

The advantages of data assimilation in parametric space rather than classic grid space

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Abstract. Data assimilation (DA), by merging observation and background information, is an important tool in the field of geosciences. However, in the presence of geophysical structures such as cyclones or ocean eddies, classic DA schemes in gridded space fail to properly estimate the structure properties, for example, their position and intensity. In this work, we propose a new DA scheme, in a reduced parametric space, which assimilates only the relevant parameters to describe the structures, with an application to a one-dimensional ocean eddy **focused on the single analysis step without the forecast step.** Comparison of DA performed in the classic gridded field and in the parametric space is made through a series of experiments with perturbed eddy parameters. Results show that DA in the parametric space can account for the nonlinearity of the eddy parameters and preserve eddy properties. **However, this** is not the case for classic DA in the gridded space. Moreover, DA in the parametric space **potentially reduces cost when reliable parameter extraction is available and the number of features is small relative to grid dimension.**

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1 Introduction

Data assimilation (DA) is widely used in geosciences to combine information from a dynamical model (the background) and observations (Carrassi et al., 2018). The classic approach consists in estimating the full state of the system, i.e., all variables on a given spatial grid.

In this study, we don't focus not on the full state but on specific geophysical structures (GSs), such as cyclones or ocean eddies. When sequential DA is applied in gridded space, linear Gaussian updates can struggle to correctly represent the position or shape of coherent structures, potentially altering their physical properties (Chen and Snyder, 2007). To address these challenges, the literature includes non-Gaussian methods such as particle filters (van Leeuwen, 2010; Poterjoy, 2016), as well as displacement-aware transformations of the state, including alignment or morphing (Ravela et al., 2007; Beezley and Mandel,

2008; Zhen et al., 2025). Feature-informed DA has also been proposed, where the characteristics of coherent structures guide the assimilation while the state remains defined in the gridded space (Srivastava et al., 2023).

The goal of our study is instead to examine a Gaussian and linear data assimilation method applied to a problem that is intrinsically non-Gaussian and nonlinear, namely the estimation of coherent structures with positional and shape mismatches. By focusing on this scenario, we aim to isolate the effects of performing assimilation directly on the structural parameters and to understand the limitations of classical linear DA methods without implementing more computationally intensive particle filters or sophisticated displacement-aware corrections.

Parametric DA belongs to the broader class of reduced subspace (see the review by Chen and Snyder (2007)), which include subspaces built from empirical orthogonal functions or autoencoders (Hannachi et al., 2007), leading modes of variability, or other parameterizations. However, these methods may struggle to accurately represent coherent GSs, whose variability is often governed by nonlinear geometrical transformations (Alvera-Azcárate et al., 2025). Unlike these approaches, which reduce dimensionality without explicitly preserving structure-specific parameters, our method directly targets the physical parameters defining the GS, allowing for a more interpretable estimation.

While previous work on feature-based, displacement-aware, or non-Gaussian DA has explored some of these aspects, the specific formulation of DA in a reduced parametric space targeting the defining parameters of coherent structures has been relatively little explored. Here, we aim to clarify its potential benefits and limitations in this context.

In this study, we propose a DA framework in which the state of the system is entirely defined by the parameters of a GS. We consider the case of an idealized ocean eddy, described by its position, radius, and amplitude. This formulation allows for a direct estimation of the statistical moments of the GS parameters within the chosen assimilation framework and ensures the preservation of the intrinsic properties of the structure. While DA in the parametric space can considerably reduce computational cost in this idealized setting due to the small dimensionality of the parameter vector, practical applications require additional computations for parameter detection, fitting, and multi-feature association, which may offset some of the computational savings.

We present a set of idealized numerical experiments designed to compare the single analysis step performed in a conventional gridded space and in this reduced parametric (object-based) space. The eddy representation and the assimilation methodology are detailed in Sect. 2, the experimental setup in Sect. 3, and the results are presented in Sect. 4. Conclusions and perspectives are discussed in Sect. 5.

2 Methodology

The objective of this paper is to compare two methods of blending two sources of information, a background and an observation, using a basic DA algorithm without the forecast step.

We consider a simplified one-dimensional cross-shore section of an ocean eddy, described by three parameters: the peak amplitude a , the peak position p , and the maximal velocity radius r such that $\mathbf{x}^{\text{param}} = (a, p, r)$. In practice, these parameters

can be estimated using standard eddy detection and fitting algorithms, such as local maxima identification combined with Gaussian profile fitting (e.g., Chelton et al., 2011).

