

1 Supporting Information for

2 **Advancing Isotope-Enabled Model for Comprehensive Understanding of**  
3 **Atmospheric Sulfur Isotope Effects: Revealing the Overlooked Isotopic Fractionation**  
4 **During Combustion and Gas Desulfurization**

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19 This supporting information document contains 19 pages with detailed descriptions of CTM  
20 and model configuration, the algorithm framework of the isotopic chemistry module, the  
21 derivation of the stable isotopic composition of a reservoir in open systems, additional data, 2  
22 tables, 3 figures, and references.

23 **Text S1 The description of CTMs and model configuration.** The three-dimensional  
 24 atmospheric chemical transport model (CTMs) solves the continuity equation for the  
 25 chemical species for representing its entire cycle in the atmosphere, including  
 26 emission, transport (e.g. diffusion, advection), chemical production/loss, dry and wet  
 27 deposition processes. The general Eulerian form of the continuity equation can be  
 28 expressed mathematically by

$$29 \quad \frac{\partial \bar{c}}{\partial t} = -\nabla \cdot (\bar{c}\bar{U}) + \nabla \cdot (K\nabla \cdot \bar{c}) + \left[\frac{\partial \bar{c}}{\partial t}\right]_{Emis} + \left[\frac{\partial \bar{c}}{\partial t}\right]_{Chem} - \left[\frac{\partial \bar{c}}{\partial t}\right]_{Dry} - \left[\frac{\partial \bar{c}}{\partial t}\right]_{Wet}$$

30 where  $\bar{c}$  is the time-averaged concentration of the chemical species of interest. The  
 31 term on the left side is the temporal change of concentration. The first two terms on  
 32 the right side are advection, and turbulent diffusion of the time-averaged flux, the last  
 33 four terms represent the emission intensity, time-averaged rate of chemical  
 34 interactions (including production and loss process), dry deposition, and wet scavenge.  
 35  $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$  is the gradient vector,  $U = (u, v, w)$  is the local wind velocity vector,  
 36 and  $K$  is a 3x3 matrix with zero values for the non-diagonal elements and with diagonal  
 37 elements  $K_x, K_y, K_z$  representing the turbulent diffusion coefficients in each transport  
 38 direction.

39 The model is initialized using gridded meteorological and emission input. The  
 40 meteorological field is processed through the interpolation of NCEP FNL (Final)  
 41 Operational Global Analysis and forecast dataset on  $0.25^\circ \times 0.25^\circ$  grids, employing the  
 42 WRF (Weather Research and Forecast model, version 3.9). Anthropogenic emissions,  
 43 including  $SO_2, NO_x, NH_3, CO, VOCs$ , black carbon, and organic carbon, are obtained  
 44 from the fusion of Hemispheric Transport of Air Pollution version-2 (HTAPv2) at a  $1^\circ$   
 45 horizontal resolution for the year 2010 (Janssens-Maenhout et al., 2015), and the  
 46 Multi-resolution Emission Inventory for China (MEIC) at a  $0.1^\circ \times 0.1^\circ$  horizontal

47 resolution for the year 2015 (Li et al., 2017). The emission inventory for the Chinese  
48 region is overwritten with the MEIC. The online mineral dust is based on the wind  
49 erosion model developed by Wang et al. (2000) and the sea salt emissions  
50 parameterizations are referenced from Athanasopoulou et al. (2008), respectively.

51 The Rapid Radiative Transfer Model for General Circulation Model (RRTMG) scheme  
52 is employed for the computation of longwave and shortwave atmospheric radiative  
53 fluxes and heating rates (Pincus et al., 2003). The Madronich F-TUV (Fast Tropospheric  
54 Ultraviolet-Visible) scheme is utilized to estimate the photolysis rate. The WSM3  
55 scheme (Hong et al., 2004) is selected for microphysics schemes. For the surface layer  
56 and boundary layer evaluations, the Monin-Obukhov scheme (Monin and Obukhov,  
57 1954) and the Yonsei University (YSU) scheme (Hong et al., 2006) are chosen,  
58 respectively.

