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# Determining the depth and pumping speed of the equatorial Ekman layer from surface drifter trajectories

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**Abstract**

Trajectories of more than 500 drogued surface drifters launched since 1979 in the equatorial ocean are analyzed by employing the results of a new Lagrangian theory of wind-driven transport along the equator forced by the prevailing Trade winds. The analysis yields robust estimates of 45 meters for the Ekman layer's depth and 1.0 meters/day for the upwelling speed of deep water into the layer.

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## 1. Introduction

The Trade winds that blow westward in the Tropics as part of the Hadley circulation are the first and basic component of the heat transport from the warm equatorial surface ocean to the cold poles, that mitigates the overall pole-to-equator temperature gradient on Earth. The mechanism that enables the poleward time-independent heat transport is the surface flow in the ocean that is directed 90° to the right/left of the wind in the northern/southern hemisphere relative to the direction of the overlying wind. This counter-intuitive flow direction results from Earth's rotation that adds the Coriolis force to the stress applied by the winds at the ocean surface. This straightforward scenario of wind-driven ocean circulation appears in all textbooks (Knauss, 1996; Talley et al., 2011) but despite its convincing simplicity, currently, no quantitative estimates are available for the parameters that control it.

The classical theory that describes the ocean response to forcing by the overlying winds was developed about 120 years ago by V.W. Ekman under the assumption of constant Coriolis frequency (Ekman, 1905). This assumption greatly simplifies the analysis by ensuring that all coefficients in the governing equations are constant. The dynamics described in Ekman's theory includes a steady flow perpendicular to the wind direction and inertial, i.e. force-free, oscillations at the local (constant) Coriolis frequency. This is in sharp contrast to the equatorial region where the Coriolis frequency vanishes at the equator and varies (linearly) with latitude, which turns the equations nonlinear so the oscillation-free flow is not steady as in Ekman's original mid-latitude theory. The poleward directed surface flows in both hemispheres along the equator imply a strong horizontal divergence along the equator which can only be balanced by the upwelling of deeper water into the wind-forced, Ekman, layer. Though the heuristic application of the mid-latitude Ekman theory to the vicinity of the equator is quite straightforward, to-date, the heuristic application could not be employed to estimate either the depth of the equatorial Ekman layer or the rate of upwelled volume of water.

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The complications that result from the inclusion of the meridional variation of the Coriolis frequency in Ekman's theory were recently resolved in a theory of wind-driven flow in which Ekman's 1905 classical theory was extended to the equatorial region (Paldor, 2024). This new theory employs the adiabaticity method (Goldstein, 1980; Paldor and Friedland, 2023) to filter out the oscillation of a water column under the sole action of the meridionally varying Coriolis force from the slow and monotonic poleward motion of a water column forced by the combination of the wind stress and the Coriolis force. The essence of the method is the formulation of the problem as the dynamics of the motion of a (quasi-)particle about the minimum of a potential while the potential itself varies with time on a slower time scale than the period of oscillations about the minimum. The wind-driven dynamics along the equator can be transformed to this special form only by substituting the pseudo angular momentum for the zonal velocity component in the governing nonlinear equations (Paldor, 2024).

Direct observations of the depth (thickness) of the equatorial Ekman layer and the rate of upwelling water to it are not available due to the poor observational definition of the layer and the extremely low speed of upwelling. In contrast, observations of drifter trajectories are both abundant and accurate so they are used in the present study to estimate these important features of the equatorial Ekman layer using the new theory of wind-driven transport in the equatorial ocean. The application of this theory to drifter observations and the data used in this study are detailed in Section 2. In Section 3 we give the results obtained by applying the theory to drifter trajectories and the study is summarized in Section 4.

