Response to Tim Garrett
by Brian Hanley – Responses in Liberation Sans

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Preamble – Response to comments from referees regarding "usual format"

Because of the comments about not following usual format, I think there needs to be a clarification of how this critique paper came about. Originally, it was submitted as a 'Commentary" type of paper. However I was told by editorial that this was too long. So on February 6th, editorial recommended it be resubmitted as a "Research" paper.

> *Perhaps the best approach would be to change the title, while still making the abstract clear that you are addressing some particular issues/errors in a particular manuscript. You then have the freedom of a research article to explain what that error is (perhaps how it may have arisen) and it's consequences for the original paper and, crucially, it's relevance for future work in this area.*

I accepted that, suggested that I revise the title to say, "Critique" instead of "Commentary", did some revisions and streamlining, and the result is what readers see. There was considerably more material involved than appears here working through various aspects. I apologize for confusion by readers.

A key point I made in that discussion with editorial was this. (**Emphasis added**.)

> *This paper is an area that is difficult to enter into, and there is no standard forum for it. The pieces and the whole, the why's and data, are being worked out. Because of this, it is impossible to get funding for, and yet **it appears central to understanding economic limits, and monetary valuation**. Consequently, I want to engage and help nurture it.*

This critique paper is the first time anyone has made a concerted attempt to step through all of the data, independently find sources of data, and validate what was provided in Lotka's Wheel. As such it is a valuable exercise and should be helpful to others. This is a basic step in science – **replication of studies** and comparing results. That is why I named these datasets replicates. The goal of these replicate datasets was to perform sanity checks. I did not expect them to be precisely the same.

Yes, there are issues found. Finding issues, if they exist, is the whole point of a replicated study. If everything checked out, this would be worthy of a commentary note saying so. However, the equations of GGK22 were not actually used as stated, and *w* is a serious error. The *W* dataset provided is based on an error of the variety I would call going to fast. I do not think this is fatal to the concept.

General remarks

The critique includes … it is left unstated in the figures what units are used for quantities, and when units in the analyses are presented, they are used inconsistently for the same quantities. There are frequent uses of very large numbers of significant digits for imprecise quantities, up to nine for an exponential growth rate in one case.

General remarks appear to primarily relate to spreadsheet

Although not specified, it appears that these remarks refer to the shared spreadsheet's graphs, not the critique paper, as the overly significant digits are only present there, and units are specified on scales in the critique-paper.

I use Linux Libreoffice Calc 24.8.1.2 which can have Excel artifacts

I use Linux almost exclusively, and currently Libreoffice Calc 24.8.1.2. When spreadsheets are accessed using Windows, some figures can have confusing artifacts. *I will make note of this in revision, to warn Microsoft Excel users.*

Units available

Units in the spreadsheet are available in the legends, and on the scales, and since this spreadsheet does not refer to any others, if there is a question, the data column can be easily identified. True, the spreadsheet X axes do not say "Year", but anyone familiar with the subject matter should know that. If it is considered important I will add year to the X axes.

Curve fit defaults are used in spreadsheets and do not matter; these are cosmetic

Yes, I leave curve fits at the defaults in spreadsheets. I do the same when using mathematical tools like Maple, Matlab, etcetera as well – until reporting them. This is good practice to avoid introducing errors from repeated truncation of precision prior to final reporting, if an equation is ported. I do not see any such "*large numbers of significant digits*" reported in the critique paper with excessive precision.

In any case, if I were to truncate the reporting of the precision in the spreadsheet, this would not affect what the internals of those curve-fitting algorithms do. Internally, the same level of precision would be kept. So at worst, this is a cosmetic matter.

w is a constant

Comment (**emphasis Hanley)**:
A primary conclusion of GGK22 as stated in the abstract is that "In each of the 50 years following 1970 for which reliable data are available, 1 exajoule of world energy was required to sustain each 5.50±0.21 trillion year 2019 US dollars of a global wealth quantity defined as the cumulative inflation-adjusted economic production summed over all history." That is the ratio W for cumulative inflation-adjusted production Y (or W (t) = 0 Y (t) dt ′ = ∑ i Y_i), to the rate of energy consumption R E, is effectively observed to be a constant for a 50-year period since 1970, that is w = W /E = 0 t Y (t) dt ′ = constant. As is argued in the GGK22 article, this constancy, **were it to hold**, it would **imply that inertia, i.e. deep**

history, plays a guiding role in the evolution of future consumptive needs by civilization, one far higher than has generally presumed.

The result that w = W /E is a constant was originally anticipated based on thermodynamic reasoning as first described in Garrett (2011) and it was also elaborated upon in GGK22. As with any hypothesis, it's validity rests on a statistical evaluation, as well as the accuracy of the statistics used. In GGK22, a half century period between 1970 and 2019 was used to evaluate w. That the statistic holds for any period before or after **can really only be speculation**, but for that period covering two-thirds of the total historical growth in civilization energy demands w was observed to be nearly constant. **It was speculated that this constancy is a fundamental feature of the global economy**: energy consumption sustains flows along the civilization networks that have previously been built through prior economic production, allowing for fraying of those networks by way of the downward correction from the nominal to the real GDP, as is commonly ascribed to economic inflation.