59 The Carbon Bond Mechanism version Z (CBM-Z) is employed for the gas-phase  
60 chemistry scheme (Zaveri and Peters, 1999). The aqueous chemistry module is taken  
61 from Regional Acid Deposition Model (RADM2) (Stockwell et al., 1997), including sulfur  
62 chemistry and a simple mechanism for wet scavenging. The dry deposition process is  
63 parameterized through the calculation of deposition velocity, employing the resistance  
64 model approach developed by Zhang et al. (2003). Recently, the Global Nested Grid  
65 Air Quality Prediction Model System (GNAQPMS) has been coupled with the CAS earth  
66 system model (CAS-ESM) (Wei et al., 2019). It shows good performance on the global  
67 budgets and spatial distribution of major atmospheric species, as validated against  
68 field observations and other model results. The oxidation processes of S(IV) to S(VI)  
69 involve gas-phase oxidation by SO<sub>2</sub> and OH radical, as well as aqueous phase oxidation  
70 through dissoluble S(IV) with O<sub>3</sub>, H<sub>2</sub>O<sub>2</sub>, NO<sub>2</sub>, and proxy acetic acid (CH<sub>3</sub>OOH), along

71 with transition metal-catalyzed O<sub>2</sub> in cloud water. The oxidation of S(IV) species by  
72 dissolved NO<sub>2</sub> is also incorporated into the sulfur aqueous chemistry, referring to the  
73 kinetic parameters proposed by Cheng et al. (2016). The bulk cloud water pH is  
74 iteratively calculated online using the electroneutrality equation, considering  
75 concentrations of anions (sulfate, sulfite, nitrate, carbonate, formic acid) and cations  
76 (NH<sub>4</sub><sup>+</sup>, Ca<sup>2+</sup>, Mg<sup>2+</sup>, Mn<sup>2+</sup>, Fe<sup>3+</sup>). Ion concentrations are determined based on Henry's  
77 law, dissociation equilibrium and ionization processes. Neutralized cations are mainly  
78 contributed by dust aerosol, with natural dust materials assumed to contain 10 wt%  
79 of Ca<sup>2+</sup>, 3 wt% of Mg<sup>2+</sup>, 0.5 wt% of Mn<sup>2+</sup> and 0.5 wt% of Fe<sup>3+</sup> (Wang et al., 2002). The  
80 gas–particle partitioning of inorganic aerosols and aerosol water content is simulated  
81 using the improved thermodynamic equilibrium module ISORROPIA II (Fountoukis and  
82 Nenes, 2007; Song et al., 2018). Refer to Table S2 for a comprehensive description of  
83 the model configuration.

84 The current implementation of the isotopic chemistry module relies on several  
85 assumptions. (1) The pollutants and cloud water are uniformly distributed within the  
86 grid cell. (2) During wet scavenging and dry deposition, no isotopic fractionation occurs,  
87 the small mass change due to isotopic substitution is insignificant relative to the mass  
88 of particle, therefore, we apply the same  $K_j$  to <sup>32</sup>S and <sup>34</sup>S due to their similar molecular  
89 mass. (3) The model does not account for variations in isotopic composition from  
90 different emission sources. Instead, we focus on model's ability to reproduce the  
91 isotope effect, thus the isotopic composition of SO<sub>2</sub> and sulfate from emission sources  
92 are simply set to 0‰. The treatment does not influence the comparison between  
93 simulated and observed sulfur isotope effect during oxidation processes.

94

95 **Text S2 The algorithm framework of isotopic chemistry module**

96 We initially track the net change of SO<sub>2</sub> and sulfate after gas-phase, heterogeneous,  
97 and cloud&aqueous chemistry module separately, and then determine the reaction  
98 fraction *f* of SO<sub>2</sub> with each specific oxidation pathway during every time step ( $\Delta t$ ) to be  
99 used as input for the isotope chemistry module. At each time step, the temperature  
100 and the temperature-dependent isotopic fractionation factor at the grid cell are  
101 updated.