## 2. Theory and Data

### 2.1. Theory

The recent extension of the wind-driven theory of ocean circulation to the equator described in Paldor (2024) has demonstrated that, as in Ekman's original theory, the oceanic response can be decomposed into a monotonic, slow, flow (which is directed poleward in the equatorial region) and fast, large

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amplitude, oscillations. In contrast to Ekman's original theory, in the equatorial region when the wind stress is directed westward the oscillations are highly nonlinear and of large amplitude (see Fig. 2 of Paldor, 2024). This new theory is applied in the present study to the trajectories of surface drifters by considering the dimensional counterpart of the non-dimensional expression derived in Eq. 9 of Paldor (2024) for the oscillation-free latitudinal motion of a water column or drifter launched near the equator:

$$\frac{dy}{dt} = \frac{1}{y(t)} \frac{-\tau^x}{H} \left(\frac{R_e}{2\Omega\rho}\right). \quad (1)$$

Here, y(t) is the distance from the equator at time $t$. The global parameters in this relation are: $\rho = 1027\ \mathrm{kg\ m^{-3}}$ (water density) $\Omega = 7.29 \cdot 10^{-5}\ \mathrm{s^{-1}}$ and $R_e = 6371 \cdot 10^3\ \mathrm{m}$ (Earth's rotation frequency and radius, respectively) so $\left(\frac{R_e}{2\Omega\rho}\right) = 4.25 \cdot 10^7\ \mathrm{m^4\ s\ kg^{-1}}$. The remaining, particular, parameters are: $\tau^x$ - the wind stress (units: $\mathrm{N\ m^{-2}}$; negative for easterly winds) and $H$ (m) - the Ekman layer's depth.

Multiplying the nonlinear relation (1) by $y(t)$ and integrating the resulting 1st order equation for $y(t)^2$ yields:

$$y(t)^2 = y(0)^2 + 2\frac{-\tau^x}{H} \cdot \left(\frac{R_e}{2\Omega\rho}\right) t. \quad (2)$$

Equation (2) can be applied to observations of drifter trajectories by inverting it to the following explicit expression for $H$:

$$H = \left(\frac{R_e}{2\Omega\rho}\right) \frac{2(-\tau^x)}{L^2 - y_i(0)^2} t_i, \quad (3)$$

where $y_i(0)$ is the distance of drifter #$i$ from the equator at $t = 0$ (i.e. the distance of the launch point from the equator) and $t_i$ is its travel time to $L = y(t_i)$, the final distance from the equator. The values of $H$ are calculated from this equation for each drifter and then averaged to yield the mean value for the particular value of $L$.

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### 2.2. Drifter trajectories

Nearly 30,000 surface drifters were released from 1979 at the ocean surface (Lumpkin et al., 2017) and the geographical trajectories of these drifters are tracked by satellites every 6 hours for periods of up to 1000 days. These (Lagrangian) observations cover the global ocean and a few percent of them were launched on both sides of the equator in the Pacific, Atlantic and Indian oceans. The slightly negatively buoyant drifter is typically drogued at 15-meter depth, so it provides an estimate of the current in the top 15 meters of the water column where the wind stress is the primary forcing (Lumpkin et al., 2017). The agreement between drifter trajectories and ocean currents demonstrated in Lagerloef et al., (1999) motivates the analysis of observed trajectories of surface drifters in order to determine the depth of the equatorial Ekman layer and the pumping (upwelling) speed of deep water into it.

The drifter trajectories used in the analysis were collected and made freely available by the NOAA Global Drifter Program (NOAA/GDP). The data were screened according to the following three criteria:

1. They were launched within $1^{\circ} \approx 110\ \mathrm{km}$ south or north of the equator (regarded as the equator).
2. The drifters remained in one hemisphere throughout the entire travel time to the final latitude (equator crossing is not allowed under westward directed wind stress).
3. The drifters were continuously tracked, with gaps no longer than one day, during their motion from the launch point to the final latitude that marks the boundary of the equatorial region (i.e. $3^{\circ} \approx 330\ \mathrm{km}$, $4^{\circ} \approx 440\ \mathrm{km}$ or $5^{\circ} \approx 550\ \mathrm{km}$).