However, as a time integral of inflation adjusted economic production, the accuracy of the calculation of W – and hence *w* = W /E – relies on a reconstruction of historical economic data. Any reexamination of the statistics and the methods is important because any reconstruction necessarily requires some subjective choices. It is also challenging. An independent reexamination of the methods used in GGK22 could shed light on better approaches. B. Hanley **provides revised reanalysis datasets from which he makes arguments that *w* is not in fact a constant, in any time period**.

...arguments that w is not in fact a constant: Not exactly. I think w is an error

What I argue is that *w* is an error, regardless of what statistics are done on some subset of *W*.

Yes, I understand why this is done in Lotka's Wheel. See section 5.1, line 275. I do not think this version of *w* is required to work with the concept.

Problem with $\hat{w} = Y / (dE/dt)$ and $Y = \hat{w} \cdot (dE/dt)$. $(dE/dt)=0$ causes divide by zero error

What these two equations say is that when *dE/dt* goes to zero:

$\hat{w} = Y / (dE/dt)$ is a meaningless expression. $\hat{w}$ = divide by zero error.

$Y = \hat{w} \cdot (dE/dt)$ is also a meaningless expression. Divide by zero times zero.

Though equation 4 ($\hat{w} = Y / (dE/dt)$) generates a divide by zero error, this is used to say real production goes to zero, when divide by zero should mean real Y goes to infinity. This should be seen as indicating an error in previous math.

No amount of statistical analysis of the 1971-2019 dataset can justify this use of a constant. It is just an error, and it generates nonsense.

Energy

To address the Hanley critique, I compare the reanalysis dataset in GGK22 to that proposed as alternative of Hanley, **what he terms a replicate dataset**. Statistics for world primary energy consumption since 1970 are stated clearly enough by both sources. Specifically as stated in the Methods of GGK22 Yearly statistics for world primary energy E_i are available for both consumption and production from the Energy Information Administration (EIA) of the US Department of Energy (DOE) for the period 1980 through 2018 and for consumption from British Petroleum (BP) for the years 1965 through 2019 (DOE, 2020; BP, 2020). A yearly composite of E_i in units of EJ yr^{-1} for the years 1970 to 2019 is created from the average of the three datasets while using single sources when only one is available. The difference between the values in the BP and EIA datasets is significant at 8.5 ± 1.5 %, but it is steady and small relative to the 180 % increase in energy consumption over the 50-year time period considered here.

In the Hanley critique, no argument is presented why the energy data used in GGK22 is problematic. The data set Hanley used was obtained from Our World in Data (Ritchie and Rosado, 2020) which refers to the Statistical Review of World Energy. No averaging is made of consumption and production as in GGK22 but this is a small consideration. Based on that dataset, consumption in 2019 was 588 ExaJoules compared to 595 ExaJoules in the GGK22 dataset. Because this difference is small, it is presumed that the two analyses are in sufficient alignment even if the GGK22 datasets is more comprehensive.

However, Hanley shows a reconstruction of energy demands in 1 CE that is three orders of magnitude smaller than that which may be inferred from GGK22 assuming w is a constant. **Where such large discrepancies exist it should be possible to perform a simple test to determine if one result or the other is unreasonable.** The Hanley critique presents the global consumption rate of energy in 1 CE as 0.0623 ExaJoules, which split among the estimated 225 million people in the world at that time would correspond to a civilization consumption rate averaging to about 9 Watts per person.

Human metabolic needs are approximately 100 Watts. To feed those needs using modern agriculture, the efficiency is about 10% implying order 1000 Watts of consumption.

Presumably ancient agriculture was less efficient, and then of course not all primary energy at that time or any time went into feeding people in a subsistence lifestyle. Militaries, buildings, roads, and sailing networks were also needed. Whatever the source of the calculation, the 9 Watts per person presented by Hanley as part of a superior dataset reconstruction is simply far to small to be plausible, to enable any form of civilization sustenance, and by two to three orders of magnitude. The problems are more of a basic scientific nature when it comes to calculation of cumulative inflation-adjusted world economic production summed over all history W.

reconstruction ... 1 CE that is three orders of magnitude smaller

The simple test that makes sense to me is to use per-capita energy in 1800 as the ceiling. This is discussed below.

Human labor is left out, hunter-gatherers, and slavery was global

I discuss how human labor is left out of energy consumption (lines 164-165m and lines 351-360). This should be a significant invisible component, that I think factors strongly in the long decline of *Y/E.*

Hunter-gatherers were a large fraction of global population in 1 CE, but what exactly this proportion was is not clear. Recent work has shown the estimation of the roughly 300 societies to be more complex than previously believed[1].

Slavery in Roman times is estimated at 10-20% overall, with 30% of the population enslaved in some regions. For comparison, in the Carolina colony of North America, 60% of the population was slaves. In colonial Boston, 10-12% of the population were slaves.

Draft animals

A ratio of one horse equivalent per 10 people may be high for the year 1 CE, particularly globally. North America had none. South America had limited use of lamas and alpacas. In the US, sources conflict, but in 1900, which was quite prosperous, the ratio was about 3 people per horse. In 1720, the ratio may have been much higher, possibly as high as 1 to 1. This declined as land became more expensive and owning horses required buying feed.

Solar energy is fundamental to agriculture & human intensive can have high yield

The energy of agriculture comes from the sun, which supplies ~1 KWh m^2-day. Plants convert 3%-6% of sunlight energy, which is ~11 KWh per m^2-year, or 109 MWh per hectare-year. A big adjustment though is that human intensive agriculture can be more productive per hectare than mechanized methods by a factor of 3 to 20 times. (Multi-cropping can be a factor.) I doubt ancient farmers produced 20 times current mechanized crop yields, but 2-5 times was probably within reach.