102 To reduce the effect of time-dependent Rayleigh equation with a large reaction  
103 fraction. The iterative time-splitting technique is employed. Within each sub-timestep,  
104 the reaction fraction is set at 1%, and a repetition involves adding equal portions of a  
105 fresh mixture. Utilizing the input mass of sulfur-containing isotopologues <sup>32</sup>SO<sub>2</sub> and  
106 <sup>34</sup>SO<sub>2</sub> at the last time step as initial value, we calculate the isotope ratio of <sup>32</sup>SO<sub>4</sub><sup>2-</sup> and  
107 <sup>34</sup>SO<sub>4</sub><sup>2-</sup> in the product, as well as the isotope ratio of <sup>32</sup>SO<sub>2</sub> and <sup>34</sup>SO<sub>2</sub> in the remaining  
108 reactant based on Rayleigh equations. We then derive the net mass change of <sup>32</sup>SO<sub>2</sub>,  
109 <sup>34</sup>SO<sub>2</sub>, <sup>32</sup>SO<sub>4</sub><sup>2-</sup> and <sup>34</sup>SO<sub>4</sub><sup>2-</sup> using the isotopic mass balance at each sub-timestep, as  
110 expressed in the following equation.

$$\Delta MC(^lX)_t = \left( MC(^hX)_{S_{t-1}} - R(^hX/^lX)_{S_t} \times MC(^lX)_{S_{t-1}} \right) / \left( R(^hX/^lX)_{P_t} - R(^hX/^lX)_{S_t} \right)$$

111 
$$\Delta MC(^hX)_t = R(^hX/^lX)_{P_t} \times \Delta MC(^lX)_t$$

112 Where  $\Delta MC(^hX)_t$  and  $\Delta MC(^lX)_t$  represent the change in molar mass of  
113 isotopologues containing heavy <sup>*h*</sup>X and light isotope <sup>*l*</sup>X of element X at time t during  
114 the chemical process, respectively.  $MC(^hX)_{S_{t-1}}$  and  $MC(^lX)_{S_{t-1}}$  represent the  
115 molar mass of reactant *S* containing heavy <sup>*h*</sup>X and light isotope <sup>*l*</sup>X of element X at  
116 time t-1, respectively.

117 During each sub-timestep, the isotope ratio of the substrate is updated with the new  
 118 mixing, and the isotope ratio of the accumulated product is weighted with the molar  
 119 mass and isotope ratio of the product at each sub-timestep. An iterative loop is  
 120 executed for the above steps.

121 **Text S3 The derivation of stable isotopic composition of a reservoir in open systems.**

122 We consider a reservoir that loses material due to any physicochemical process with  
 123 isotopic fractionation at a constant temperature. At any instant, the stable isotope  
 124 ratio is given by

$$125 \quad R = \frac{N^*}{N} \quad \dots(S1)$$

126 Taking the logarithm of both sides of eq. (S1)

$$127 \quad \ln R = \ln N^* - \ln N \quad \dots(S2)$$

128 After differentiating, eq. (S2) becomes

$$129 \quad \frac{dR}{R} = \frac{dN^*}{N^*} - \frac{dN}{N} = \frac{dN^*}{NR} - \frac{dN}{N} \quad \dots(S3)$$

130 At any instant, let  $dN_e^*$  and  $dN_e$  are being lost and  $dN_r^*$  and  $dN_r$  are being added to the  
 131 reservoir. So, the net change in  $N^*$  and  $N$  is

$$132 \quad \begin{aligned} dN^* &= dN_r^* - dN_e^* \\ dN &= dN_r - dN_e \end{aligned} \quad \dots(S4), (S5)$$

133 Dividing eq. (S4) by eq. (S5)

$$134 \quad \frac{dN^*}{dN} = \frac{dN_r^* - dN_e^*}{dN_r - dN_e} = \frac{\frac{dN_r^*}{dN_r} \times \frac{dN_r}{dN_e} - \frac{dN_e^*}{dN_e}}{\frac{dN_r}{dN_e} - 1} = \frac{\beta R_r - \alpha R}{\beta - 1} \quad \dots(S6)$$

135 Where  $\alpha$  is defined as  $(dN_e^*/dN_e)/(N^*/N)$ , and  $\beta$  is  $dN_r/dN_e$ , the ratio of the  
 136 amount of material added to that lost.

