The authors have worked on all of my comments and most adequately covered. Unfortunately, the key equation (now Eq. 9 former Eq. 6) is in my opinion still not correct. As the authors obviously do not take into account how the DOD(AOD) at CALIPSO is calculated.

Imagine a visually thin layer of dust that is missing an AOD of 0.15 – naturally, you need a larger LR to obtain the missing AOD (because there is less integrated backscatter). If, on the other hand, you imagine an optically thick dust layer that is missing 0.15, then it is sufficient to raise the LR only slightly to extract the missing AOD from the already large backscatter. It is not the absolute difference that matters, but the relative difference. I tried to explain/describe the issue in more detail below.

<u>Explanation of problem with updated lidar ratio which cannot simply be made from the difference of the AOD (DOD).</u>

The authors use:

$DOD_{Calipso} = \int_{z_{bottom}}^{z_{top}} \sigma(z)dz = LR\_Calipso \int_{z_{bottom}}^{z_{top}} \beta(z)dz$ (1)

$DOD_{diff} = DOD_i - DOD_{CALIPSO} = DOD_i - LR\_Calipso \int_{z_{bottom}}^{z_{top}} \beta(z)dz$ (2)

But as you see, the difference in DOD does not only depend on the LR but also on the Integral of the backscatter coefficient. Thus, it is NOT the same if you have a DOD_difference of 0.15 in case of an optical depth of 0.5 OR 2.0:

I try to explain this for the example of DOD_i =0.5 and DOD _difference 0.15:

$$DOD_i = 0.5\ with\ DOD_{diff} = 0.15 \rightarrow DOD_{Calipso} = 0.35$$

With your formula this yields to

$$LR_{updated} = LR_{CALIPSO} \cdot \left(1 + DOD_{diff}\right) = 50.6$$

BUT remember Eq. 1 (above):

$$\begin{aligned} DOD_{Calipso} = 0.35 = \int_{z_{bottom}}^{z_{top}} \sigma(z)dz \\ = LR\_Calipso \int_{z_{bottom}}^{z_{top}} \beta(z)dz \\ = 44 * \int_{z_{bottom}}^{z_{top}} \beta(z)dz \rightarrow \int_{z_{bottom}}^{z_{top}} \beta(z)dz = \frac{0.35}{44} = 0.008 \end{aligned}$$

With your updated LR and Eq. (1) you now get an **DOD of 50.6*0.008=0.4 which is NOT 0.5**

If you want to get the same DOD as measured by MIDAS or POLDER-3/GRASP you need to calculate ratios:

$LR_{updated} = LR_{Calipso} * \frac{DOD_i}{DOD_{Calipso}} = \frac{DOD\_i}{\int_{z_{bottom}}^{z_{top}} \beta(z)dz}$ (3)

$LR_{diff} = LR_{updated} - LR_{Calipso} = LR_{Calipso}(\frac{DOD_i}{DOD_{Calipso}} - 1)$ (4)

For the example chosen above it would mean

$$LR_{updated} = LR_{Calipso} * \frac{DOD_i}{DOD_{Calipso}} = \frac{DOD_i}{\int_{z_{bottom}}^{z_{top}} \beta(z)dz} = \frac{0.5}{0.008} = 62.5 \text{ sr}$$

Which is significantly different from the results of your approach you use.

This gives $DOD\_i = DOD\_Calipso$: **62.5*0.008=0.5**

The main relation which is given and you should use is:

$$\frac{LR_{updated}}{LR_{Calipso}} = \frac{DOD_i}{DOD_{Calipso}} \quad (5)$$

You can repeat the same exercise with an DOD_i of 2.0 and again a DOD _difference 0.15:

$$DOD_i = 2.0 \; with \; DOD_{diff} = 0.15 \rightarrow DOD_{Calipso} = 1.85$$

With your formula this yields to

$$LR_{updated} = LR_{CALIPSO} \cdot \left(1 + DOD_{diff}\right) = 50.6$$

BUT remember Eq. 1:

$$DOD_{Calipso} = 1.85 = \int_{z_{bottom}}^{z_{top}} \sigma(z)dz$$

$$= LR\_Calipso \int_{z_{bottom}}^{z_{top}} \beta(z)dz$$

$$= 44 * \int_{z_{bottom}}^{z_{top}} \beta(z)dz \rightarrow \int_{z_{bottom}}^{z_{top}} \beta(z)dz = \frac{1.85}{44} = 0.042$$

With your updated LR you now **get a DOD of 0.042*50.6=2.1275 which is NOT 2.0**

If you apply Eq. (3) and (4) for the example chosen above it would mean

$$LR_{updated} = LR_{Calipso} * \frac{DOD_i}{DOD_{Calipso}} = \frac{DOD_i}{\int_{z_{bottom}}^{z_{top}} \beta(z)dz} = \frac{2.0}{0.042} = 47.62 \text{ sr}$$

and thus

$$LR_{diff} = 3.6$$

And therefore, **completely different from you chosen approach**, but gives $DOD\_i = DOD\_Calipso$: 47.62***0.042=2.0**

Because of this issue, I have meanwhile doubt that the content of the paper is correct and thus publishable. Therefore, I hope the authors can comment on this!