55 For our idealized experiments, however, the parameters are prescribed manually, rather than obtained from a detection algorithm, in order to control the uncertainties and systematically study the assimilation behavior. The physical profile along the cross-section is then given by the mapping

$$\mathbf{x}^{\text{grid}} = h(\mathbf{lon}; \mathbf{x}^{\text{param}}) = a \exp \left(-\frac{1}{2} \left(\frac{\mathbf{lon} - p}{r} \right)^2 \right), \quad (1)$$

60 where $\mathbf{lon}$ is the vector of longitudinal positions along the cross-section. Here, h serves as a mapping function from the parametric space $\mathbf{x}^{\text{param}}$ to the physical gridded space $\mathbf{x}^{\text{grid}}$. It is not an observation operator nor a numerical dynamical model, but simply defines the physical profile corresponding to a given set of parameters.

Two spaces are considered for the eddy representation: the *parametric space* (param), of dimension $n = 3$ corresponding to (a, p, r) , and the *gridded space* (grid), of dimension m equal to the length of the longitude vector. The mapping function h allows conversion from the parametric to the gridded space.

65 In practice, parameters estimated from observations are subject to fitting and detection errors, as real oceanic eddies are not strictly Gaussian and may interact with neighboring structures. In our experiments, we mimic these uncertainties by prescribing errors manually. Two sources of uncertainty are considered: (1) representation and fitting errors, corresponding to deviations from the ideal Gaussian shape, and (2) background errors due to dynamical model uncertainties. We explicitly account for the second source through an ensemble that allows estimation of the background error covariance $\mathbf{B}$, while the first source
70 is applied to the observed eddy to combine fitting and mapping uncertainties through $\mathbf{R}$. This controlled setup allows us to examine the behavior of Gaussian linear DA in a scenario that is intrinsically nonlinear and non-Gaussian.

Considering background eddy parameters $\mathbf{x}^{b, \text{param}}$ and its associated surface height $\mathbf{x}^{b, \text{grid}} = h(\mathbf{lon}; \mathbf{x}^{b, \text{param}})$, we define the updated state vector:

$$\mathbf{x} = \mathbf{x}^b + \boldsymbol{\eta}^b, \quad (2)$$

75 with a stochastic error $\boldsymbol{\eta}^b \sim \mathcal{N}(\mathbf{0}, \mathbf{B})$ assumed to be Gaussian distributed with the background covariance matrix $\mathbf{B}$. This updated state vector is related to the observation by:

$$\mathbf{y} = \mathcal{H}(\mathbf{x}) + \boldsymbol{\varepsilon}, \quad (3)$$

where $\mathbf{y}$ is the observation vector, $\mathcal{H}$ is the observation operator, and $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the stochastic observation error assumed to be Gaussian distributed with the observation covariance matrix $\mathbf{R}$. Similarly to the background eddy, the observed eddy is
80 first defined by its parameters $\mathbf{y}^{\text{param}}$ and their associated surface height $\mathbf{y}^{\text{grid}} = h(\mathbf{lon}; \mathbf{y}^{\text{param}})$.

In our case, since the state vector is fully observed in both spaces, the operator $\mathcal{H}$ is the identity matrix, leading to a simple linear observation equation.

2.1 Tutorial case study

85 For the sake of simplicity, we first suppose that the parameters are fully independent and that the detection algorithm gives same values of uncertainties $\sigma^2 = [\sigma_a^2, \sigma_p^2, \sigma_r^2]$ for the background eddy and its observation, such that the error matrices $B^{\text{param}} = R^{\text{param}} = \sigma^2 I_3$, where σ is the standard deviation.

Since the uncertainties of the surface height in the reconstructed grid space depend on the highly nonlinear parameters, we employ an Ensemble Optimal Interpolation (EnOI). Considering the background ensembles $X^{b,\text{param}}$ and $X^{b,\text{grid}} = h(\text{lon}; X^{b,\text{param}})$ and the observation ensembles Y^{param} and $Y^{\text{grid}} = h(\text{lon}; Y^{\text{param}})$, the EnOI scheme is the following, with the update of the analysis ensemble mean $\bar{X}^a$

$$\bar{X}^a = \bar{X}^b + K(\bar{Y} - \bar{X}^b) \quad (4)$$

and of the analysis covariance P^a

$$P^a = (I - K)B, \quad (5)$$

95 where K is the Kalman gain, given by $K = B(B + R)^{-1}$, where $B = \text{Cov}(X^b, X^b)$ and $R = \text{Cov}(Y, Y)$.

