I went back to Mancktelow and Grasemann (1997) where it is stated that no solution exists to the heat conduction-advection equation if one assumes that the temperature increases linearly with depth when depth goes to infinity. I agree with this. However, the suitable solution that we seek here is that corresponding to a given heat flux coming from the mantle, i.e., at some depth (the thickness of the lithosphere?). So this correspond to imposing a temperature gradient at a given depth (without making any assumption about what happens below). In the Earth the base of the lithosphere may be considered as a boundary between two domains, one where conduction dominates over advection and the other where advection dominates over conduction.

The point I made in my earlier review of this (very good) manuscript is that there exists an analytical solution to the steady-state conduction-advection heat equation with a basal heat flux (or gradient) condition. The equation that I provided was quite cryptic. Below I give it in a clearer form and demonstrate how it can be derived. I do so because I think this is an important (and general) point to make for our quantitative interpretation of thermchron data.

The conductive-advective heat transport equation at steady-state represents a balance between two terms, one representing advection of heat by rocks travelling towards the surface and the other representing their conductive cooling:

$$-v\frac{\partial T}{\partial z} = \kappa\frac{\partial^2 T}{\partial z^2} \tag{1}$$

where T is temperature, z is depth, v is rock advection velocity towards the surface and κ heat diffusivity.

Assuming the following boundary conditions:

$$T(0) = 0 \text{ and } \frac{\partial T}{\partial z}(L) = G \tag{2}$$

we can express the same equation in dimensionless form:

$$-P_e\frac{\partial T'}{\partial z'} = \frac{\partial^2 T'}{\partial z'^2} \tag{3}$$

$$T(0) = 0 \text{ and } \frac{\partial T'}{\partial z'}(1) = 1 \tag{4}$$

using the following variables:

$$z' = z/L,\ T' = T/GL \text{ and } P_e = \frac{vL}{\kappa} \tag{5}$$

The general solution to this second-order equation is:

$$T(z') = C_1 e^{-P_e z'} + C_2 \tag{6}$$

Adding the surface boundary condition:

$$T(0) = 0 \rightarrow C_1 + C_2 = 0 \tag{7}$$

yields the following form:

$$T(z') = C_1(1 - e^{-P_e z'}) \tag{8}$$

Adding the second boundary condition:

$$\frac{\partial T'}{\partial z'}(z' = 1) = 1 \rightarrow C_1 = \frac{1}{P_e e^{-P_e}} \tag{9}$$

we obtain the steady-state solution in dimensionless form:

$$T(z') = \frac{1}{P_e} \frac{1 - e^{-P_e z'}}{e^{-P_e L}} \tag{10}$$

or in its original (dimensional) form:

$$T(z) = \frac{G\kappa}{v} \Big(\frac{1 - e^{-vz/\kappa}}{e^{-vL/\kappa}} \Big) \tag{11}$$

which satisfies the initial equation and the two boundary conditions.

In the following figure, I show a numerical solution to the transient equation:

$$\frac{\partial T}{\partial t} - v\frac{\partial T}{\partial z} = \kappa \frac{\partial^2 T}{\partial z^2} \tag{12}$$

assuming an initial condition at conductive steady-state:

$$T(z) = \frac{z\ T_l}{L} \tag{13}$$

where $T_l = 500°\text{C}$, $L = 30$ km, $\kappa = 25$ km/Myr^2, $v = 1$ km/Myr and $G = T_l/L$. I used an implicit finite difference scheme to solve this equation. I show ten steps during the transient stage, as well as the initial (conductive) solution and the analytical steady-state solution derived above for comparison. We see that the numerical solution does indeed converge towards the analytical solution as time increases.

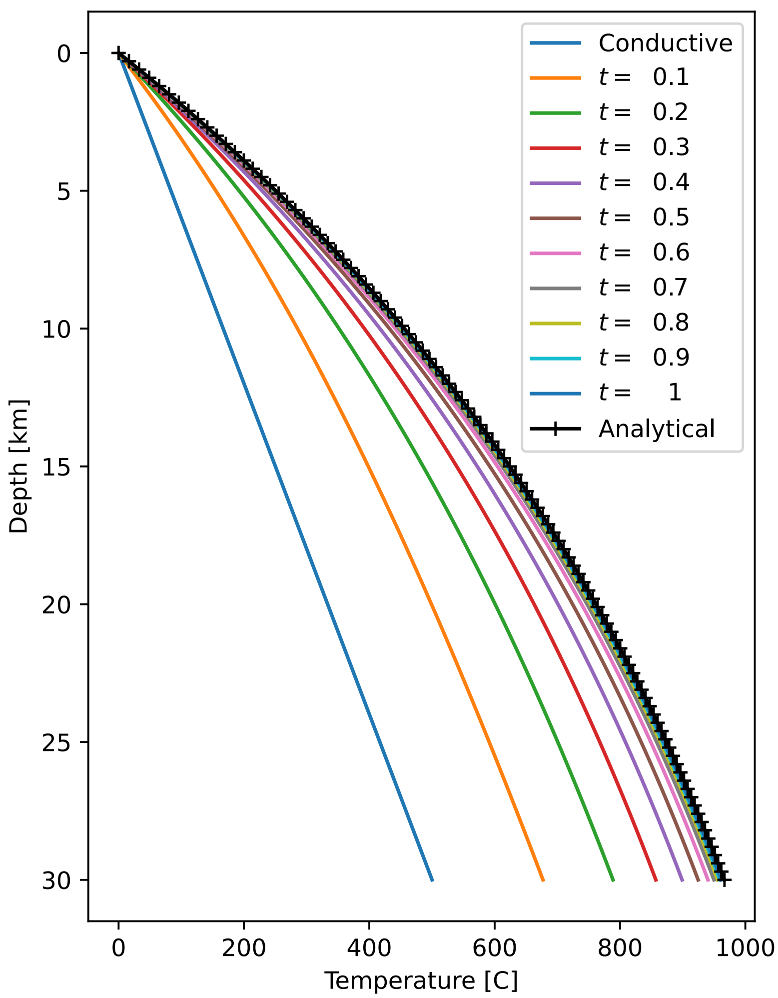

Figure 1: Comparison between numerical transient solution (colored lines) and analytical steady-state solution (black line with crosses).