Curation of "wild" lands by human populations

Europeans, like native American tribes, used to curate their "wild" lands to maximize game and trees that were useful to them. I recently got an education in Northern California tribal methods when spending time on a small ranch that had some "untouched" land. That land had been cared for with fire to keep grazing available for deer, and brush that favored turkeys and rabbits. Oak trees with preferred high acorn yields had been planted and fertilized with fire baked oyster and mussel shells from the bay region, likely through trade. Pinon pines had been planted, etcetera. Thse kinds of techniques are high yield for the labor expended, and work well as long as the population density isn't too high.

Note that the concept of "interest" comes from the "interest" that the landowner got from animals born on their land.

Sailing ships

Yes, it is true that sailing networks had some impact. I don't have a value for energy of sail in the global economy. I didn't find someone that had done that.

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1 Zhu D, Galbraith ED, Reyes-García V, Ciais P. Global hunter-gatherer population densities constrained by influence of seasonality on diet composition. Nat Ecol Evol. 2021 Nov;5(11):1536-1545. doi: 10.1038/s41559-021-01548-3. Epub 2021 Sep 9. PMID: 34504317; PMCID: PMC7611941.

Concentration on 1 CE low value ignores the high confidence E value of 1800 CE.

In this sequence from A to C, it should be obvious by inspection that the significant difference is not in the *Y* datasets, regardless of any possible issues. It is the difference in E datasets that generates the huge *Y/E* discrepancy, and by extension the *W/E* discrepancy. Note that the 1800 to present *E* data in figure 1-B are high confidence. It is this E difference that drives the W/E

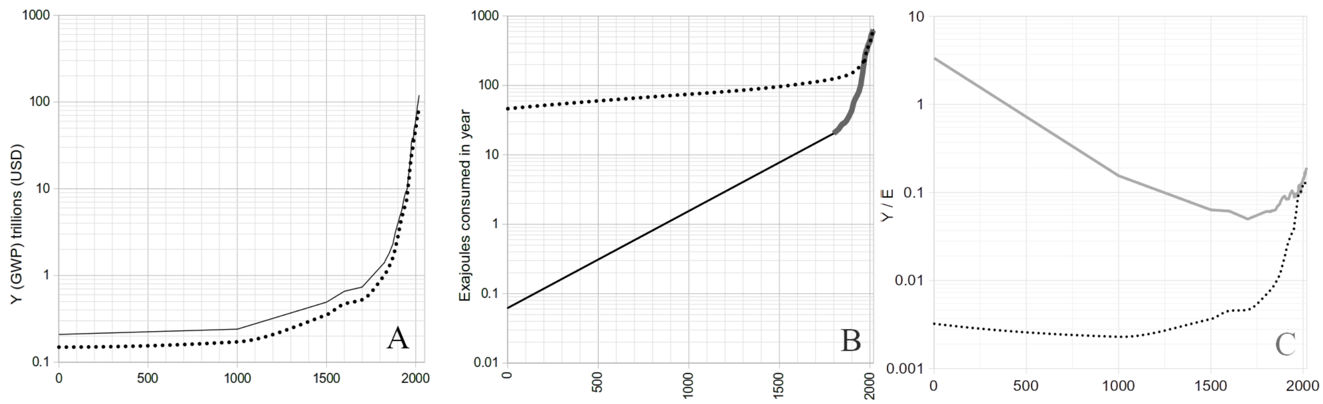
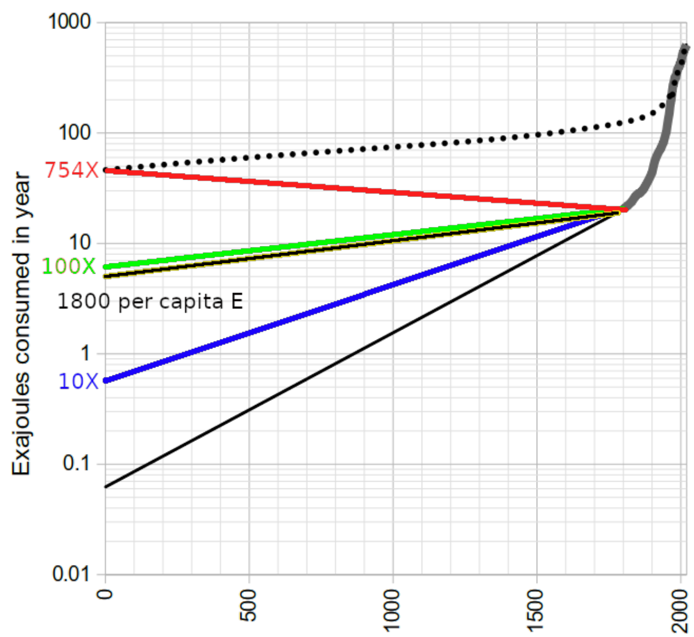


Figure 1-B energy datasets – revised based on alternative assumptions



In this 1-B figure, multiple alternative exponential interpolations are shown. Red upper curve meets the Lotka's Wheel 1 CE value. Green shows interpolation to 100X the *E_Rep* value, and blue shows an interpolation to 10X the *E_Rep* value. In black is shown the interpolation of a steady level of 1800 CE energy per capita.

