Author's response to Anonymous Referee 1 Comment 1

July 14, 2025

General comments

We thank the referee for their constructive comments. A point-by-point reply is reported below, with referee comments in orange and our replies in black.

[The VGS theory, while well-motivated, is adapted from oceanographic contexts and relies on assumptions about pressure dependence that have not been directly tested under glacial conditions.]
and
[The study draws on seismic data from only five sites, which limits the spatial resolution and generalizability of the inferred effective pressure fields. It is not surprising, at least to me, that PIG exhibits Coulomb-like behavior as I'm not aware of any studies that contradict this. As a point of curiosity, I am interested to see how this methodology performs in other environments where basal conditions are debated, such as the interior of the Greenland Ice Sheet or alpine glaciers (obviously outside the scope of this study!)]
While we agree that the VGS theory needs further testing under glacial conditions, we did adjust the compressional viscoelastic time constant τ_{p} to account for the difference in exerted overburden pressure. We are currently working on applying the same methodology to Thwaites Glacier, the results of which will be presented in a follow-up publication. A brief discussion of the points above will be added to the revised manuscript.

Specific comments

[I would like to see more detail on the logic behind the formulation of the custom prior distributions. While I appreciate the justification for the Cmax prior shown in the Supplement, the distributions for porosity and grain size are less clear. Since these appear to be new compilations from the literature, it would be helpful to include the underlying data (in the Supplement would be sufficient) and to show how those priors were constructed from the compiled observations. Additionally, in cases where porosity was estimated from active seismic data (e.g., Blankenship et al., 1987), it's worth noting that those estimates assumed no dependence on effective stress. This could introduce some circularity when those values are used to constrain priors in a model that explicitly incorporates effective stress. Clarifying these points would strengthen the study.]
A supplementary table outlining the grain size and porosity data will be added to the revised manuscript. The porosity prior was primarily informed by borehole data and sediment cores. Similar to the porosity of sands and glass beads used to validate the VGS, the seismic estimates from, e.g., Blankenship et al. [1987] function in a supplementary capacity. We will clarify this in the revised manuscript and incorporate the referee's suggestion.

[The use of independent prior distributions may oversimplify the relationships among subglacial sediment properties, particularly where physical coupling through compaction or consolidation is expected.]
and

[Secondly, grain size, porosity, and effective stress are not independent in natural systems, but are physically coupled through compaction, consolidation, and sediment mechanics. If I understand the methodology correctly, parameter sets were sampled independently from their prior distributions, grain size and porosity, for example, and then used to calculate effective stress via Buckingham's VGS theory. However, relationships between these variables have been described in the sediment mechanics literature and impose constraints on what combinations are physically reasonable. I am concerned that treating them as statistically independent in the prior sampling may lead to internally inconsistent sediment states. While the Bayesian framework helps downweight poor-fitting combinations, would explicitly incorporating physically based constraints or coupled priors could improve the robustness of the analysis in a meaningful way?]
In general, the porosity is inversely related to the mean (or median) grain size, but this relationship is convoluted by other properties such as the particle size uniformity [e.g., Wang et al., 2017, Atapour and Mortazavi, 2018, Gupta and Ramanathan, 2018, Díaz-Curiel et al., 2024]. While it is correct that the parameter sets were sampled independently, and using coupled priors would improve the robustness of the analysis for our most extreme parameter combinations (e.g., high porosity and large grain size), the relationship between porosity and grain size outside these extreme parameter combinations, and therefore the formulation of such a coupled prior, is less clear. As the Bayesian framework already downweights the extreme parameter combinations through the chosen independent prior distributions (as correctly identified by the referee), and the minimum misfit and MAP parameters are generally consistent with the porosity-grain size relationship described in the literature [e.g., Díaz-Curiel et al., 2024], we do not expect a significant change in the posterior probabilities. We will add a brief discussion of this to the revised manuscript.

[Regarding ut, I respect the uncertainty that leads the authors to use a log-uniform prior, but as I recall, the Zoet-Iverson slip law includes a prediction for ut based on sediment properties (most notably grain size) which already has a relatively narrow range in this study. Given that, it doesn't seem reasonable to expect ut values near 10^4 m/yr as equally likely as, say 10^2? There are also at least two other studies I can recall that provide calculated values of ut in different configurations: Helanow et al. (2020; DOI: 10.1126/sciadv.abe7798) for sliding over rough, rigid beds and Hansen et al. (2024; DOI/10.1029/2023GL107681) for frozen sediments over till. Some discussion of this would be helpful, as it's not clear whether the wide prior range used here is physically justified.]
Zoet and Iverson [2020] report $u_{t,noN}$ values in the range 56.36 to 363.52 MPa^{-1} m yr^{-1}. Because Hansen et al. [2024] use the same bed material (Horicon till sourced from same location) but with plowing clasts removed, they use the model parameters given in Table S1 in Zoet and Iverson [2020] except for a smaller clast radius $R = 0.0045$ m (instead of $R = 0.015$ m or $R = 0.030$ m), leading to $u_{t,noN} = 1120.17$ MPa^{-1} m yr^{-1}. Given these significant uncertainties and that $u_{t,noN}$ depends on several other uncertain parameters, we argue that $u_{t,noN}$ is best represented by a log-uniform prior (currently covering the range 3.16 to 3155.76 MPa^{-1} m yr^{-1}). We will include these additional details in the revised manuscript. Note that the regularised Coulomb law used in Helanow et al. [2021] is not the same as in Zoet and Iverson [2020].