137 Substituting the value of  $dN^*$  from eq. (S6) to eq. (S3)

138 
$$\frac{dR}{R} = \left( \frac{\beta R_r - \alpha R}{\beta - 1} \right) \times \frac{dN}{NR} - \frac{dN}{N} = \frac{dN}{N} \times \left( \frac{\beta R_r}{\beta - 1} - \frac{\alpha}{\beta - 1} - 1 \right)$$
 ... (S7)

139 or

140 
$$\frac{dR}{\left( \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R \right)} = \frac{dN}{N}$$
 ... (S8)

141 Taking integration on both sides

142 
$$\frac{1}{-\left( \frac{\alpha}{\beta - 1} + 1 \right)} \int \frac{d \left\{ \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R \right\}}{\left\{ \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R \right\}} = \int \frac{dN}{N}$$
 ... (S9)

143 The condition for integrating this equation is

144 
$$\frac{\beta R_r}{\beta - 1} \neq \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R, \text{ it gives that } \frac{dN_r}{dN_e} \neq \left( \frac{\alpha - 1}{R_r - 1} \right)$$

145 At  $\beta=1$ , this equation is not solvable, hence first we take the case where  $\beta \neq 1$ .

146 Case 1: on integrating eq. (S8), we get

147 
$$\frac{1}{-\left( \frac{\alpha}{\beta - 1} + 1 \right)} \times \ln \left\{ \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R \right\} = \ln N + k$$
 ... (S10)

148 After applying boundary conditions, at  $t=1, N=N_0, R=R_0$ , we evaluate  $k$  and substitute

149 in eq. (S10); we get

150 
$$\frac{1}{-\left( \frac{\alpha}{\beta - 1} + 1 \right)} \times \ln \left\{ \frac{\left( \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R \right)}{\left( \frac{\beta R_r}{\beta - 1} - \left( \frac{\alpha}{\beta - 1} + 1 \right) \times R_0 \right)} \right\} = \ln \frac{N}{N_0} = \ln f$$
 ... (S11)

151 Here  $f$  is the fraction of the material left in the reservoir relative to the initial amount,

152 which is defined as  $(N^* + N) / (N_0^* + N_0)$

153 Further simplifying eq. (S11) becomes

154 
$$R = R_0 f^{-\left( \frac{\alpha}{\beta - 1} + 1 \right)} + \frac{\beta}{\alpha + \beta - 1} \times R_r \times \left[ 1 - f^{-\left( \frac{\alpha}{\beta - 1} + 1 \right)} \right]$$
 ... (S12)

155 or

156  $R = R_0 f^\rho + \frac{\beta}{\alpha + \beta - 1} \times R_r \times [1 - f^\rho]$  ... (S13)

157 Here  $\rho = -\left(\frac{\alpha}{\beta - 1} + 1\right)$

158 Substituting  $\beta=0$ , eq. (S13) converts to the simple Rayleigh equation, which cross-  
 159 checks the derivation.

160  $R = R_0 f^{(\alpha-1)}$  ... (S14)

161 Case 2: When  $\beta=1$ ,  $dN = 0$  hence from eq. (S5),  $dN_r = dN_e$ . Equation (S3) becomes

162  $dR = \frac{dN_r^* - dN_e^*}{N} = R_r \frac{dN_r}{N} - \alpha R \frac{dN_e}{N} = \left(\frac{R_r - \alpha R}{N}\right) dN_r = \left(\frac{R_r - \alpha R}{N_0}\right) dN_r$  ... (S15)

163 After rearranging and integrating, this becomes

164  $-\frac{1}{\alpha} \int \frac{d(R_r - \alpha R)}{R_r - \alpha R} = \frac{1}{N_0} \int dN_r$  ... (S16)

165 After solving fully and applying boundary conditions as in the previous case, we get

166  $R = R_0 \exp[-(\alpha N_r / N_0)] + (R_r / \alpha) \times \{1 - \exp[-(\alpha N_r / N_0)]\}$  ... (S17)

