Multiscale statistical analysis of thermal and non-thermal components of seawater pCO_2 in the western English Channel: scaling, time-reversibility, and dependence

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Reply to Referees' Comments

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We thank the Associate Editor and reviewer for their time and commitment throughout the review process. Please find our responses to the concerns below. The reviewer's comments are shown in blue, and the additions made to the manuscript are highlighted in green.

Reviewer's comment

While I appreciate the considerable effort the authors have devoted to revising this manuscript, I find their response to my first question unsatisfactory. The authors claim to have analyzed five years of data; however, Table 1 clearly shows that the primary variable, pCO_2, contains over 60 % missing data. Such a substantial data gap raises serious concerns about the robustness and interpretability of the statistical analyses presented. The authors must more thoroughly address these limitations and assess their potential impact on the study's overall conclusions.

In their reply, the authors argue that the statistical methods applied remain valid even with missing values. However, this response does not sufficiently address the core concern - namely, how much the results might differ if a more complete or longer-term dataset were available. The authors further contend that "because the dataset is relatively large and we are not estimating inferred statistics over very long temporal scales (e.g., greater than one year), the results remain robust, with averages computed over large numbers of observations." This justification is unconvincing and appears overly simplistic. Oceanic state variables such as pCO_2 are strongly influenced by seasonal variability, suggesting that results could change considerably if data from all months or additional years were included. Moreover, interannual variations - potentially driven by anthropogenic influences - may further affect the observed patterns and trends.

We did not fully understand the reviewer's comments on this subject during the previous round. We hope we now have a better understanding of the reviewer's reservations, and that they will find our revision more satisfactory. In our response here, we separate seasonal and interannual variations.

With regard to interannual variability, we agree with the reviewer that the results obtained could be different if a longer database were available, and that interannual variations, with their potential anthropogenic influences, have an impact on the results. However, this is the case for all studies conducted over a given period: it is clear that we do not have data for another period to know what the interannual influences are. It is therefore clearly not possible for us to quantify this, but we have added comments on this subject in the conclusion, to remind readers that interannual variations, with their potentially different forcings, could have an influence on the reported results.

This has been added in the text: "The statistics reported here may depend on the forcing associated with interannual variability and its potential anthropogenic forcing. Further analyses, performed over different periods of time, will be needed in future works, in order to compare with the results presented here. Also, longer..." (lines 379–381) and "although each time scale is represented by sufficient statistics, the data are not fully homogeneous on a monthly basis. Appendix A shows that December and March were undersampled compared with other months. At the seasonal scale, however, the distributions are more balanced. This temporal heterogeneity represents one limitation of our analysis and highlights the need for future comparisons with datasets containing fewer missing values. Appendix B examines the effects of missing data on the analysis of PDF quotients, restricting the computation to observations for which all parameters were simultaneously available. The influence of missing values is limited in this case, and the results are broadly consistent with those presented in the main analysis." (lines 386–392).

Although the title of the study emphasizes that five years of data were analyzed, the assumption that statistical convergence has been achieved (lines 201-202) is not adequately substantiated. The authors should clarify the criteria used to determine that the available dataset is sufficiently large and representative to support robust statistical inference.

We have added a new Appendix that addresses the issue of missing values and the representativeness of statistics at sub-annual scales. First, despite the fact that there are missing values, some of the methods we use employ increments between two measurement points separated by a given interval. All intervals are thus visited, and statistics are presented for all intervals. The attached figure, which we have added to this Appendix, shows the number of time increments at scale τ for all scales between 1 day and 365 days. We can see that there are between about 4,000 and 30,000 values considered depending on the scale. This allows us to have relatively good confidence in the convergence of the statistics for all these scales. This has been adjusted in the text: "Further discussion of this issue, including details on the number of data points at each scale, is provided in Appendix A." (lines 91–92).

