Supplementary Material:

Equilibrium-Approximated Solutions to the Reactive Lauwerier Problem: Thermal Fronts as Controls on Reactive Fronts in Earth Systems

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An Extended Form of the Conservation Equations:

Aquifer temperature. Assuming radial symmetry and that heat transfer through the layers confining the aquifer is governed by conduction, the heat equation in polar coordinates is given by:

$$\frac{\partial T}{\partial t} = \frac{\alpha_{\mathrm{b}}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \alpha_b \frac{\partial^2 T}{\partial z^2}, \quad \text{for} \quad \begin{cases} z \le -\dfrac{H}{2} \\ z \ge \dfrac{H}{2} \end{cases}, \qquad (S.1)$$

where T represents the temperature, t is time, r and z are the radial and vertical coordinates, respectively, with the origin located at the injection well center, and H is the thickness of the aquifer (see Fig. 1). The quantity $\alpha_{\mathrm{b}} = K_{\mathrm{b}}/C_{\mathrm{pb}}$ represents the thermal diffusivity, where the subscript b refers to bulk rock, K is the thermal conductivity, and C_{p} is the volumetric heat capacity (Stauffer et al., 2014).

Assuming that heat transfer within the fluid in the aquifer is governed by advection and thermal dispersivity, and the heat-transport equation can be written as

$$C_{\mathrm{p_b}}\frac{\partial T}{\partial t} = -C_{\mathrm{p_f}}\frac{1}{r}\frac{\partial (ruT)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(D_{\mathrm{T}} r\left(\frac{\partial T}{\partial r} + \frac{\partial T}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(D_{\mathrm{T}}\left(\frac{\partial T}{\partial r} + \frac{\partial T}{\partial z}\right)\right), \quad \text{for} \quad -\frac{H}{2} \le z \le \frac{H}{2}. \qquad (S.2)$$

Here, the subscript f refers to fluid, D_{T} is the isotropic thermal dispersivity, $u(r)$ is the fluid velocity (or Darcy flux), which can be calculated from the total volumetric flow rate, Q, using $u = Q/(H2\pi r)$ (assuming uniformity of u along the z-direction of the aquifer; Andre and Rajaram, 2005; Chaudhuri et al., 2013).

Assuming full thermal mixing in the *z*-direction of the aquifer allows the establishment of the 'depth-averaged' Eq. 3 in the main text. In this case, the heat exchange between the aquifer and the confining rocks is integrated into the heat exchange term (**Θ**).

Reactive Transport. Similarly, the solute transport equation, incorporating advection, dispersion, and reaction in the aquifer, is given by

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(D_{\mathrm{s}} r\left(\frac{\partial c}{\partial r} + \frac{\partial c}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(D_{\mathrm{s}}\left(\frac{\partial c}{\partial z} + \frac{\partial c}{\partial r}\right)\right) - \Omega(r,t), \quad \text{for} \quad -\frac{H}{2} \le z \le \frac{H}{2}, \qquad (S.3)$$

where c represents the solute concentration [M L^{-3}], D_{s} is the isotropic solute dispersivity, and Ω is the reaction term (Chaudhuri et al., 2013; Szymczak and Ladd, 2012). The equations describing the reaction rate, Ω, solubility dependence on the temperature, $c_{\mathrm{s}}(T)$, and the porosity change are provided in Section 2.3 in the main text (Eqs. 6, 8 and 9, respectively).

In the case of planar flow and Cartesian coordinates, the equations B.1-B.3 above take the following form,

$$\frac{\partial T}{\partial t} = \alpha_{\mathrm{b}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right), \quad \text{for} \quad \begin{cases} z \le -\dfrac{H}{2} \\ z \ge \dfrac{H}{2} \end{cases}, \qquad (S.4)$$

$$C_{\mathrm{p_b}}\frac{\partial T}{\partial t} = -uC_{\mathrm{p_f}}\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(D_{\mathrm{T}}\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(D_{\mathrm{T}}\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial z}\right)\right), \quad \text{for} \quad -\frac{H}{2} \le z \le \frac{H}{2}, \qquad (S.5)$$

and

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(D_{\mathrm{s}}\left(\frac{\partial c}{\partial x} + \frac{\partial c}{\partial z}\right)\right) + \frac{\partial}{\partial z}\left(D_{\mathrm{s}}\left(\frac{\partial c}{\partial z} + \frac{\partial c}{\partial x}\right)\right) - \Omega(r,t), \quad \text{for} \quad -\frac{H}{2} \le z \le \frac{H}{2}. \qquad (S.6)$$