The latitudes $3^{\circ}$ and $4^{\circ}$ were used in previous studies to define the boundaries of the equatorial region (Brady and Bryden, 1987; Lagerloef et al., 1999; Johnson et al., 2001) but in the present study we also used $L = 5^{\circ} \approx 550\ \mathrm{km}$ to verify the robustness of the calculated averages to the selected values of $L$. For $L = 2^{\circ} \approx 220\ \mathrm{km}$ the singularity at $L^2 \rightarrow y_i(0)^2$ in Eq. (3) yields highly erratic and too high value of $H$.

Of the nearly 30,000 drifter trajectories archived in AOML archive as of 8/2024, over 1500 drifters reached the final latitude ($3^{\circ}, 4^{\circ}$ or $5^{\circ}$) and out of them about 700 drifters remained in one hemisphere.

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The number of drifters in the Atlantic and Pacific oceans that reached each of the final latitude is given in the 2nd column of Table 1 that includes, in addition, the mean launch distances of $y_i(0)$ (3rd column) and mean travel times of $t_i$, (4th column) to the final latitudes (noted in the rows of this table). The Indian Ocean is excluded from the analysis due to its positive mean annual wind stress (see Sect. 2.3).

### 2.3. Wind stress

The daily wind stress values over the oceans, $\tau^x$, used in this work are available at NOAA/CoastWatch site in $0.125°$ spatial resolution for the period 1999-2009. We calculated the averaged wind stress in the region of the Indian, Atlantic and Pacific oceans in a zonal strip that straddles the equator between $-L$ and $+L$, where $L$ corresponds to 3°, 4° or 5°. These averages are given in the 5th column of Table 1 for the Atlantic and Pacific oceans but not for the Indian ocean where the calculated mean values are positive (so $H < 0$) and small (less than $+0.01\ \mathrm{N\ m^{-2}}$) probably due to the strong seasonal forcing by the Monsoon system that induces eastward directed zonal winds throughout part of the year in this ocean (Hastenrath and Polzin, 2004; Zhang et al., 2022).

| $L$ (degrees) | Number of drifters | Mean $y_i(0)$ (degrees) | Mean $t_i$ (days) | $\tau^x$ $(Nm^{-2})$ | Mean $H$ (m) | $W = \frac{H}{t_i} \cdot \frac{L-y_i(0)}{L}$ (m/day) |
|---|---|---|---|---|---|---|
| 3.04 | 610 | 0.29 | 29.67 | -0.0261 | 51.10 | 1.56 |
| 4.04 | 576 | 0.29 | 43.43 | -0.0264 | 42.48 | 0.91 |
| 5.04 | 531 | 0.29 | 58.12 | -0.027 | 37.16 | 0.60 |

**Table 1: Drifter characteristics in the Pacific and Atlantic oceans and the zonal wind stresses there. The shown values of $L$ are larger by a few kilometers compared to the distances corresponding to $3°, 4°$ or $5°$ since a drifter is determined to be "at $L$" with an offset of up to 6 hours after its passage of that point. Less than 10% percent of the relevant drifters were launched in the Indian ocean which is not included in this table and in the analysis since the annual mean wind stress in the Indian Ocean is directed eastward, which is inconsistent with a poleward directed net motion.**

## 3. Results

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Four representative drifter trajectories are shown in Fig. 1 and they demonstrate the richness of observed trajectories near the equator, the intricate combination of oscillations with slow poleward propagation and the occurrence of equatorial crossing in many trajectories.

Substituting the values of $y_i(0)$ and $t_i$ for each drifter in Eq. (3) and averaging these values over all relevant drifters yields the values of $H$ given in the 6th column of Table 1. The histograms of the $H$ values for each value of $L$ are shown in Fig. 2 so the value of $H$ is best estimates by: $H = 44 \pm 7 \approx 45$ m.

