Supplement

Figure S1. Comparison of two methods for calculating PBL top pressure in ERA5: both using the bulk Richardson number (see Methods), with one using properties from the lowest model level, following standard ECMWF practice, and the other from the 2 m level. (a) Joint histogram of PBL heights calculated from both methods, at the hottest time of day over 40°N–65°N land in JJA 2001-2021. The 1:1 line is shown in dashed black, and the correlation coefficient given above the panel. (b) Map of PBL top pressures at the hottest time of day on June 1st, 2001 based on the lowest model level. (c) as in (b), using the 2 m-based PBL top pressure.
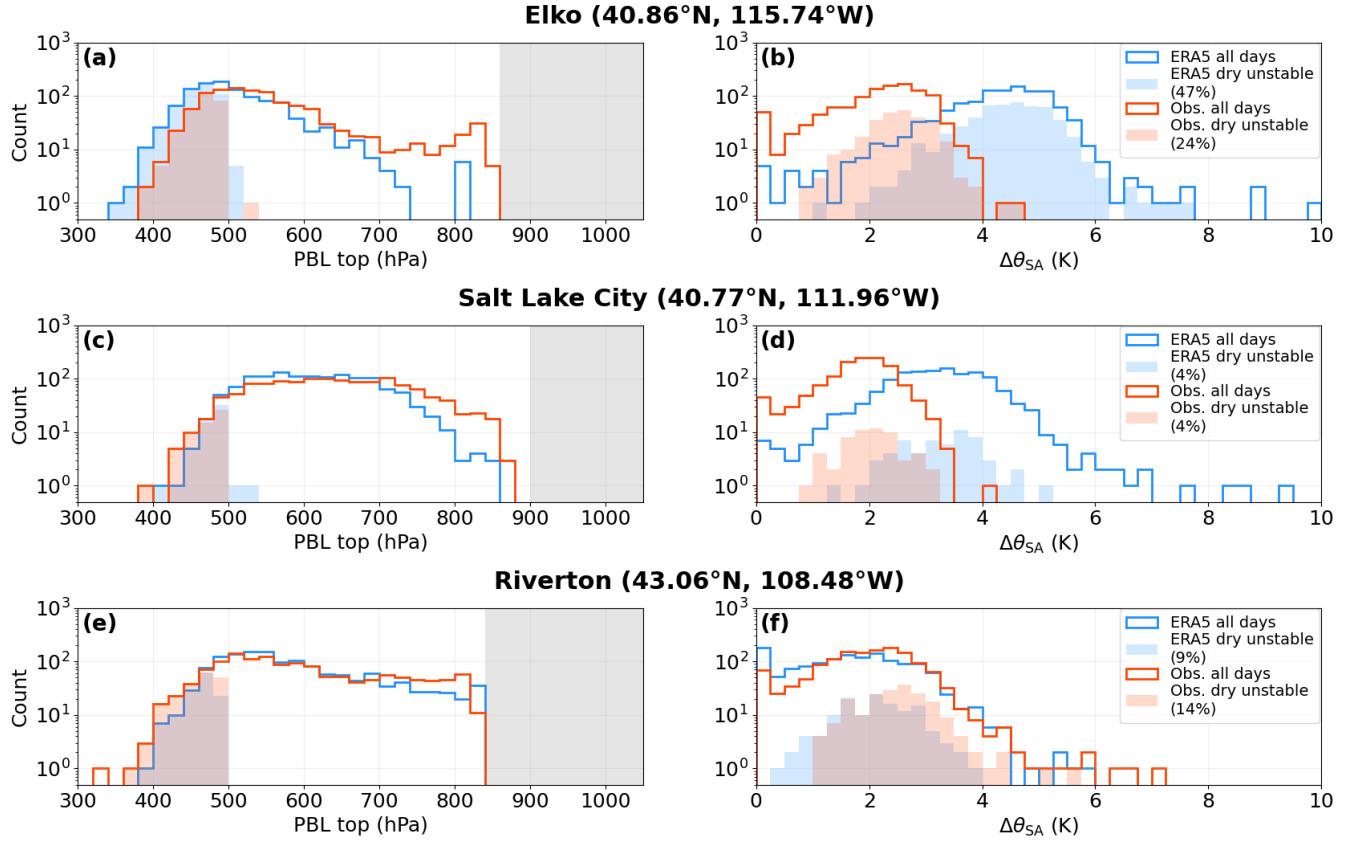
Figure S2. Characteristics of dry adiabatic bound exceedances at three sounding stations. **(a, b)** As in Fig. 3c, e. **(c,d)** and **(e, f)** As in (a, b) for the Salt Lake City and Riverton locations, respectively.