The data assimilation is performed in both spaces, indicated with the superscript `param` and `grid`, using the following cost function J

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2}(y - x)^T R^{-1}(y - x), \quad (6)$$

100 The Figure 1 illustrates the setup of our experiments comparing assimilation in the parametric space and the classical gridded space.

2.2 Realistic case study

In a more realistic scenario, one often starts from gridded observations rather than direct parameter values. Let

$$105 \quad y^{\text{grid}} = h(\text{lon}; y^{\text{param},*}) + \varepsilon^{\text{grid}} \quad (7)$$

be the observed surface height along the cross-section, where $y^{\text{param},*}$ are the single observed eddy parameters. Similarly, the background in gridded space is

$$x^{b,\text{grid}} = h(\text{lon}; x^{b,\text{param},*}) + \eta^{b,\text{grid}}, \quad (8)$$

with $x^{b,\text{param},*}$ the background parameter.

110 For gridded DA, y^{grid} is assimilated directly using the exponentially correlated error matrices:

$$B_{ij}^{\text{grid}} = R_{ij}^{\text{grid}} = \sigma_h^2 \exp\left(-\frac{|\text{lon}_i - \text{lon}_j|}{\lambda}\right), \quad (9)$$

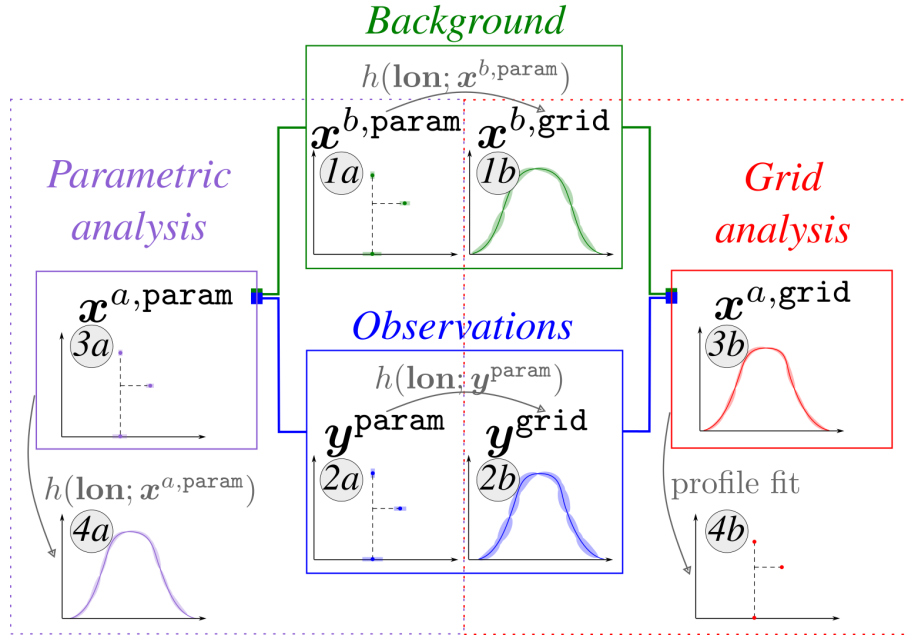


Figure 1. Schematic of the data assimilation workflow comparing parametric (purple) and gridded (red) spaces. For each approach, the sequence background (green box, panel 1) $\rightarrow$ observation (blue box, panel 2) $\rightarrow$ analysis (panels 3) is shown. Panels labeled “a” correspond to parametric space, “b” to gridded space. Background and observation parameters, respectively x^{param} and y^{param} , are mapped to physical profiles, respectively x^{grid} and y^{grid} , via the function h . Observations y^{param} , y^{grid} may differ from the background. The parametric analysis can be mapped back to the gridded space (panel 4a), and profile fitting in the gridded space allows retrieval of parameter estimates (panel 4b). This figure illustrates how uncertainty in one parameter (among a , p , r) propagates through the assimilation step in each representation.

where σ_h^2 is the surface height variance and λ the correlation length. In the gridded space, y^{grid} is directly assimilated.

For parametric DA, the parameters y^{param} and $x^{b,\text{param}}$ are first estimated from y^{grid} and $x^{b,\text{grid}}$ via a fitting procedure of the function h , and the covariance matrices R^{param} and B^{param} are derived from the fitting process. y^{param} is then assimilated in the parametric space. This workflow tests whether the parametric approach remains advantageous once the inverse nonlinear mapping from gridded observations is accounted for.