[It would be helpful to emphasize more clearly in the introduction or discussion that the method presented here is primarily applicable to soft-bedded glacier systems, since the acoustic impedance contrast relies on wave propagation through a granular medium. This is an important distinction, especially considering that some of the tested sliding laws were originally formulated for rigid or mixed bed topographies. I think an open question remains in glaciology regarding how these different sliding laws apply across regions with spatially heterogeneous basal conditions (e.g., Maier et al., 2021, https://doi.org/10.5194/tc-15-1435-2021). The result that a fast-flowing, soft-bedded glacier like Pine Island Glacier exhibits Coulomb-style sliding is not surprising to me, given the preponderance of experimental and field evidence in the literature. But in light of continued and recent discussion in the literature (e.g., Law et al., 2024, https://doi.org/10.48550/arXiv.2407.13577) it would be worth emphasizing the both the utility and the limitation of the geophysical datasets to constrain the slip law.]
The referee is correct that, strictly speaking, the Viscous Grain-Shearing theory only applies to granular material. However, as outlined in detail in our response to the second referee, whenever we are using

a sliding law originally formulated for hard beds (e.g., Budd, Schoof), we assume a granular, relatively undeformable material that cannot support tangential friction at its interface with the ice (here referred to as *rigid bed*). We will clarify this in the revised manuscript.

We agree with the referee that spatially heterogeneous basal conditions remain an open research question and spatially variable parameters (grain size, porosity, as well as sliding law parameters) should thus be explored in future studies. We will comment on this in the revised manuscript.

References

Hadi Atapour and Ali Mortazavi. The effect of grain size and cement content on index properties of weakly solidified artificial sandstones. *Journal of Geophysics and Engineering*, 15(2):613, feb 2018. doi: 10.1088/1742-2140/aaa14a. URL https://dx.doi.org/10.1088/1742-2140/aaa14a.

D. D. Blankenship, C. R. Bentley, S. T. Rooney, and R. B. Alley. Till beneath ice stream b. 1. properties derived from seismic travel times. *Journal of Geophysical Research*, 92:8903–8911, 1987. ISSN 01480227. doi: 10.1029/JB092iB09p08903.

Jesús Díaz-Curiel, Bárbara Biosca, Lucía Arévalo-Lomas, David Paredes-Palacios, and María J. Miguel. On the Influence of Grain Size Compared with Other Internal Factors Affecting the Permeability of Granular Porous Media: Redefining the Permeability Units. *Lithosphere*, 2024(1): lithosphere_2023_231, 01 2024. ISSN 1941-8264. doi: 10.2113/2024/lithosphere_2023_231. URL https://doi.org/10.2113/2024/lithosphere_2023_231.

Aniket Gupta and A. L. Ramanathan. Grain texture as a proxy to understand porosity, permeability and density in Chandra Basin, India. *SN Applied Sciences*, 1(1):1, 2018. ISSN 2523-3971. doi: 10.1007/s42452-018-0001-3. URL https://doi.org/10.1007/s42452-018-0001-3.

D. D. Hansen, K. L. P. Warburton, L. K. Zoet, C. R. Meyer, A. W. Rempel, and A. G. Stubblefield. Presence of frozen fringe impacts soft-bedded slip relationship. *Geophysical Research Letters*, 51(12):e2023GL107681, 2024. doi: https://doi.org/10.1029/2023GL107681. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2023GL107681. e2023GL107681 2023GL107681.

Christian Helanow, Neal R. Iverson, Jacob B. Woodard, and Lucas K. Zoet. A slip law for hard-bedded glaciers derived from observed bed topography. *Science Advances*, 7:2–9, 2021. ISSN 23752548. doi: 10.1126/sciadv.abe7798.

Ji-Peng Wang, Bertrand François, and Pierre Lambert. Equations for hydraulic conductivity estimation from particle size distribution: A dimensional analysis. *Water Resources Research*, 53(9):8127–8134, 2017. doi: https://doi.org/10.1002/2017WR020888. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2017WR020888.

Lucas K. Zoet and Neal R. Iverson. A slip law for glaciers on deformable beds. *Science*, 368, 4 2020. ISSN 10959203. doi: 10.1126/science.aaz1183.