Response to Reviewer Comments

We thank the reviewers for their continued careful reading of the manuscript and for the constructive feedback. We have addressed each point below and made corresponding revisions to the manuscript.

Major Comment 1: Excessive figures and runtime comparisons

We agree that several figures were redundant. We have removed Figures 9 and 10 (L∞ norm results for the 2D inclusion benchmark) and replaced them with a single sentence noting that the errors measured in the L∞ norm behave essentially the same as in the L_1 norm. We have similarly removed Figure 15 (the comparison of slab profiles at 40 My), replacing it with a sentence noting that the slab profiles are visually indistinguishable between the staircase and variational methods.

Regarding the runtime figures: we appreciate the reviewer's concern. We wish to clarify that in the previous version of the manuscript, the runtimes reported were cumulative totals over all 5,000 timesteps. In this revised version, the figures have been updated to show the mean Stokes solver runtime per timestep**,** averaged over 5,000 timesteps, which reduces the reported values by a factor of 5,000. The purpose of the runtime figures is not to benchmark the absolute performance of the solvers in isolation, but to compare the *relative* performance of the three free surface discretisation methods under identical conditions. Since each method runs within exactly the same code and infrastructure, these relative comparisons are meaningful regardless of the absolute values. We have updated the figure captions in Sections 5.4 and 5.5 to make the per-timestep averaging explicit.

Major Comment 2: Literature connectivity and the sticky air error attribution

We agree that the manuscript was insufficiently connected to the broader literature, and we have made two substantive revisions.

First, regarding the mention of alternative methods only in the conclusion: we have moved the discussion of immersed boundary methods and CutFEM to the Introduction (Section 1), where they now appear alongside the existing paragraph citing Larionov et al. (2017) and Verzicco (2023). We have also removed the characterisation that these methods are limited to stationary boundaries, which was inaccurate. The revised text acknowledges that immersed boundary methods have been applied to moving and deforming boundaries across a wide range of applications since at least the 1990s, including rotating turbine blades and blood vessel dynamics.

Second, regarding lines 279–280 and the attribution of sticky air errors to insufficient viscosity contrast: the reviewer is correct on both counts. The claim as written was unsupported and is in fact contradicted by the figures on the following page, which show that the largest errors are concentrated at the boundary rather than in the far field. We have revised this passage to align with the established literature, including Deubelbeiss and Kaus (2008) and Heister et al. (2017) — both of which are already cited in the manuscript — which demonstrate that near-boundary errors in the sticky air method are caused by the viscosity discontinuity itself and are not substantially reduced by increasing the viscosity contrast. The revised caption to Figure 7 already correctly states this, and we have updated the body text at lines 279–280 to be consistent with this interpretation. We note that Figure 7 now also cites Heister et al. (2017, Section 3.3) directly, which contains an extended discussion of these boundary artefacts and relevant prior literature.

Minor Issues

- **Paragraph indentation after equations:** We have removed the spurious blank lines between equations (2), (5), (7), (9), (10), and subsequent text in the LaTeX source, so that text continuing a sentence after an offset equation is no longer incorrectly indented as a new paragraph.
- **Line 149 ("optimised"):** We have replaced "optimised" with "a stationary point is sought for", accurately reflecting that this is a saddle point problem rather than a minimisation or maximisation.
- **Section 5.1, equivalence of zero-viscosity inclusion and free surface:** We have added a clarifying remark acknowledging that the equivalence between a zero-viscosity inclusion and a free surface boundary condition holds in this specific test case because the surrounding fluid is incompressible and the outer Dirichlet boundary conditions fix the total enclosed volume. We note that this equivalence does not hold in general, and have added a sentence to this effect.
- **Line 285:** We have inserted "norms" after $L\infty$.
- **Line 287:** We have corrected the viscosity ratios. The three ratios compared are 100, 200, and 1000 (consistent with the caption of Figure 8), and these are now listed in ascending order.
- **Equations (11) and (12):** We have added absolute value signs to the error terms, giving $|P_i - P_{i,a}|$ as appropriate.
- **Line 296:** The references to Figures 9 and 10 have been updated to appear in numerical order; as noted above, these figures have been removed and replaced with a single sentence.
- **Section 5.2 setup ordering:** We have moved the description of the model setup to precede Equation (14), so that the physical context is established before the exact solution is presented.
- **Eq. (14):** We thank the reviewer for catching this. The formula as written was incorrect and suggested unbounded growth of topography. The correct expression is the decaying exponential already given in Equation (13); Equation (14) previously contained a sign error in the exponent. This has been corrected.
- **Figure 16 caption ("rate"):** The y-axis label has been corrected to "distance" (km) to be consistent with what is actually plotted. The word "rate" has been removed from both the caption and the figure title.
- **Line 436:** These two references were inadvertently included and do not correspond to any citation in the surrounding text. They have been removed.

We agree with both structural suggestions.

- **Single subsections:** We have removed the subsection headings 2.2.1, 4.1.1, and 5.2.1, integrating their content directly into the parent sections 2.2, 4.1, and 5.2 respectively.
- **Parallel structure between Sections 2.1 and 2.2:** We have restructured Sections 2.1 (sticky air) and 2.2 (staircase method) so that both are presented at the same level of hierarchy, with no subsections, and with parallel organisation: in each case, we first describe the method, then enumerate its limitations. The two sections are now structurally symmetric.