The Lotka's Wheel *E* dotted curve 1 CE value of 46.172 EJ has to drop by a factor of 2.7 times to get down to the high confidence *E_Rep* 1800 CE value of 20.35. This does not appear reasonable.

Let's try using per-capita energy of 1800 as a guide, and then simply hold per-capita E steady. This should be an overestimate, that can be considered an upper limit.

1800 CE: 20.35 EJ / 953.56 million Pop = 0.021341939 EJ/mPop

1 CE: 225.82 mPop · 0.021341939 EJ/mPop = 4.82 EJ

Interpolation to 4.8 EJ is shown in yellow, just below the 100X interpolation.

If I accept this overestimate limit of 4.8 EJ in 1 CE, that does not make the problems with *Y/E* and *W/E* go away, it makes them smaller, but not in a way to change my critique significantly.

Reasonable 1 CE value range: Low of* 1.7 *EJ, high of 3.75 EJ

I can accept raising global energy consumption, but not by a factor of around 750X. Perhaps split the difference between the 4.8 EJ that is obtained by using the 1800 CE energy per capita, and my 1 draft animal per 10 people of ancient Rome, then make the range +/- half the midpoint to ceiling or floor. This would yield, roughly 2.7 EJ globally as the midpoint, with a high of 3.75 EJ and a low of 1.7 EJ.

Market Exchange Rate (MER) versus Purchasing Power Parity (PPP)

The approach outlined in the Methods section of GGK22 is that Economic production is tallied and averaged using World Bank (WB) and United Nations (UN) statistics for the years 1970 to 2019 (The World Bank, 2019; United Nations, 2010) and expressed here in units of trillions of market exchange rate, inflation-adjusted "real"-year 2019 dollars. So there are two key points here, which are that the production statistics must be adjusted for inflation and expressed in Market Exchange Rate (MER) values.

The reason to do so was described in greater detail in a prior article referenced in the methods by Garrett et al. (2020) and in the paper the critique references by Garrett (2011). In the Hanley et al. critique, no criticism is made of statistics used or the choice to focus on real, MER dollars so presumably that is not an aspect in contention.

However, Hanley mixes and matches datasets to reconstruct a long time series for economic production that is inconsistent in its choice of units for each year, and so cannot be used for the purpose of addition to calculate W. The economic production statistics they use since 1990 appear to have as their original source the Penn World Tables, although details are not given. I**f this is the case, the both key points of MER, inflation-adjusted dollars are satisfied**.

For data between 1960 and 1990, the Hanley critique states use of data from the Federal Reserve (NYGDPMKTPCDWLD). These are expressed in MER dollars but they do not account for inflation to convert from current-year nominal to fixed-year real dollars. This is explicitly stated in the Federal Reserve dataset but not acknowledged by Hanley. The description in Hanley is difficult to follow but it appears a scaling multiplier is applied to the Penn World Table datasets to match the two economic production datasets post-1990, and that the Federal Reserve data set is used as the basis for a longer time series. Not surprisingly, by not adhering to the requirement of using real 2MER dollars for all years the result is an average 44% difference between the GGK22 and Hanley datasets. The difference is washed away by Hanley with "The discrepancy is due to a change in how OWID GWP and its data sources are calculated versus other datasets, and the difference is consistent enough."

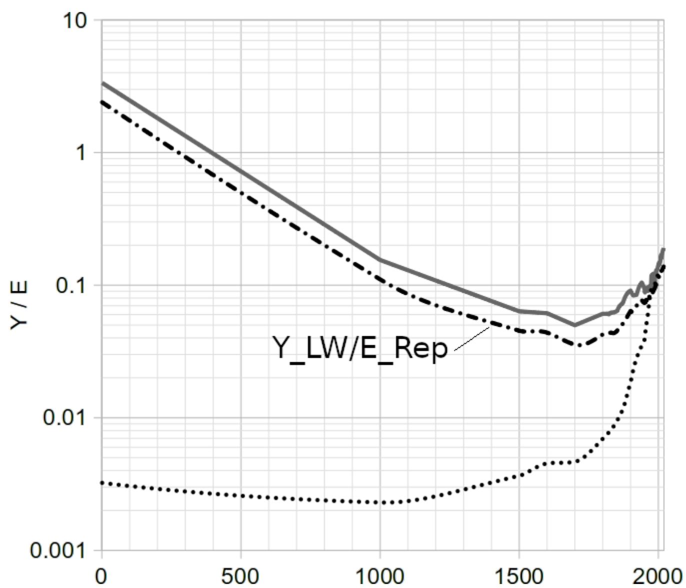
This [Hanley "*The discrepancy is due to a change in how OWID GWP and its data sources are calculated versus other datasets, and the difference is consistent enough*."] is misleading. For the purpose of calculating W, and error will be cumulative, in this case introduced by not considering the growing devaluation of purchasing power through inflation. Most importantly, for the purpose of calculating W , it makes no sense to sum dollars from one year to the next where they are expressed in the different units of current dollars from different years. The error is like adding 1 pint to 1 quart and saying there are two quarts.

Take Y_Rep off the table and the replication of results problem persists

The simple step of taking the Hanley critique *Y_Rep* data off the table and replacing it with *Y_LW* (the Lotka's Wheel dataset) and dividing by the *E_Rep* data, shows that the concerns voiced do not matter for the point being made – that there is a major discrepancy that appears on its face to falsify the *W* dataset provided by GGK22, and falsify the belief that the speculation that the 50 year modern period can simply be extended back into the past is true. There is an effect on the present from the past, but it is more complex.