167 Here  $N_r$  indicates the total number of lighter isotopic molecules added thus far.

168 **Table S1 Compiled observation data of  $\delta^{34}\text{S}$  values of coal, sulfate aerosol, and**  
 169 **wet precipitation across different Chinese provinces**

| Province       | $\delta^{34}\text{S}$ of coal | $\delta^{34}\text{S}$ of wet precipitation | $\delta^{34}\text{S}$ of aerosol |
|----------------|-------------------------------|--|----------------------------------|
| Beijing        | 7.4±16.1                      | 4.7±1.8                                    | 6.1±1.3                          |
| Tianjin        |                               |  | 4.1±1.6                          |
| Inner Mongolia | 0.8                           |  |                                  |
| Heilongjiang   | 3.2±1.8                       |  | 6.4                              |
| Liaoning       | 9.4±3.2                       |  | 5.9                              |
| Jilin          | 1.7                           |  | 8.1                              |
| Hebei          | 3.5±6.6                       |  |                                  |
| Henan          | 6.4±1.0                       | 2.1±2.3                                    |                                  |
| Shandong       | 7.6±2.9                       |  |                                  |
| Gansu          | 7.3±3.5                       |  |                                  |
| Zhejiang       | 4.0±1.5                       |  | 6.2±2.7                          |
| Jiangsu        | 3.8±1.3                       |  | 5.5±1.0                          |
| Anwei          | 3.8                           |  |                                  |
| Fujian         |                               |  | 3.5±1.1                          |
| Chongqing      |                               | 4.3±1.1                                    |                                  |
| Jiangxi        | -4.4±1.3                      |  |                                  |
| Sichuan        | 8.8±1.3                       | 3.9  |                                  |
| Shaanxi        | 7.9±4.6                       | 12.3±1.1                                   |                                  |
| Shanxi         | 12.1±10.8                     |  |                                  |
| Hubei          | 12.1                          |  |                                  |
| Hunan          | -6.6±5.7                      |  |                                  |
| Guangdong      | 9.6±1.0                       |  | 5.2±1.6                          |
| Guizhou        | -7.5±0.1                      | -2.8±9.8                                   | 1.2                              |
| Yunnan         | -0.7                          |  |                                  |

170 The dataset (Mean±std.) are compiled from references.

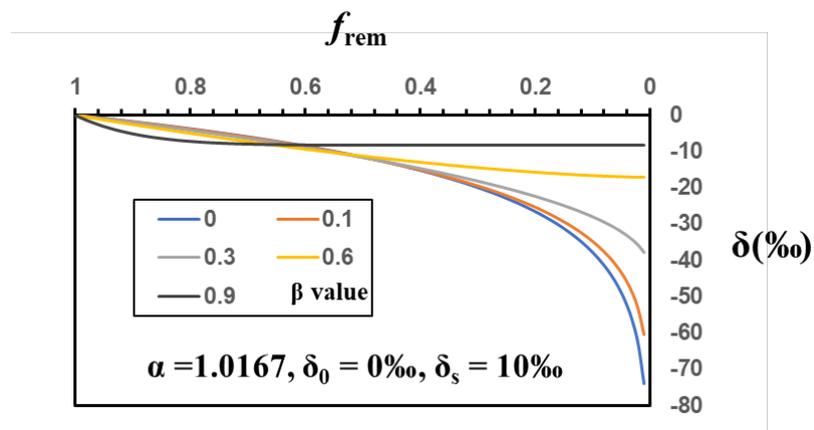
171 The compiled data from various Chinese provinces are cited from sources such as Beijing (Fan et  
 172 al., 2020; Wei et al., 2018; Han et al., 2016b; Hong et al., 1993; Maruyama et al., 2000; Ohizumi et  
 173 al., 1997; Mukai et al., 2001; Han et al., 2016a; Zhu et al., 2016), Tianjin (Han et al., 2022; Ding et  
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184 et al., 2001; Xiao et al., 2009; Xiao et al., 2014), Yunnan (Hong et al., 1993)  
185