3 Experiments

The comparison of DA in both spaces is tested with a first set of three experiments, each focusing on a single uncertain parameter, while the other two parameters are set to their true values. In each experiment, the uncertain parameter differs between the observation and the background, whereas the other two parameters are identical in both. Specifically, the uncertain parameter in the three experiments is respectively: amplitude a (Experiment #1), position p (Experiment #2), and radius r (Experiment #3). This setup allows us to isolate the effect of perturbing a single parameter on the assimilation results. In these

Table 1. Values of the eddy parameters in each experiment setting.

Experiment	Eddy	Amplitude a [m]	Position p [km]	Radius r [km]
#1	Background	1.0 ± 0.7	0	7
	Observation	2.0 ± 0.7	0	7
#2	Background	1.0	-5 ± 7	7
	Observation	1.0	5 ± 7	7
#3	Background	1.0	0	6 ± 4
	Observation	1.0	0	14 ± 4

idealized experiments, the background and observation errors are set to the same magnitude. This controlled setup ensures that the analysis is expected to lie between the background and the observation, allowing a clear evaluation of the assimilation performance in parametric versus gridded space. An ensemble of 10000 members is considered both for the background and the observation. The ensemble members for both the background and the observation in the `param` space are generated by randomly perturbing the parameters a , p and r around σ^2 . The ensemble members in the gridded space are generated by applying Eq. (1) to the parametric ensemble. The Table 1 gives the parameterizations for both experiments.

A second set of one experiment is conducted to test the sensitivity of the parametric DA scheme to realistic operational cycles, see Sect. 2.2. Based on the Experiment #2 parametrization for $y^{\text{param},*}$ and $x^{b,\text{param},*}$, the associated surface heights for both observed and background eddies given by Eqs. (7) and (8) in the `grid` space were perturbed with the Gaussian kernel given Eq. (9) with $\sigma_h = 0.2$ m and $\lambda = r/2$. This experiment is called Experiment #4 and correspond to a position-perturbed experiment after parametric eddy fitting.

For all the four experiments, two comparisons are performed: the reconstruction of the surface height and the estimation of the eddy parameters. To compare the surface height reconstruction, the assimilated parameters are projected into the `grid` space through Eq. (1). To evaluate the estimation of eddy parameters, the assimilated gridded products are projected into the `param` space through a Gaussian profiled fitting.

4 Results and discussion

4.1 Tutorial case study

The surface height profiles as a function of the longitude are shown in Figs. 2, 3 and 4 for the amplitude, position and radius perturbation experiments, respectively. In Fig. 2a), we find the same analyzed profiles for both `param` and `grid` spaces whose height is the exact mean of the background and observation heights. The amplitude of the analyzed eddies is estimated to be at 1.5 m, without changes in the position or the radius. Figure 2b) also shows same results in both `param` and `grid` spaces for

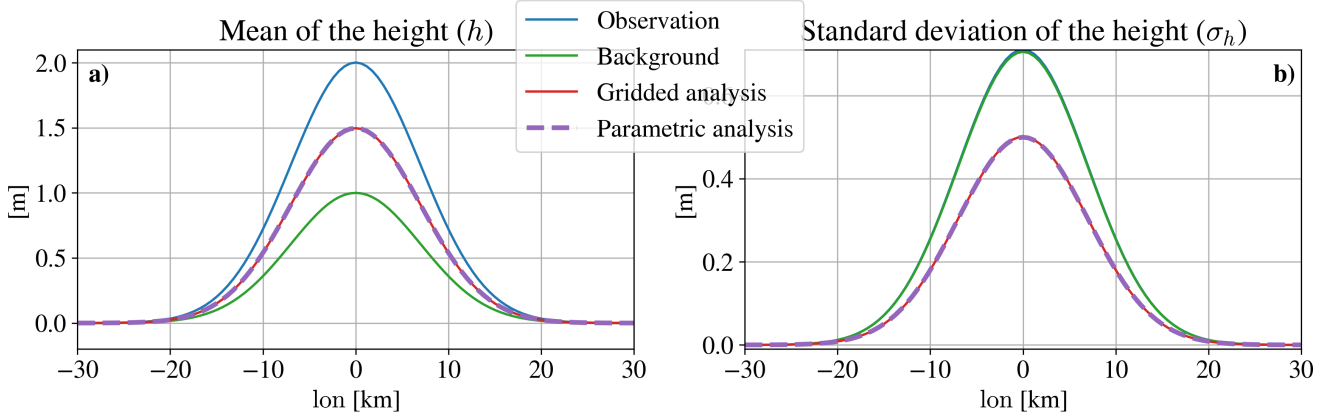


Figure 2. Results for the amplitude perturbation experiment. Left: **Mean of the** surface height of observed (blue) and background (green) eddy, and resulting height from gridded assimilation (red) and parametric assimilation (dashed purple) along the longitude. Right: same label for the standard deviation $\sigma_h = \sqrt{\text{diag}(\mathbf{P}^a)}$ of surface height along the longitude.

the estimation of the a posteriori error of the data assimilation given in Eq. (5). Those same results between `grid` and `param` assimilation are due to the fact that amplitude is a linear parameter of the function h defined in Eq. (1).