That said, it is my belief that this thermodynamic approach is so compelling that what is needed is to try to figure out why, rather than throwing out Garrett's fundamental hypothesis.

Figure 1-C with addition of Y_LW/E_Rep



Points addressing the MER vs PPP concerns.

To perform a proper replication study, I had to find what I considered the most correct dataset independently. This was the Our World in Data (OWID) group's data, which is based on Maddison project and World Bank. This represents the career work of multiple teams that deal with the issues raised by Garrett. My confidence in OWID work is very high.

Inflation is accounted for in OWID dataset

Per the OWID documentation, "*This data is adjusted for inflation and for differences in living costs between countries*."[2]

Data between 1960 and 1990, using NYGDPMKTPCDWLD is accounted properly

The purpose of using this NYGDPMKTPCDWLD dataset was to be able to do a better job of interpolating the 10 year intervals than a linear or exponential interpolation.

The dataset provided by OWID has three 10 year intervals from 1960 to 1990. The NYGDPMKTPCDWLD was scaled to fit at each end of the three intervals. This automatically corrected for inflation, because each end was corrected for inflation by definition. Using these scalars, the NYGDPMKTPCDWLD data were used to populate the interval spans.

I needed to do this because without it I would lose significant detail between 1970 and 1990. That would result in a gross mismatch within those intervals when comparing with Garrett's datasets. That this was successful is visible by inspection in figures 1-E and figure 5, from 1970 to 1990. While my *W_Rep* values are higher, they mirror the variation of *W_LW* values.

PPP vs MER background

For deep time estimations, one must make use of PPP techniques almost exclusively. Why is this? If one thinks about it, how can an archeologist make use of, say, records from Rome to figure out what people had in terms of money? It is not possible to have a real market exchange rate into currencies that did not exist. What can be done is to find out, for instance, how many denari were paid for <u>some basket of products necessary for living</u>, and luxury products as well. This falls under the umbrella of how PPP is done.

44% difference between the GGK22 and Hanley datasets

I assume that the 44% number comes from the mean of the dataset ratios I show in column C of the spreadsheet.

If I use area under the curves, GGK22 ΣY = \$3,380 trillion and Hanley ΣY = \$4,419. The ratio of Hanley/GGK22 = 1.31, or 31% larger. It correlates well enough to the mean of the ratios. This is still close enough, as we saw above in the Figure 1-C curve using GGK22's *Y* dataset.

I do not agree that I intended to "wash away" this difference. I reported the difference, although not in the critique-paper. What I intended to convey is that these datasets are close enough that these quantitative changes will not qualitatively change results in a meaningful way. I show this in the figure 1-C graph above.

Additionally, I already showed that quantitative differences did not make a large qualitative impact on *W* in figure 2 of the critique, reproduced below. Figure 2 also provides a good segue into the next section.

Equation 5 difficult to understand.

For the reconstruction of GDP for earlier years, Eq. 5 is difficult to understand and highly contrived given e ln x = x. The reconstruction refers to Our World in Data datasets for GDP statistics pre-1960,

2 https://ourworldindata.org/grapher/global-gdp-over-the-long-run

which in turn appear to use the same Maddison database used by GGK22. This database appears to have been adjusted for inflation, which is good.

Overthinking?

I suspect this puzzlement is due to overthinking it. I simply presented a standard exponential interpolation method. If I had said, "exponential interpolation" this issue might not have arisen. I was just being careful to show exactly how the exponential interpolation was done.

The second equation is just solving the exponential on the left for *a*.

Given that generally speaking, population rose exponentially, and was a primary driver of GDP, I think that an exponential interpolation is required.

Adjustment from PPP to MER dollars

However, what is not considered is the second necessary adjustment to convert from purchasing power parity dollars used in the Maddison database to market exchange rate dollars. Specifically, as stated by GGK22: The dataset is adjusted for inflation and to convert from currency expressed in purchasing power parity dollars to market exchange units using as a basis for adjustment the time period between 1970 and 1992 for which concurrent market exchange rate (MER) and purchasing power parity (PPP) statistics are available.

An argument might be made that PPP dollars are preferable to MER dollars. I don't believe this would be appropriate for physical reasons. Whatever the choice, there must be consistency when creating a continuous time series of economic production. The final dataset for Y Hanley uses is a mix of real, inflation-adjusted, MER, and PPP data. Without consistent careful adjustment for inflation and to MER dollars across the full 2000 year time series, the numbers in each year cannot be added to create the central quantity W.

Any summation must use numbers with the same units. Not all dollars are the same.

PPP to MER is a red herring

First, I find it hard to believe that it is possible to meaningfully convert PPP to MER in deep time. One might be able to do it to some degree going back 800 years, perhaps. This is as far as Schmelzing got with interest rates[3]. But one has to ask the question, what does MER mean when applied to global numbers?

PPP was created to correct the errors inherent in MER figures. After conversion, without knowing what the MER figures were, how does one reconstruct MER? To reconstruct MER figures from PPP, one would need to have the original MER figures *for each country in that time,* or at least the exchange rates by year. MER to PPP conversions, like exchange rates, are not static. They change by the year. (Reported usually by the month.) It just doesn't make sense to do this, or to think that figures in one century could be applied to figures in another century. I think this is a mistake.