**Table S2 The detailed information of model configurations**

| Module  | Parameterizations/schemes  |
|---|--|
| Input global analysis data                        | NCEP GDAS/FNL data   |
| Gas-phase chemical mechanism                      | CBM-Z mechanism (Zaveri and Peters, 1999)  |
| Aqueous-phase chemical mechanism and wet scavenge | RADM mechanism (Chang et al., 1987)  |
| Anthropogenic emissions                           | HTAPv2 and MEIC  |
| Dust, sea salt                                    | wind erosion model (Wang et al., 2000)<br>sea salt online parameterization (Athanasopoulou et al., 2008) |
| Photolysis scheme                                 | Madronich F-TUV  |
| Microphysics scheme                               | WSM3 (Hong et al., 2004)   |
| Longwave radiation scheme                         | RRTM (Mlawer et al., 1997)   |
| Shortwave radiation scheme                        | Dudhia scheme (Dudhia, 1989)   |
| Surface layer                                     | Monin-Obukhov (Janjic) scheme (Monin and Obukhov, 1954)  |
| Land surface schemes                              | NOAH (Ek et al., 2003)   |
| Planetary boundary layer physics                  | YSU scheme (Hong et al., 2006)   |
| Cumulus   | Kain-Fritsch scheme (Kain and Kain, 2004)  |
| Vertical layers                                   | 20   |

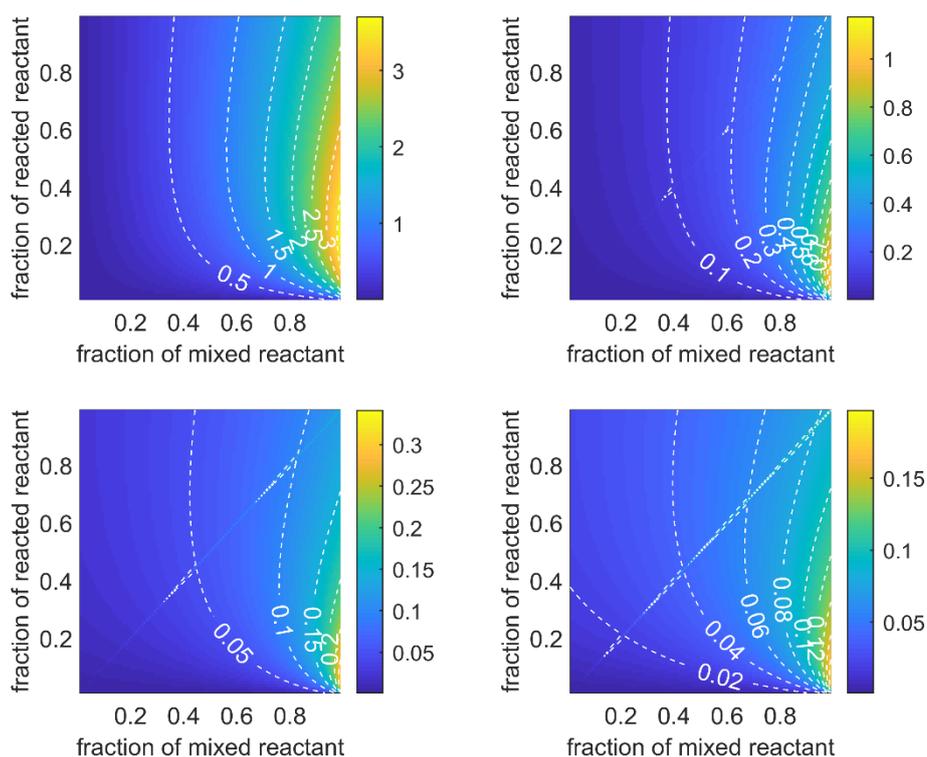
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191 **Figure S1.** An example illustrating the change in isotopic composition ( $\delta$ ) of a reservoir  
192 related to the remaining fraction ( $f_{rem}$ ) of the original material, considering various  
193 values of  $\beta$ , which represents the ratio of the instantaneous amount added to the  
194 removal during an infinitesimal time interval  $\Delta t$  ( $\beta \neq 1$ ). The isotopic composition of the  
195 external source ( $\delta_s$ ) is assumed to be 10‰, and the fractionation factor is set to 1.0167.