However, we agree with the reviewer on the potential importance of seasonal variability, and we understand that it is also necessary to verify that each season is well represented in the sampling, so as to avoid giving more statistical importance to one season over another. Figure B below shows the number of values present in the analyzed time series, by month of the year and by season. We can see that certain months, such as March, November, and December, are less frequently present than October or September. This difference becomes less pronounced when we consider seasonal granularity, where the four seasons are relatively more balanced. This is the potentially most significant impact of these missing values, and we have mentioned it both in the Appendix on missing values and in the conclusion.

Here is the new Appendix section: "Most of the analysis methods employed in this study account for the issue of missing values, a common problem when working with high-frequency in situ observations. For instance, spectral analysis, time-reversal symmetry analysis, and the PDF quotient of increments all incorporate the temporal variability of $p\mathrm{CO}_2$ by considering differences between observations separated by a given time scale τ. This approach ensures that, despite occasional data gaps—sometimes extending over relatively long periods—a sufficient number of samples remains available at each time scale. Figure A1 *(Fig. A in this letter)* illustrates the number of $p\mathrm{CO}_2$ increments available for all considered scales between 30 minutes and 365 days. Despite approximately 60 % missing data in the time series (the highest rate among our variables; Table 1), more than 3900 temporal increments are available for timescales shorter than one year. This provides sufficient statistical support for our analyses, which rely predominantly on mean values and are restricted to these timescales.

On the other hand, the distribution of the available $p\mathrm{CO}_2$ data over the five-year period, shown as a function of month and season in Fig. A2 *(Fig. B in this letter)*, is not perfectly homogeneous. Some months, such as December and March, are under-sampled relative to others, which likely results in an under-representation of their dynamics in the aggregated statistics. At the seasonal scale, the distribution is more balanced, although autumn contributes proportionally more data than the other seasons. This sampling structure should be considered when interpreting the results. A full assessment of its influence would require a continuous time series covering the entire period." (Appendix A; lines 425–439).