3 https://www.bankofengland.co.uk/working-paper/2020/eight-centuries-of-global-real-interest-rates-r-g-and-the-suprasecular-decline-1311-2018

In addition to this problem, when PPP approaches are used to estimate GDP worldwide in the first place, how can you differentiate between nations? What would such a conversion even mean?

Since PPP normalizes to the physical real world economy, I think that PPP is the more accurate. I did not criticize the use of MER in modern times, as my primary goal was to perform a sanity check, and globally, MER or PPP are close enough.

Background on CPI

Fundamental to the concept of inflation is that to calculate it requires a consumer price index of some kind. This index is a basket of commodities recognized as fundamental to individuals living in a civilization. However, when one commodity inflates too much, the economy substitutes another that is cheaper. Because of this, the concept of substitution is key. What this also means is that inflation has built-in limits.

The US CPI has gone through many changes over the past century[4].

Inflation can (rarely) be very high[5]. From my experience in nations that experienced hyper-inflation, in each of those nations, people turned to other nation's currencies at least to some degree. The US dollar is a common alternative currency, but the Euro, Swiss Franc, and British pound were also present. Essentially, demand evolves a "basket of currencies" response.

Note that in one of the examples, post-WW2 Hungary, the inflation was deliberately engineered to make paying off existing debt cheaper, and encourage entrepreneurship. This strategy was successful, and the hyper-inflation ended. (As is usually the case, this capsule discussion elides significant complexity and context.) I'll stop here going down that rabbit-hole.

1 CE value of *W*

As a note, the value of W obtained by GGK22 for year 1CE was not a summation of a much longer dataset as stated in Section 1.5.2 of the critique. The method was stated in the Methods section of GGK22 to be something quite different, an inference from the GDP at that time assuming that W and population were growing equally fast. Note too the discussion of uncertainties that might accompany this assumption. The value for cumulative production in 1 CE. W(1) is obtained by assuming that W was growing as fast as population at that time at rate and that Y (1) = R W W (1). Population data from 1 CE and 1 century before and after suggest that global population was 170 million and growing at 0.059 % yr −1 (United States Census Bureau, 2018).

GGK22 method inferring from GDP that *W* & population grew equally fast is an error

If $Y_i = 0$ this means that humans ceased to exist. If humans do not exist all of this is moot. So, Y_i cannot be zero nor less than zero. For any time in which humans exist, there will be a Y_i.

4 https://www.bls.gov/cpi/additional-resources/historical-changes.htm#
5 https://www.investopedia.com/articles/personal-finance/122915/worst-hyperinflations-history.asp

W will always increase. Equation 1 requires that for any non-zero *Y, W* must increase each year, because *Y* less than or equal to zero is ruled out.

Because *W* is the summation of all previous Y_i, ***W* will always be larger and faster growing than *Y*.** *Y* only grows by $Y_i – Y_{i-1}$ while *W* grows by Y_i. Proportionally, *W* growth slows down as *i* rises, but the amount by which *W* grows is always greater than the amount by which *Y* grows.

Y can decrease YoY continuously, asymptotic to zero, and *W* will continue to increase continuously during this time.

Because the energy humans themselves contribute to the economy is not accounted for in energy accounting today, this is also left out of energy accounting in the past. This should create anomalies in the distant past of high GDP to energy consumption (high *Y/E*).

YoY, *W/E* can decrease, stay the same, or increase.

For *W/E* to decrease YoY, then E_i/E_{i-1} must be a ratio greater than the ratio of Y_i/W_{i-1}; for *W/E* to stay the same, then E_i/E_{i-1} must be equal to the ratio of Y_i/W_{i-1}; for *W/E* to increase, E_i/E_{i-1} must be less than the ratio of Y_i/W_{i-1}.

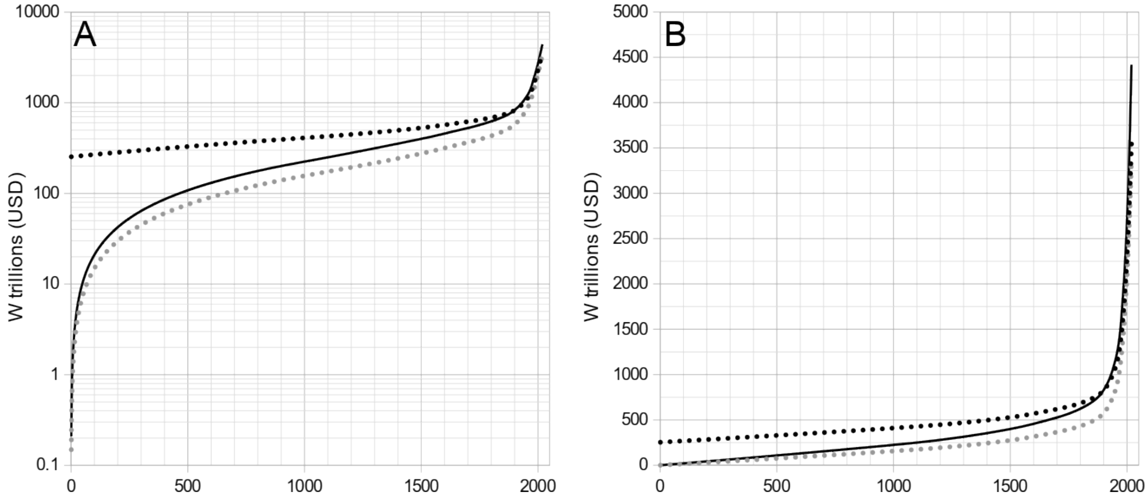
Therefore, all cases except for *W/E* increasing, must have E_i/E_{i-1} greater than 1.