Equation (2) can also be employed to calculate the poleward, oscillation-free, velocity of a drifter on its way from $y_i(0)$ to $L = y(t_i)$ from the drifter's average speed during its travel: $V = \frac{L-y_i(0)}{t_i}$. Thus, the volume divergence (per unit length in $x$) that results from the anti-parallel, poleward directed, volume fluxes of 2 water columns that are initially conjoined along the equator and move poleward is $2HV = 2H\frac{L-y_i(0)}{t_i}$. The vertical volume flux (per unit length) due to Ekman pumping during $t_i$ is: $2LW$, where $W$ is the pumping speed. Equating the vertical and horizontal fluxes yields $WL{=}H\frac{L-y_i(0)}{t_i}$ or $W = \frac{H}{t_i}\frac{L-y_i(0)}{L}$. The mean values of $H$ and $t_i$ in Table 1 then yield the mean value of $W \approx 1.0\ \mathrm{m/day}$ for the three values of $L$.

The estimated $H$ and $W$ values along the equatorial Atlantic and Pacific Oceans are noted in Fig. 3 on a qualitative, textbook, cartoon of the wind forcing and resulting oceanic flow patterns.

## 4. Summary and Discussion

The mean estimates $H = 45\ m$ and $W = 1\ m/day$ calculated here based on surface drifter trajectories are more robust compared to prior estimates derived from standard hydrographic observations. Table 1 shows that the present estimates of $H$ vary with $L$ by a few *meters* and those for $W$ by about 0.5 $\frac{m}{day}$. These variations are smaller than those of estimates based on standard hydrographic data that can vary by a factor of up to 3 (Wyrtki, 1981; Brady and Bryden, 1987; Lukas and Lindstrom, 1991; Weingartner

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and Weisberg, 1991). As an example, the $H = 45\ m$ value reported here exceeds the estimate of 30-40 m proposed in Lukas and Lindstrom (1991) but the *O*(10%) variation of the present estimate is significantly smaller than *O*(80%) variation in the latter estimate. In view of the crucial role played by the poleward flow of warm equatorial water in mitigating the large radiative pole-to-equator temperature gradient (Czaja and Marshal, 2006; Hartmann, 2016) a reliable quantification of the initiation of this flow is important for understanding Earth's climate.

The successful application of the new theory of wind-driven equatorial transport in the ocean developed in Paldor (2024) lends credence to the relevance of this theory to observations in the equatorial ocean. It can be argued that the calculation and successful application of the oscillation-free speed of poleward wind-driven motion on the equator is of similar significance to ocean dynamics as the development of the expression for the steady transport in the original $f$-plane Ekman theory.

**Author contribution**:

NP: Initiation of project, writing various drafts and theoretical analysis

YD: Data collection and analysis, editing and production of display items.