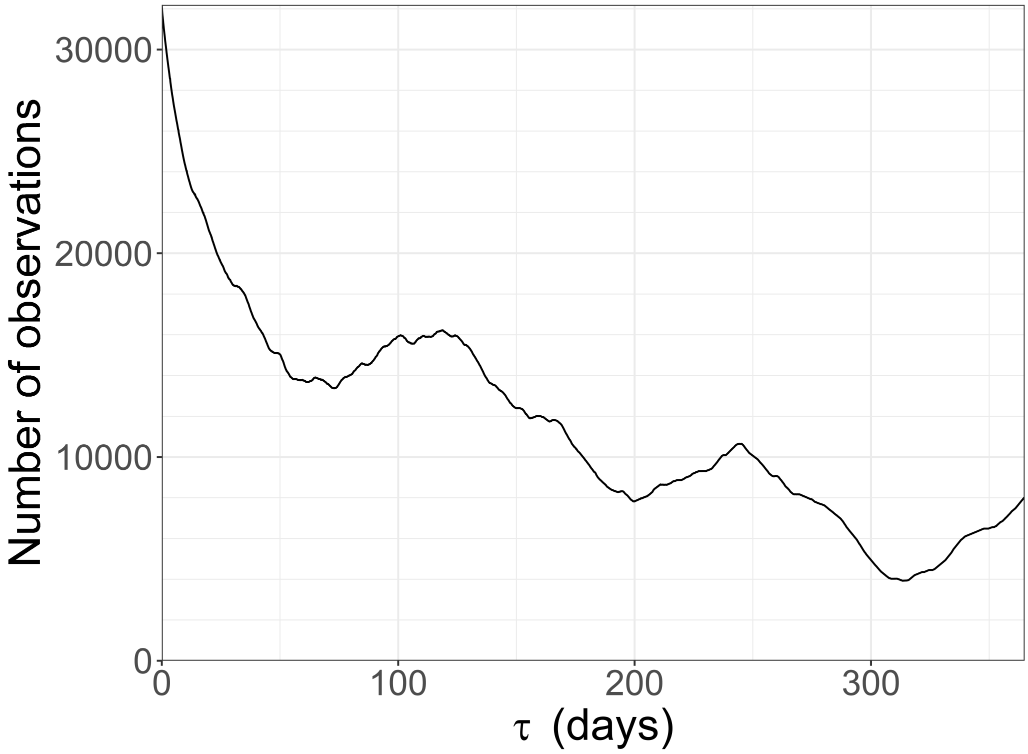

Figure A: Number of observed temporal increments of $p\text{CO}_2$ as a function of the considered timescale τ.

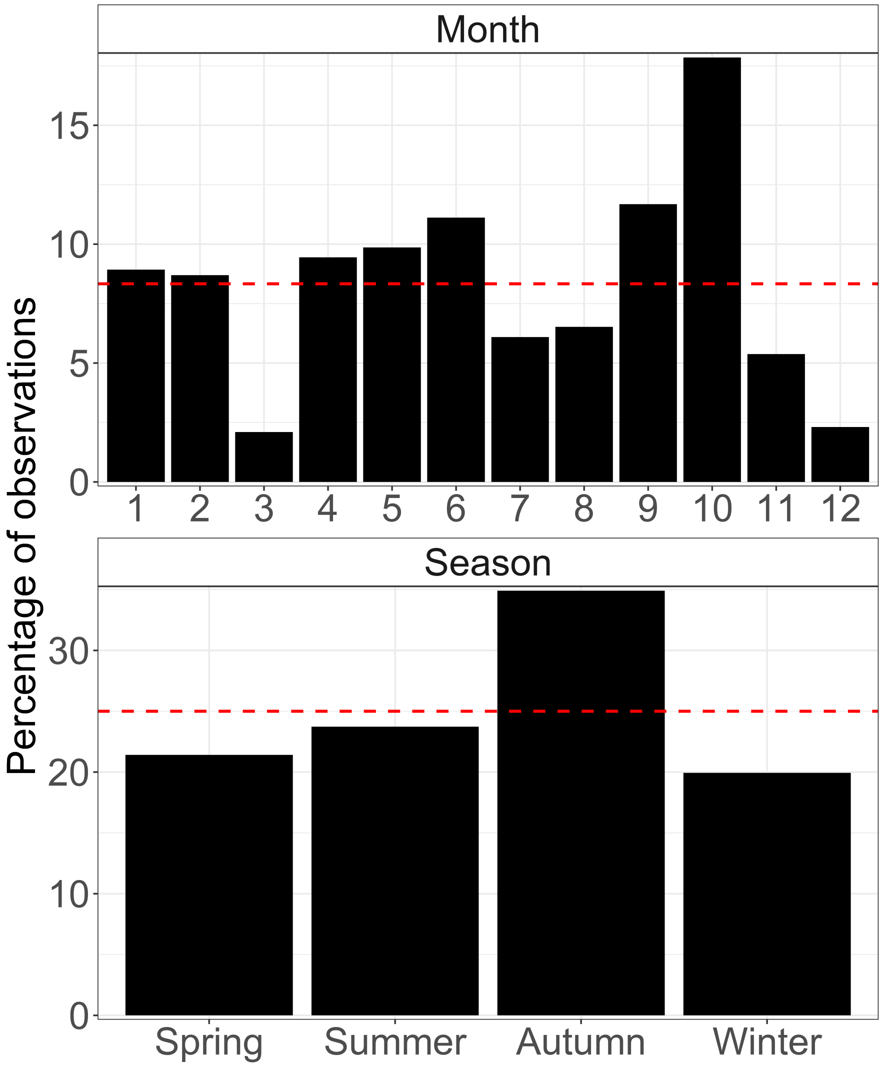

Figure B: Distribution of the available pCO_2 data by month and season. The red dashed line represents the expected values for a uniform sampling.