If *W/E* decreases YoY, this means that YoY *Y/E* decreased.

If *W/E* stays the same YoY, then E_i/E_{i-1} must be greater than 1, and YoY *Y/E* increased.

For E_i/E_{i-1} less than 1, YoY, *W/E* must be increasing and *Y/E* must be increasing by a higher YoY ratio.

Figure 2 discussion. We can agree GGK22 values of W are not from eq. 1.



W curve. A. Log scale. B. Linear scale. The upper black dotted curve is the Lotka ' s W heel supplement W dataset. The lower gray dotted curve is W_{LW} for 1-2019 CE created per equation 1. Solid curve is W Rep for 1-2019 CE, also created per equation 1. What is visible here is that the algorithm of equation 1 when applied to generate W from Y_{LW} and the Y_{Rep} , data produces the expected curves that begins with the year 1 CE value. The provided Lotka's Wheel supplement column labeled as W does not.

We agree that the *W* curve supplement dataset provided by GGK22 was not produced using equation 1 ($W(t) = \Sigma Y_j$) that was provided.

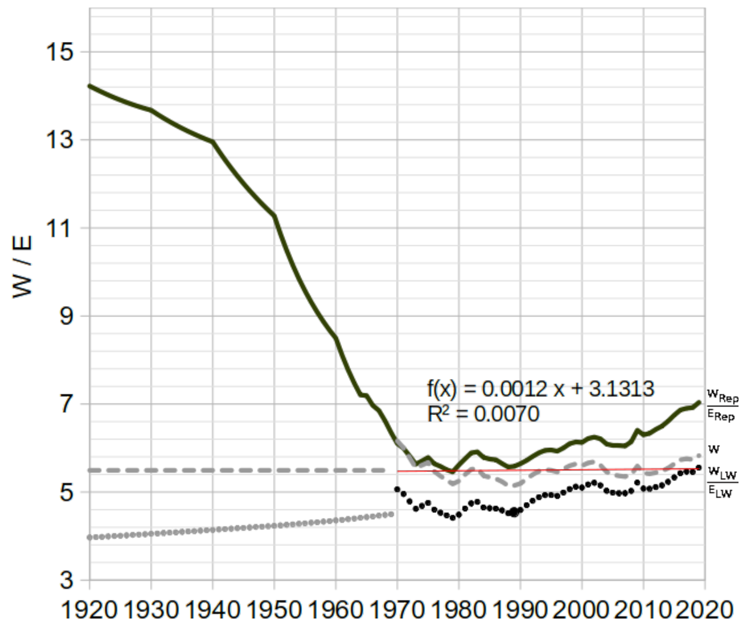
I think it would be nonsense to say that deeper history does not exist. Thus, in Figure 2 (reproduced just above)the first roughly 500 years is a throw-away. However, as I discuss in lines 51-55, there is not a good formal basis for defining that deep time economic history.

Yes, the method stated in GGK22 says how this value was obtained. I am aware of this. I should have said that the only way to reconcile the *W* curve data with equation 1 is to assume deeper history.

Claims of misrepresentation of *W* are false.

While there are inevitable uncertainties in the reconstruction of W as with any other, the yearly values of W since 1970 that are emphasized here cover two-thirds of total growth, so the calculations are more strongly weighted by recent data that are presumably most accurate. Thus, calculation of W, most particularly the conclusion that w is nearly a constant, can be shown to be relatively insensitive to uncertainty in the older statistics (Garrett et al., 2020).
Even the datasets provided in the supplement of GGK22 are misrepresented, although it is unclear how or why. Figure 1E pretends to reproduce the GGK22 data. However, it shows a distinct positive trend in the time series between 1970 and 2019 for w = W /E. This is false. Figure 2c for w in GGK22 shows a different time series with no clear trend. The value presented by Hanley for w in 1970 is about 5 (although no units are given). In the GGK22 dataset provided in the supplement it is 6.1 inflation-adjusted year-2019 MER USD per ExaJoule per year. The difference may seem small but it is significant where constancy is claimed.

Figure 1-E with GGK22 W dataset displayed – Note fig 5 annotates 1-E



Here I added the gray dashed curves from the *W* dataset provided in the GGK22 spreadsheet. That data does not quite have a slope of zero, as shown.

Simply put, I cannot explain the *W* data from GGK22. It conflicts with my replication effort. The W_{LW} curve uses equation 1, which was provided as the definition of *W*, and I summed the GGK22 supplied *Y* data.

The data is what it is. I only report it.

Using GGK22 provided equation 1 to create the *W* dataset is not "pretending".

Either equation 1 provided in GGK22 is the definition of *W* or it is not. Providing a definition and then ignoring it in favor of a dataset produced in some other way renders any discussion meaningless. All that I have done is to use the definitions provided and implement them. I don't think that can be classified as pretending.

Figure 2 is the only place that I showed the GGK22 *W* dataset – a heavy black dotted curve. Otherwise, I ignored it, because it obviously does not follow the rules defined in GGK22 for producing it.

However, I can add the GGK22 W data to figures 1-E and 5 to highlight the discrepancy if desired.

Section 2.2 math – the rate growth equation (interest calculation).

The mathematics in section 2.2 makes no sense. How is economic production Y equal to a growth rate g? How can a dimensioned rate r be added to a dimensionless number unity? It also appears to refer to calculus but not in any way that is familiar or that leads to results that are comprehensible.