**Competing interests**: The authors declare that they have no conflict of interest.

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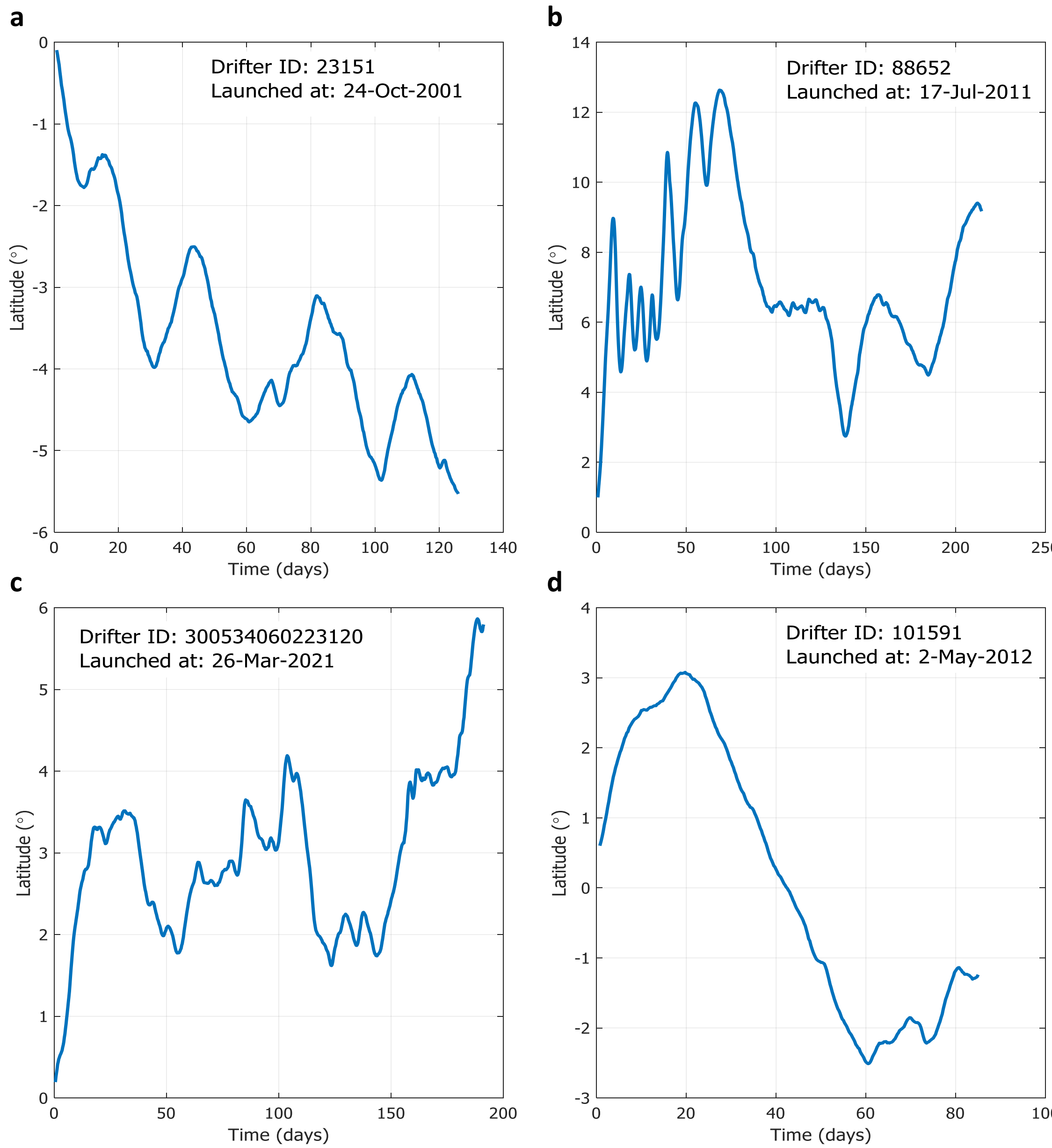


**Figure 1: Four drifter trajectories originating within $1^{\circ}$ of the equator analyzed in this study. a) A typical southern hemisphere trajectory that clearly shows oscillations and a mean poleward flow; b) A fast northern hemisphere trajectory that reaches $4^{\circ}$ in just a few days; c) A slow northern hemisphere trajectory that reaches $4^{\circ}$ in more than 100 days; d) A trajectory that reaches $3^{\circ}$ but not $4^{\circ}$ prior to crossing the equator so it is included in the analysis of $L = 3^{\circ}$ since it crosses the equator only after reaching $3^{\circ}N$.**