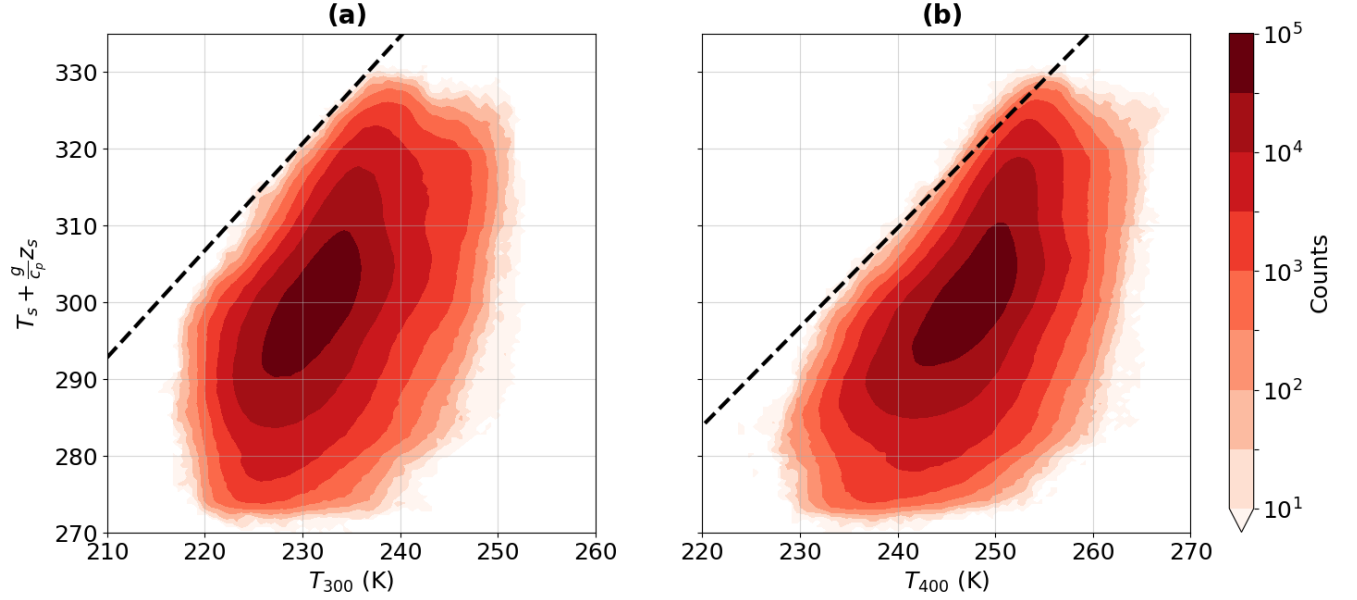
Figure S3. Dry adiabatic upper bound using different reference levels in the free troposphere. **(a)** As in Fig. 2a, except using the 300 hPa level as a reference level. The dashed line shows the DSE-based dry adiabatic bound, $T_s + gz_s/c_p = T_{300} + gz_{300}/c_p \simeq T_{300}[1 + g\overline{z_{300}}/(c_p\overline{T_{300}})]$ (see ZB23, the overbar denotes a time and spatial average over JJA 2001-2021, 40° N–65° N land), where $\overline{z_{300}}/\overline{T_{300}} = 40.3$ m K^{-1}. **(b)** as in (a) except using the 400 hPa level as a reference. The dashed line shows $T_s + gz_s/c_p = T_{400}[1 + g\overline{z_{400}}/(c_p\overline{T_{400}})]$ with $\overline{z_{400}}/\overline{T_{400}} = 29.7$ m K^{-1}.