The simplest equation for interest rate growth.

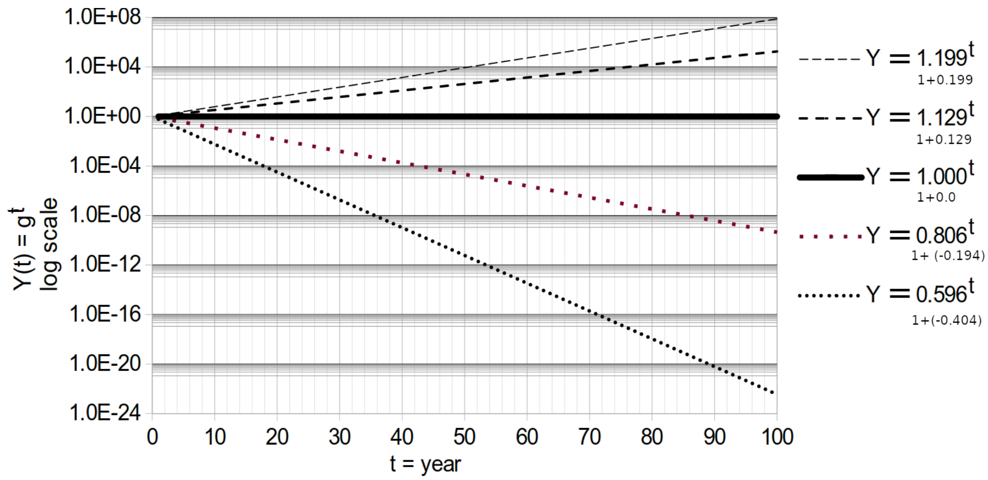
First, let me point out that this growth equation is an empirically founded confirmation of Garrett's hypothesis that the future is dependent on what comes before it. This equation encapsulates this fundamental empirical fact of capitalism, that the creation of more value depends on the capital one has to do it with. Financiers represent their holdings of capital as units of currency, just as GDP does. For these reasons, this is the most correct equation type to use for behavior of Y.

Second, I agree that the GDP value of Y is not directly equal to the growth rate curve itself, unless $Y_0 = 1$. I apologize for misspeaking there. I should have written $Y(t) = Y_0 \cdot g^t$.

The expressed growth rate equation was intended to boil the equation down to its essence, to make the point that an equation of this type needs to be used, and not the average value w.

The units are whatever Y is, which for GDP is measured in units of currency.

Figure: growth equation graph $Y(t) = Y_0 \cdot g^t$



In this figure, the equation for $Y(t) = Y_0 \cdot g^t$, (where $Y_0 = 1$) is shown for various values of g. In the legend, just below each value of g is shown the 1+ r components, where r is the growth rate. The value of 1 is a constant that essentially stands for "what came before". This thermodynamic concept of Garrett's is front and center in the concept of interest and yield for investments.

Hopefully this clarifies how this equation works and why it is chosen. To restate, it supports and fits into the thermodynamic concept of growth.

In this figure, the flatline heavy black curve for $Y = Y_0 \cdot 1.0^t$ is the equation for a "steady state" economy where GDP does not change from one year to the next. This is one of a family of equations that come from the $Y = Y_0 \cdot g^t$ equation, because r can conceivably vary in our use, although I do not show this occurring in the above figure.

For calculus use, the growth equation $Y = Y_0 \cdot g^t$ is needed as the equation for Y

In equation 3 of Lotka's Wheel, which is presented in my section 1.3, Garrett presents an equation for Y. My contention is that the reduction of Y to the constant ***w*** multiplied by *dE/dt* is an error. Because of this error, a nonsensical result was obtained: that GDP could be any number of currency units and *be reduced to zero by inflation*.

It does not matter if *W* appears linear or not. It does not matter what kind of statistics are done to derive values for *w*. What I am saying is that the correct equation for *Y* must still be used. In Garrett's *dE/dt* = 0, presumed steady-state scenario, it is this equation for *Y* that gives us the *dY/dt* value of zero that one would expect, and everything behaves sensibly.

Using the correct equation, with $g = 1.0$, then $Y(t) = Y_0 \cdot 1.0^t$ has a derivative, $Y_0 \cdot 1.0^t \cdot ln(1.0)$, that is zero. And, *dE/dt* does not directly enter into it.

One might expect (as Garrett does in Lotka's Wheel) that both *dE/dt* and *dY/dt* would be zero at the same time, except that the data suggests that when $dE/dt = 0$ then *dY/dt* should be low, but still above zero, at least for a while.

Concluding remarks

...As a suggested path forward, I suspect that a reconstruction of the timeline of civilization energy consumption in the distant past might be made using a combination of ice-core carbon dioxide concentrations, a carbon cycle model, and an assumption of a mean fuel makeup that does not minimize the widespread reliance on wind, water, and sunlight. People built where they could sail, navigate rivers, and grow crops because they offered forms of primary energy that could be efficiently accessed.
Tim Garrett

I think the suggestions for a path forward that Garrett gives in his concluding remarks are good ones. I also think this should be done in concert with archeologists and others that have wrestled with these issues before.

That said, perhaps we can work out a suggested provisional method for extending into the past prior to 1 CE. Such provisional method(s) would be superseded later, and could be an interesting comparison. If Tim is open to it I could make some suggestions.