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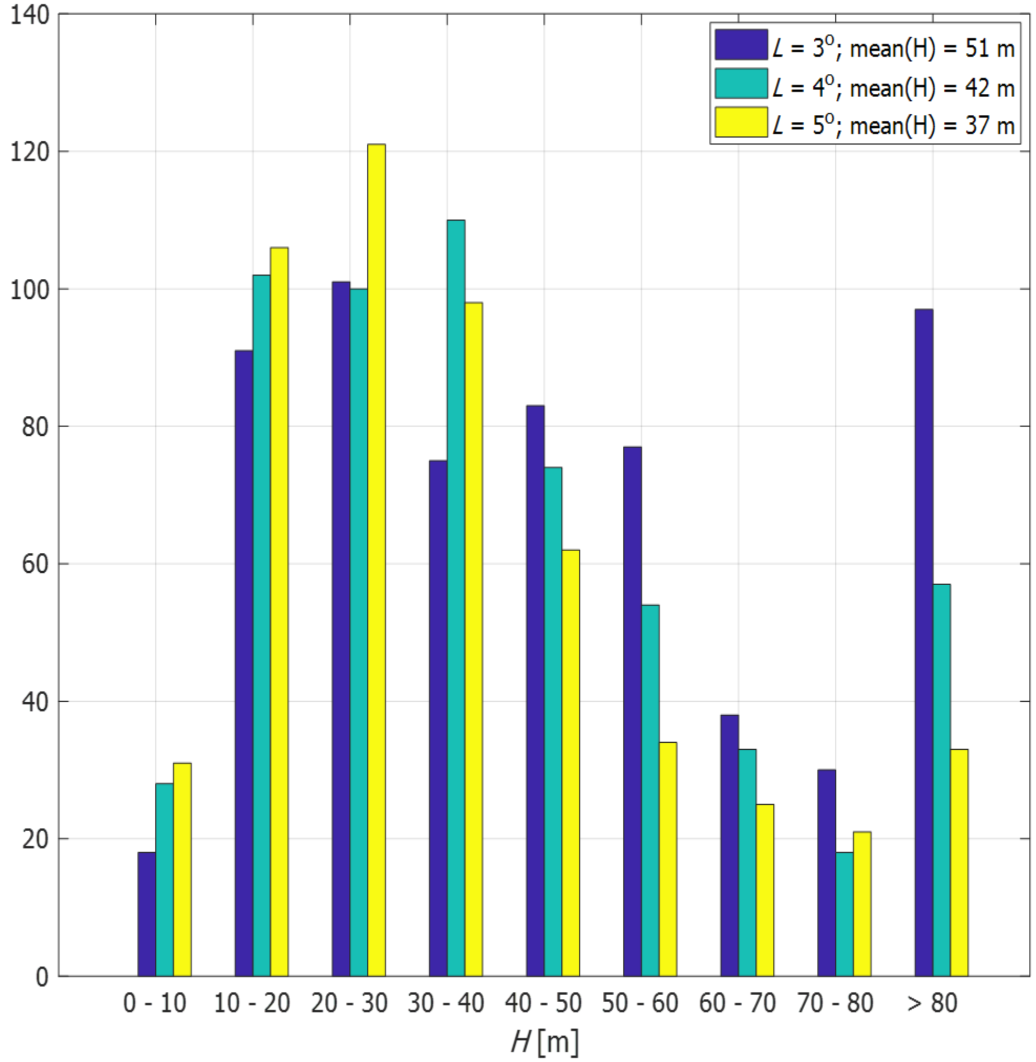


**Figure 2: The histograms of $H$-values for the 3 values of $L$. For $L = 3^\circ$ the tail of $H > 80$ m is as high as the maximum cell of $H = 20 - 30$ m consistent with singularity of Eq. (3) at $L^2 \approx y_i(0)^2$.**

