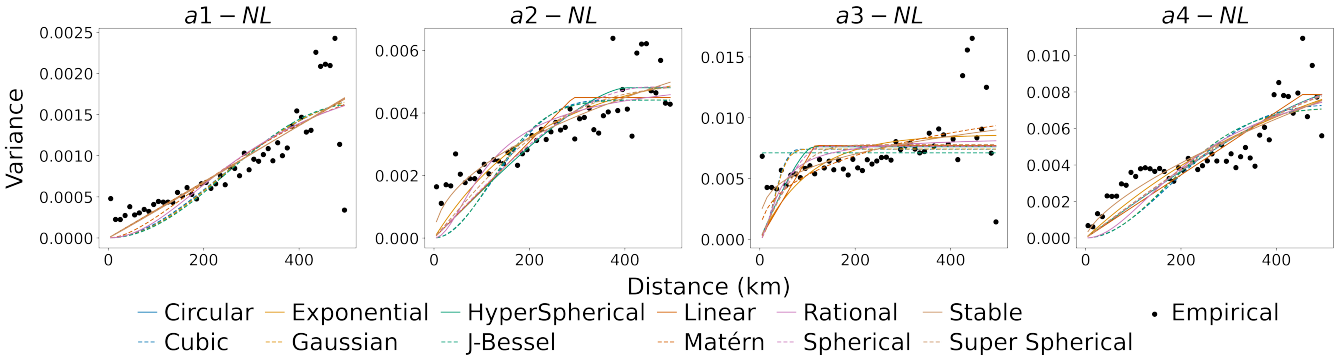
These OK-based candidate models were then compared against RBF interpolation. A suite of kernel functions was tested in RBF to assess the sensitivity of performance to kernel choice. Finally, as described in the main article, approximately 10% of rain-gauge stations (15 stations) were withheld for cross-validation. Mean absolute errors (MAEs) computed at these stations were used to rank the candidate models. As summarised in Table S3, the models yielding the lowest MAE for each CSGD parameter were selected to generate the final spatial fields of CSGD parameters.



(a)



(b)



(c)

Figure S1. Variogram fitting results for CSGD parameter sets using selected variogram models: (a) climatological, (b) linear conditional, and (c) nonlinear conditional parameters.

Table S1. Summary of R^2 values of the selected variogram models used in OK interpolation fitted to CSGD parameter sets.

Variogram model	Climatological			Linear conditional			Non-linear conditional			
	μ	σ	δ	α_2	α_3	α_4	α_1	α_2	α_3	α_4
Circular	0.87	0.81	0.88	0.90	0.86	0.79	0.68	0.78	0.35	0.78
Cubic	0.87	0.81	0.89	0.89	0.86	0.80	0.70	0.79	0.36	0.77
Exponential	0.84	0.77	0.87	0.90	0.85	0.78	0.67	0.78	0.35	0.78
Gaussian	0.87	0.80	0.89	0.89	0.86	0.80	0.70	0.78	0.36	0.77
Hyper Spherical	0.87	0.81	0.88	0.90	0.86	0.79	0.68	0.78	0.35	0.78
J-Bessel	0.65	**0.85**	**0.89**	0.86	**0.87**	0.80	**0.71**	**0.79**	0.37	0.77
Linear	**0.88**	0.82	0.89	0.90	0.86	0.78	0.68	0.78	**0.38**	0.77
Matérn	0.86	0.80	0.89	0.89	0.86	0.80	0.70	0.78	0.36	0.77
Rational	0.87	0.80	0.88	0.90	0.86	0.80	0.70	0.78	0.36	0.77
Spherical	0.87	0.81	0.88	0.90	0.85	0.79	0.68	0.78	0.35	0.78
Stable	0.87	0.80	0.89	**0.90**	0.86	**0.80**	0.70	0.79	0.36	**0.78**
Super Spherical	0.87	0.81	0.88	0.90	0.86	0.78	0.68	0.78	0.35	0.78

Table S2. Summary of WLS statistics of the selected variogram models used in OK interpolation fitted to CSGD parameter sets.

Variogram model	Climatological			Linear conditional			Non-linear conditional			
	μ	σ	δ	α_2	α_3	α_4	α_1	α_2	α_3	α_4
Circular	16.3	234.6	4.3	38.9	20.7	64.3	2.6	17.6	116.3	61.1
Cubic	16.1	246.2	3.7	42.3	19.4	53.2	2.0	17.7	114.8	56.3
Exponential	21.6	300.4	5.0	**38.6**	21.3	65.2	2.8	17.6	117.6	61.2
Gaussian	16.5	254.8	3.7	42.9	19.4	**52.7**	2.0	17.8	113.9	**56.2**
Hyper Spherical	16.3	234.6	4.3	38.9	20.7	64.3	2.6	17.6	116.3	61.1
J-Bessel	46.9	220.5	**3.4**	57.0	19.5	56.3	2.1	18.0	116.0	56.3
Linear	**14.3**	**216.9**	3.8	40.4	**19.3**	61.7	2.6	17.9	115.7	58.6
Matérn	16.9	249.7	3.6	41.1	19.3	52.7	2.0	17.8	**113.4**	56.2
Rational	16.6	255.7	3.7	39.8	19.4	52.8	**2.0**	17.8	113.9	56.2
Spherical	17.4	246.9	4.7	38.8	21.0	64.3	2.6	17.5	116.3	61.1
Stable	16.5	254.8	3.7	38.8	19.4	55.3	2.0	**17.5**	113.9	57.3
Super Spherical	16.3	234.5	4.3	38.7	20.7	64.7	2.7	17.5	116.6	61.5

Table S3. MAE of alternative spatial interpolation methods applied to the CSGD parameter sets.

Interpolation method		Climatological			Linear conditional			Non-linear conditional			
		μ	σ	δ	α_2	α_3	α_4	α_1	α_2	α_3	α_4
OK	*argmax* R^2	0.10	0.23	0.42	**0.06**	2.15	**0.06**	0.12	2.27	0.58	**0.06**
	argmin WLS	0.10	0.39	0.42	0.06	48.25	0.41	0.27	**0.04**	**0.08**	0.44
RBF	Cubic	0.05	0.06	0.04	0.09	0.06	0.07	0.03	0.07	0.10	0.07
	Gaussian	0.13	0.47	0.13	0.23	0.80	0.54	0.04	0.15	1.02	0.56
	Inverse	0.13	0.47	0.13	0.23	0.79	0.54	0.04	0.15	1.02	0.56
	Linear	**0.03**	**0.05**	**0.02**	0.06	**0.04**	0.07	0.02	0.05	0.09	0.07
	Multiquadric	0.03	0.05	0.02	0.06	0.04	0.07	**0.02**	0.05	0.09	0.07
	Quintic	0.05	0.08	0.08	0.13	0.08	0.08	0.03	0.09	0.12	0.09
	Thin plate	0.04	0.05	0.03	0.07	0.05	0.07	0.02	0.05	0.09	0.07