Summary of Revisions

Manuscript Structure and Length

- Restructured and shortened from ~41 to ~26 pages
- Replaced thesis-like presentation with a more focused paper structure
- Condensed background material on existing methods
- Reduced numerical experiments to those most clearly demonstrating key properties

Title

- Updated to make clear the method is specifically for finite difference discretisations (variational finite difference approach)

Description of the Method (Section 3)

- Rewritten to more clearly explain how the continuous variational formulation is discretised on the staggered grid
- Now explicitly describes how volume integrals are approximated cellwise, how φ enters each term, and how discrete operators G, D, and M relate to the standard staggered-grid discretisation
- Stress elimination procedure leading to the final pressure-velocity system set out explicitly
- Now begins earlier in the manuscript, immediately following a condensed introduction to existing methods
- Sections 3.1 and 3.3 merged so that the general case including body forces and traction boundary conditions is presented from the outset
- Body force term now included consistently throughout
- Section 3.5 on non-Cartesian meshes removed

Relationship to Other Methods (Sections 2.1 & 2.2)

- Added brief discussion of related finite element approaches and how they relate to the present method
- Sections 2.1 and 2.2 shortened and reframed to motivate the variational scheme rather than read as a standalone review
- Suggested reference incorporated

Relationship to Larionov et al. (2017)

- Introduction clarified to specify contributions of this paper relative to Larionov et al. (2017): implementation in a geodynamic FD code with multigrid, and systematic comparison against sticky air and staircase methods

Implementation Details (Section 4)

- Section 4.1 revised: other volume fraction methods now briefly named, with a note that only Lagrangian tracking is used in this study
- Section 4.2 on air tracers removed
- Implementation details specific to StagYY reduced, with focus shifted to results
- Discussion of fast air flow and its effect on timesteps acknowledged and relevant text adjusted

- Statements about numerical instabilities toned down to avoid overstating generality beyond the specific StagYY context

Results

- Reduced to key benchmarks in 2D and 3D
- Several figures merged or removed; overlapping solid lines replaced with markers or symbols
- Scaling tests consolidated into fewer figures
- Error and mesh size axes in convergence plots now use logarithmic scales
- Non-convergence issue in the tracer distribution test discussed more fully
- Subduction benchmark revised, with near-surface viscosity errors connected more explicitly to Schmeling et al. (2008)
- 3D plume benchmark section removed

Summary and Conclusions

- Rewritten in prose rather than bullet points and shortened
- Typo fixed ("incompressible" corrected to "compressible")

Minor Corrections

- Equation punctuation corrected throughout
- Typographic error "on on the latter" corrected