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Dear GMD Editorial Board,

Here are our responses to the anonymous reviewer 1 comments our manuscript received from GMD. In bold are direct responses to the reviewers questions, and in blue are the section number and the text that was added/amended to address the reviewers comments in the manuscript.

[1] The manuscript should more clearly position GIC relative to commonly used operational stochastic schemes (e.g., SPPT, SKEB), explaining conceptual differences and complementary roles.

**Both SPPT and SKEB are choices of noise that is added to governing equations to account for subgrid processes. After choosing a stochastic parameterization with colored noise to add to a model the GIC is used to improve the convergence of the numerical time stepping scheme used to solve the stochastic governing equations**
Expanded [1]: Recent studies have focused on addressing this indeterminacy by introducing a stochastic element into parameterizations [**?**, **?**]. Numerous stochastic parameterization has become popular in operation models, for example Stochastically Perturbed Parametrization Tendencies (SPPT) Palmer et al. (2009) and Stochastic Kinetic Energy Backscatter (SKEB) Berner et al. (2009). Naively applying numerical time stepping methods developed for deterministic systems to systems with these stochastic parameterizations may produce non-physically relevant solutions. The most common discretizations in stochastic analysis are the Itô and Stratonovich interpretation, these interpretations result in different solutions to the same system. Similarly different numerical time stepping schemes may converge to these different solutions.

[2] Provide clearer implementation guidance for computing $\frac{dg}{du}g$ in complex models, including when symbolic differentiation, automatic differentiation, or numerical approximations are suitable.
**The form of $g[u]$ would determine what methods might be appropriate for computing $\frac{dg}{du}g$ and as there are no restriction on the forms $g[u]$ might take a wide variety of methods might be applicable to any choice of $g[u]$. To reflect this we have added a note to the manuscript about how the efficiency of using the GIC would be affected.**
Added a note to [3.4] that $\frac{dg}{du}g$ could be approximated and that the efficiency of this approximation would affect the efficiency of GIC vs using a higher order scheme

[3]Strengthen the explanation of the observed critical timestep behavior and convergence order transitions, ideally through a simplified stability or scaling argument.
**We are unaware of any scaling or stability argument that can explain the critical timestep behavior. It is our understanding that the addition of the GIC has a dissipative effect at each point in space which is analogous to smoothing the noise process in time**
[3.1]For non-zero $\alpha$, the addition of the GIC reduces the final error and increases the critical time step after which the scheme achieves the theoretic best convergence rate of 1.0. The critical time step is increased with the addition of the GIC, which acts as a dissipative term at each point in space, functioning similar to smoothing the noise in time. For time steps which do not resolve the

colored noise, the GIC smooths the solution which better represents the underlying smoothness of the colored noise.

[4] Add a complete parameter and configuration table for the 2D experiments, and ensure reproducibility instructions meet GMD standards. [3.1] added Table 1, and Table 2
Include a summary comparison table listing all numerical schemes (Euler, Milstein, KP, KP2, RK2), their convergence orders with and without GIC, and the interpretation (Itô or Stratonovich) they approach. [3.4] Table: Summary of Numerical Time Stepping Methods added

[5] Clarify in figure captions the error norm, number of realizations, and the definition of error bars. Added: clarification: 100 realizations of the noise for each $\alpha$ to each figure caption

[6] If possible, provide a short workflow or example demonstrating how GIC can be embedded into a simplified atmospheric or ocean model component to enhance practical applicability. **This would unfortunately be outside the current funding scope**

Thank you for the insightful feedback and for helping us improve the presentation of our work. Sincerely,

William Johns, Lidong Fang, Huan Lei, Panos Stinis