NOTE:

Reviewers' original comments are in Cambria type with yellow highlighting.

Text from the revised manuscript is in bold italic font.

Page/Line/Figure numbers given by reviewers were left unchanged in their text and refer to the originally submitted version of the manuscript

[Line XXX] indicates line number in the annotated manuscript

Reply to Reviewer #1

I thank the authors for their thorough response, the revisions have significantly improved the manuscript. By retraining the SR models with structural constraints, the authors successfully resolved the dimensional inconsistency between A and Ksat. The resulting formulas are now more logically sound and physically rigorous. The updated expressions (F1–F3) are more transparent, and the added statistical evidence regarding Ksat and recession coefficients provides a convincing physical justification for the model's behavior. The additional comparison with deep learning-based symbolic regression confirms that the identified relationships are robust and not dependent on a specific optimization paradigm. The manuscript now meets the standards for publication in HESS.

We sincerely thank the reviewer for the encouraging assessment of our revised manuscript. We are grateful for the reviewer's constructive and thoughtful comments, which have helped us strengthen the manuscript substantially.

Reply to Reviewer #2

I commend the authors for their diligent and thorough revisions. My previous concerns regarding dimensional consistency, model interpretability, and evaluation thresholds have been adequately addressed. The manuscript has been substantially improved and is now in very good shape. I only have a few minor corrections and discussion points for the authors to consider before publication.

We sincerely thank the reviewer for the positive evaluation of our revised manuscript and for acknowledging the improvements made in response to the previous comments.

1. Physical implication of the additive vs. multiplicative structure (Section 5)

While the separation of A and \operatorname{K}_{sat} into independent terms successfully resolves the strict unit mismatch (as the empirical coefficients now consistently absorb the respective dimensions), it implies that the delays caused by catchment size and soil permeability are purely additive (i.e., independent parallel or sequential components). From a classical hydraulics perspective, travel time is fundamentally a multiplicative function of a length scale and inverse celerity. Specifically, considering that characteristic flow distance roughly scales with the square root of the catchment area (\operatorname{A}^{0.5}, e.g., Hack's law) (Hack, 1957) and subsurface flow velocity is proportional to hydraulic conductivity (\operatorname{K}_{sat} per Darcy's law) (Freeze and Cherry, 1979), the travel time could follow a coupled, multiplicative relationship (Time\ \propto Distance/Velocity\ \propto\operatorname{A}^{0.5}\cdot\operatorname{K}_{\ operatorname{sat}}^{-1}).

I suggest expanding the discussion (e.g., in one section of Discussion) to briefly touch upon this difference. It would be insightful to hear the authors' thoughts on why the SR algorithm might have favored an additive form over a multiplicative one—for instance, was it due to the complexity penalties applied to nested operators during the SR search, or does the additive structure genuinely offer a superior empirical fit? Furthermore, does this SR-derived additive structure have a plausible physical justification (e.g., representing distinct delay mechanisms such as horizontal channel routing vs. vertical infiltration), or does it act primarily as an empirical approximation of a classically coupled process? Adding this perspective would deepen the current discussion and highlight the fascinating boundaries bet

References

Freeze, R.A., Cherry, J.A., 1979. Groundwater. Prentice-Hall, Englewood Cliffs, N.J.
Hack, J.T., 1957. Studies of longitudinal stream profiles in Virginia and Maryland (Report No. 294B), Professional Paper. https://doi.org/10.3133/pp294B

We thank the reviewer for this insightful comment. Catchment area A is an indirect surrogate for characteristic flow length, whereas catchment-aggregated K_{sat} does not directly represent the Darcy velocity along a specific subsurface flow path. Therefore, imposing a strict multiplicative scaling relationship for A and K_{sat} may be overly idealized in this regionalization context. The additive structure may be more suitable because the

optimized parameter N is not a process-based travel time for a single hydraulic pathway, but an flow-duration parameter that integrates multiple sources of catchment delay. In this sense, the SR algorithm may have favored the additive form because it allows different controls to contribute independently to the persistence of event flow. Specifically, the area-dependent term may primarily represent horizontal routing, drainage network organization, and catchment-scale storage effects, whereas the K_{sat}-dependent term may represent effects of runoff generation, flow partitioning, infiltration capacity, and subsurface transport processes. Thus, the additive structure can be viewed as a parsimonious approximation of the aggregated catchment response rather than a strict hydraulic travel-time equation. The coefficients in the equations can be interpreted as scaling factors that absorb dimensional conversion and scale effects. We added the discussion on this point in the manuscript.

[Section 5.2, lines457-460]

Overall, the additive structure among A, K_{sat}, and f_{SWE} reflect the separate contributions of different catchment-scale delay processes to the event flow through different pathways and at different time scales. Their additive combination therefore provides a parsimonious empirical approximation of the integrated flow-duration response.

2. Typographical and Grammatical Corrections

Please correct the following minor language issues throughout the manuscript:

Abstract: "default to 5 days" should be "defaults to 5 days".

Introduction: Change "gaged catchment lacking..." to plural "gaged catchments lacking...". Additionally (Line 89), the term "ungagged catchments" appears to be a typographical error and should be corrected to "ungauged" or "ungaged" catchments.

Section 3.1: "A smaller N result in..." should be "A smaller N results in...".

Section 3.2: "The function space is consisted of..." is grammatically incorrect; please change to "The function space consists of...".

Section 4.3: "This may be because the SR formulas were calibrated... at the CONUS scale, their region-specific performance..." is a run-on sentence. Please replace the comma with a semicolon (e.g., "...scale; thus, their...").

Section 5.2: "...making more rainfall excess to rout through..." contains a typo. "rout" should be corrected to "route".

We thank the reviewer for carefully identifying these typographical and grammatical issues. All suggested corrections have been made in the manuscript.