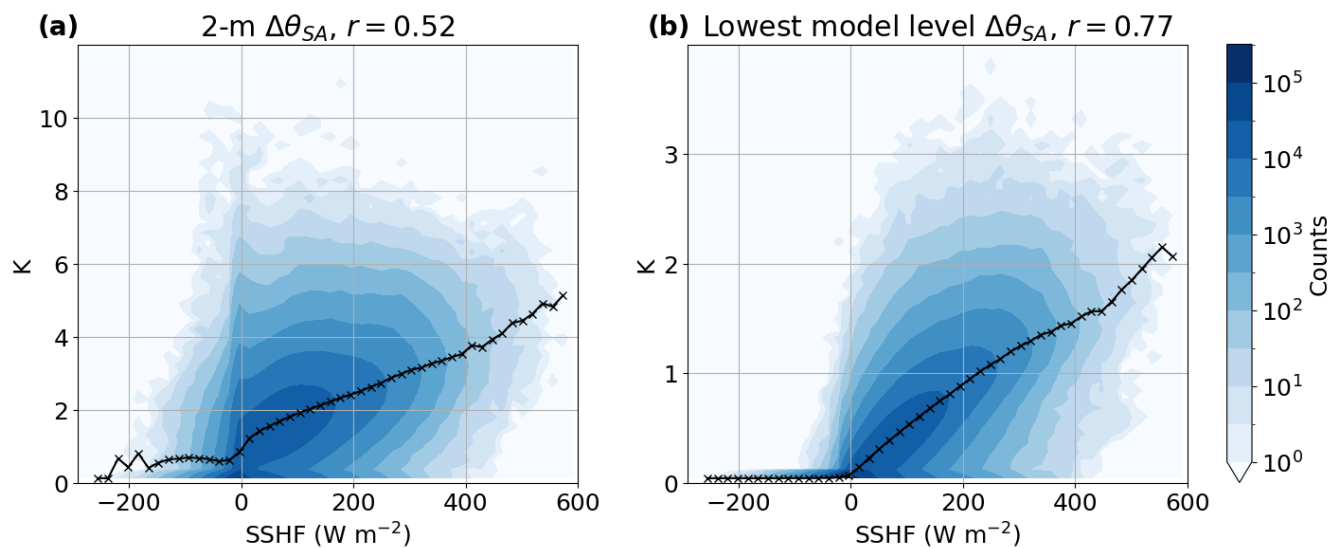
Figure S4. Relationship between surface sensible heat flux (SSHF) and superadiabatic layer strength. **(a)** Joint histogram of SSHF and $\Delta\theta_{\mathrm{SA}}$ (shading) over midlatitude land, at the hottest time of day during JJA 2001 in ERA5. The correlation coefficient between the two variables is given above the graph. The mean $\Delta\theta_{\mathrm{SA}}$ in each bin of SSHF is shown with black crosses. **(b)** As in (a), except that we use θ at the lowest model level, rather than the 2-m level, to calculate $\Delta\theta_{\mathrm{SA}}$. This version is thus less affected by the boundary layer parameterization.
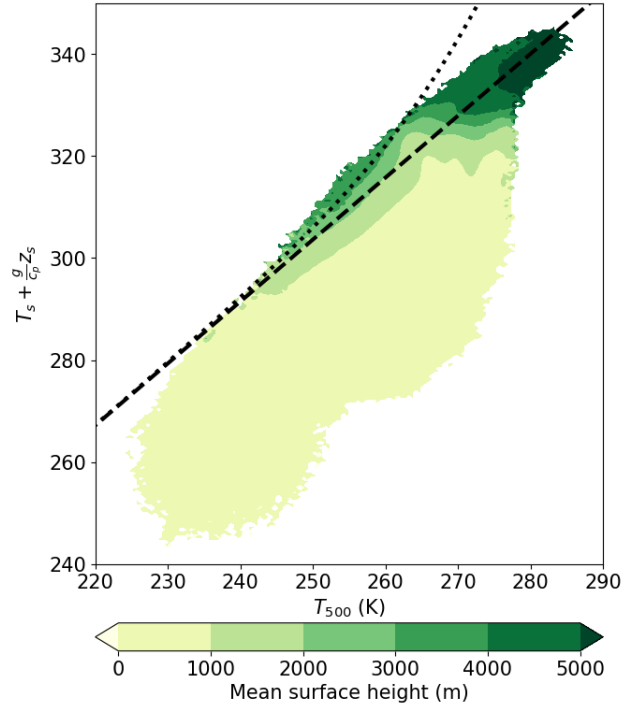
Figure S5. Mean surface height in the T_{500}–DSE_s phase space of Fig. 4. The dashed line and dotted line are as in Fig. 4.