Reply to Referee #1 (Ms. Qin Huang)

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Thank you very much for your detailed review of our manuscript and for providing valuable feedback. Below, we have listed your questions and comments (*italicized*), followed by our responses and proposed revisions to the manuscript (shown in violet). We believe that these changes improve the clarity and quality of our work.

1. DA method comparison - This study uses LETKF for data assimilation, following Sun et al. (2023), which makes sense for a high-dimensional system like L96. Several other studies in this area (e.g., Miyoshi & Sun 2022; Kawasaki & Kotsuki 2024; Nagai et al. 2024) use EnKF, particularly with the lower-dimensional L63 model. It could be helpful to briefly clarify the rationale for choosing LETKF here, and short comments on whether the assimilation method affects control outcomes (even qualitatively) could be of interest to readers.

We use the LETKF for data assimilation for the following reasons: (i) as you mentioned, the LETKF reduces computational costs for high-dimensional systems such as the Lorenz 96 model, mainly because it performs assimilation locally and operates in a low-dimensional ensemble space, avoiding expensive full-space computations and (ii) since we plan to use the SCALE-LETKF in future work, adopting the LETKF here provides a smoother transition. **In the revised version, we will include the first reason in the first paragraph of Section 2.2.**

Control method comparison - While the manuscript positions the proposed method as a bottom-up alternative to top-down strategies like MPC, it does not include direct performance comparisons. A brief discussion of how the approach compares relative to recent MPC-based or other CSE studies could help contextualize its contributions. If direct comparisons are not feasible, outlining conceptual trade-offs or implementation differences would help clarify the novelty and practical significance.

Thank you for pointing this out. We agree that our manuscript does not provide a direct performance comparison between our bottom-up strategy and top-down approaches such as Model Predictive Control (MPC). However, directly comparable MPC-based studies with the Lorenz 96 model have not been published yet. Therefore, we briefly outline the conceptual trade-offs and implementation differences here. Our bottom-up approach may be particularly suitable when intervention options are limited or discrete, while top-down methods like MPC are generally more effective when intervention options are continuous or nearly unlimited. In addition, our method is relatively simple to implement and offers interpretability, whereas MPC methods, though often more computationally intensive, can provide mathematically optimal control solutions under specified constraints. In the revised paper, we will summarize these advantages and limitations in the Summary and Discussion section.

Optimal control - The current method selecting intervention scenarios by minimizing the maximum ensemble outcome is effective but not optimal. While other top-down strategies determine inputs using optimization minimizing costs, this utilizes limited intervention criteria.

Yes, we agree that our bottom-up approach is not optimal. Top-down approaches typically determine optimal control inputs under cost constraints. Therefore, if sufficient computational resources and time are available, control-theoretic methods such as MPC would indeed be the most effective. Our study instead offers a complementary or backup approach for cases where computational power is limited.

Perturbation magnitude - In Fig. 4 (one-site) and Fig. 8 (two-site), the reported perturbation magnitudes (642.1 and 674.9, respectively) seem quite large. It would help to clarify the units or scale used, are these relative to system variability, or absolute values in state units?

Thank you for this comment. Since the perturbation magnitude (i.e., intervention energy) is a time-integrated quantity, it is more useful to consider the intervention size. The intervention size, u, is itself dimensionless but can be compared with the other terms in Eq. (1). In fact, the intervention sizes, $u = 1.6$ for Fig. 4 as well as $u = 2.0$ for Fig. 8, are not particularly small when compared with the parameter $F = 8$ or the typical range of the state variable, $-12 \lesssim X_i \lesssim 16$ (Fig. 1a). On the other hand, a typical displacement caused by the intervention during the time step of $\Delta t = 0.01$ is $u \times \Delta t = 0.02$ for $u = 2.0$. This is not particularly large when compared to the typical range $-12 \lesssim X_i \lesssim 16$. In Sun et al. (2023), the typical size of one-step displacement is $\alpha D_0 = 0.2 \times 0.1989 = 0.03978$ (see their Fig. 5). Therefore, the intervention size used in this study is comparable to, or smaller than, that used in the previous work. **We will mention these points in the revised manuscript**.

Also, Fig. 4 references "operation energy" - what exactly does this refer to?

We apologize for the confusion. The quantity “operation energy” refers to the intervention energy associated with changes in the intervention. However, it was mistakenly shown only in the figure and is no longer used in the present version of the manuscript. **We will delete it in the revised manuscript.**

The average number of changes (22.4, 22.1) are shown as non-integers, since interventions are presumably discrete in time, why are these fractional? Is this an average over ensembles or multiple trials?

Yes exactly, the average number of changes is shown by a fractional because it is an average over ten 100-y segments.

Overall, these suggestions are meant as optional additions - the manuscript is already very complete and well-structured. Including a bit more comparative context could further enhance clarity for readers unfamiliar with the broader control and data assimilation literature.

Thank you for your warm and helpful comments. We will improve the clarity of the paper in the revision.

References

Sun, Q., Miyoshi, T., and Richard, S.: Control simulation experiments of extreme events with the Lorenz-96 model, Nonlinear Process. Geophys., 30, 117–128, https://doi.org/10.5194/npg-30-117-2023, 2023.