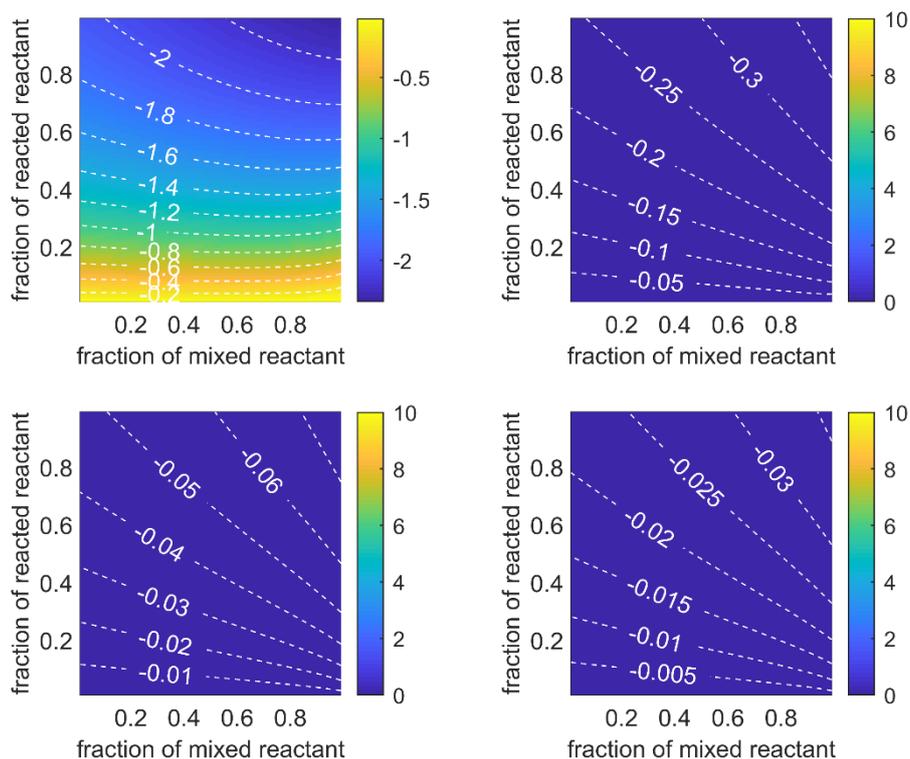
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197

198 **Figure S2.** Simulation deviation of the isotopic composition of reservoir for varying  
 199 sub-steps (a) 1, (b) 10, (c) 50, and (d) 100, relative to reference simulation with  
 200 integration method in the open system associated with simultaneous reaction and  
 201 mixing processes. Each simulation involves the addition of a compound with an initial  
 202  $\delta$ -value of 10.0‰ to the substrate (initially  $\delta = 0\text{‰}$ ), while the chemical reaction  
 203 exhibits an isotopic effect with a fractionation factor of  $\alpha = 1.0167$ . The fractions of  
 204 added and removed products range from 1% to 99% of the initial amount of substrate  
 205 and are uniformly divided at every time sub-step. The color scheme and contour lines  
 206 illustrate the  $\delta$ -value differences between the respective simulations and the  
 207 referenced simulation.

208



209

210 **Figure S3.** Simulation deviation of the isotopic composition of the product for varying  
 211 sub-steps (a) 1, (b) 10, (c) 50, and (d) 100, relative to reference simulation with 1000  
 212 sub-steps. Each simulation involves the addition of a compound with an initial  $\delta$ -value  
 213 of 10.0‰ to the substrate (initially  $\delta = 0\text{‰}$ ), while the chemical reaction exhibits an  
 214 isotopic effect with a fractionation factor of  $\alpha = 1.0167$ . The fractions of added and  
 215 removed products range from 1% to 99% of the initial amount of substrate and are  
 216 uniformly divided at every time sub-step. The color scheme and contour lines illustrate  
 217 the  $\delta$ -value differences between the respective simulations and the referenced  
 218 simulation.

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