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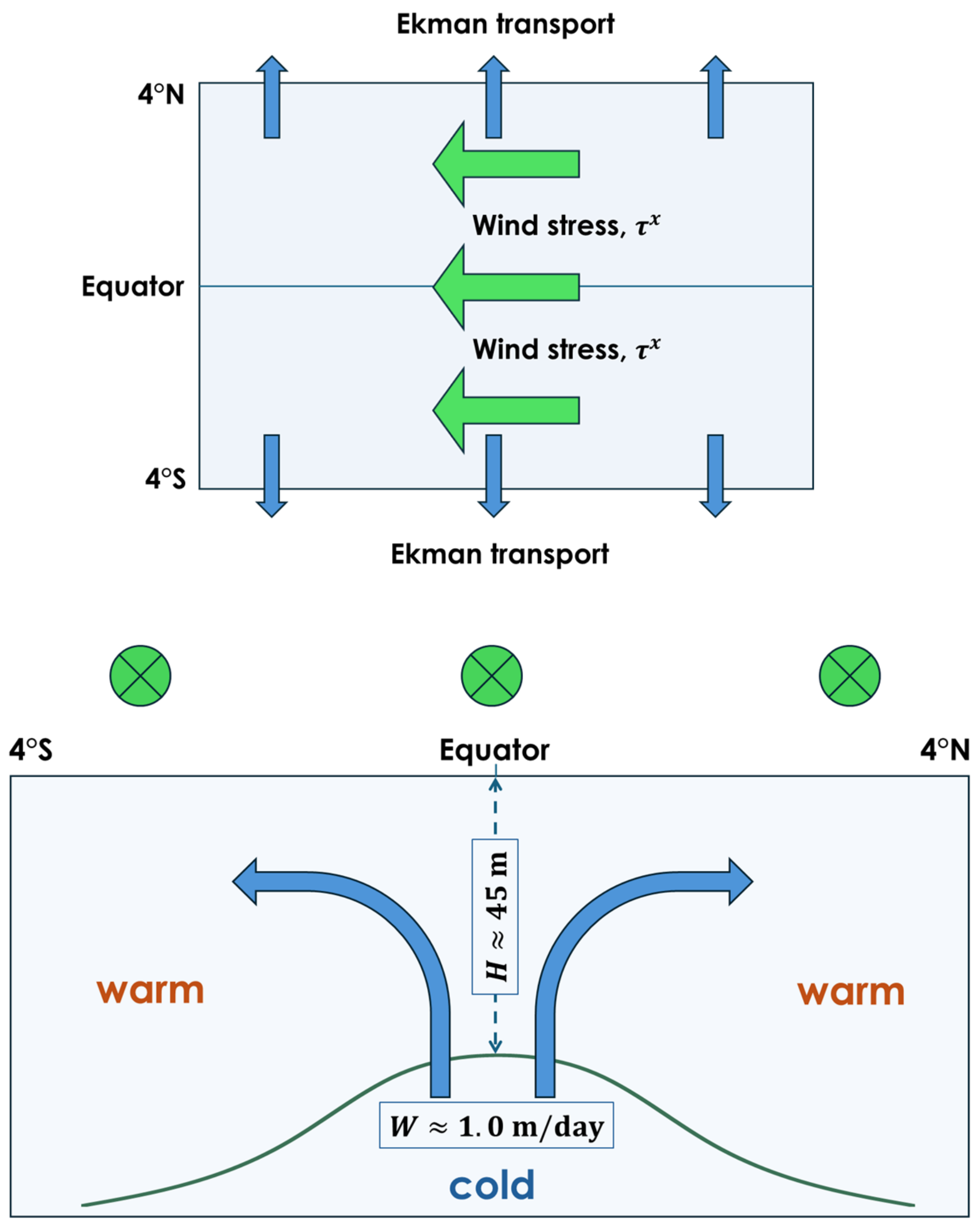


**Figure 3: A sketch relating the poleward directed wind-driven surface flow along the equator under westward directed wind stress (upper panel) which is compensated by the upwelling of water from below (lower panel). The upper panel is a planar view and the lower panel is a latitude-height cross-section viewed from the east. The $H \approx 45$ m and $W \approx 1.0$ m/day estimates are the main results of this study.**

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