By contrast, in Fig. 3a), the analyzed height profile in the `grid` space displays a flattened and widened shape compared to the one in the `param` space. Indeed, though the position is accurate in the gridded space, the amplitude is underestimated and the radius is overestimated (0.8 m and 9 km, respectively). This result reflects the nonlinear effect of the position p in the definition of the function h in Eq. (1). The analyzed profile in the `param` space displays similar amplitude (1.0 m) and radius (7 km) to the background and to the observation, and the position is the exact mean of the two, i.e. 0 km. In Fig. 3, the analyzed standard deviation in the `grid` space is also underestimated compared to the `param` space, which seems more realistic.

The effect of the parameters' nonlinearity is also depicted in Fig. 4a) for Experiment #3 in the case of the radius perturbation. The `grid` height analyzed profile displays a narrower peak with a wider tail compared to the `param` profile. A further analysis shows that the `grid` analyzed profile is no longer Gaussian, its slope having decreased. On the contrary, in the `param` space, the estimated radius corresponds to the mean of the observed and background radii. Moreover, the profile is still Gaussian in the `param` space. The estimated amplitude and position are accurate in both spaces and correspond to the observation and background parameters. Similarly to Experiment #2, Figure 4b) shows an underestimation of the `grid` standard deviation compared to the `param` one.

By deforming the surface height profile of the eddy, the `grid` DA affects the dynamic properties of the reconstructed eddies. In Experiment #2, the `grid` analyzed eddy has a widened and flattened surface height compared to the background and its observation, **leading to a modified geostrophic current field , an oceanic current in which the pressure gradient given by the height horizontal gradient is balanced by the Coriolis effect.** It leads to a reduction in the rotational velocity, therefore to an underestimation of the Eddy Kinetic Energy (EKE) , **proportional to the square of the geostrophic current over the eddy area.**

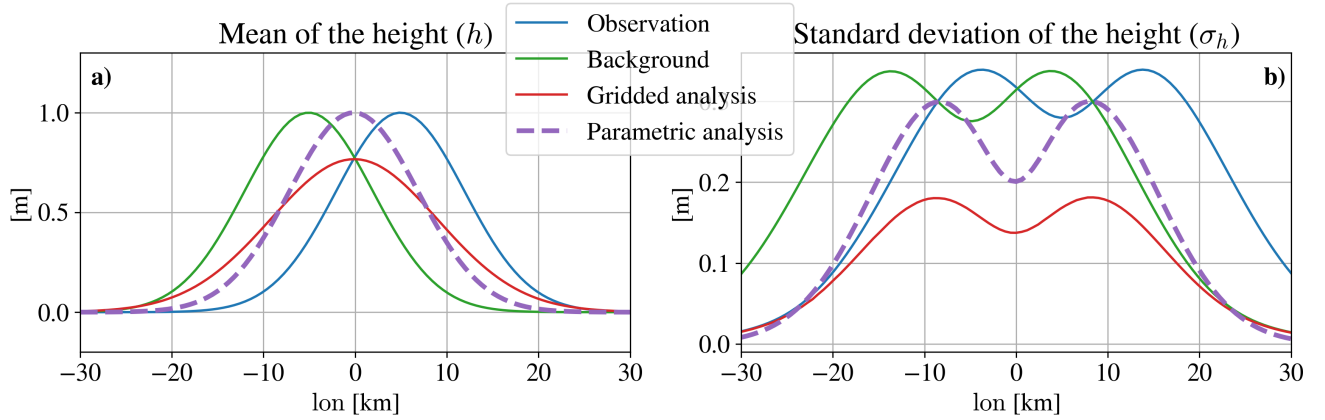


Figure 3. Results for the position perturbation experiment. Left: **Mean of the** surface height of observed (blue) and background (green) eddy, and resulting height from gridded assimilation (red) and parametric assimilation (dashed purple) along the longitude. Right: same label for the standard deviation $\sigma_h = \sqrt{\text{diag}(\mathbf{P}^a)}$ of surface height along the longitude.

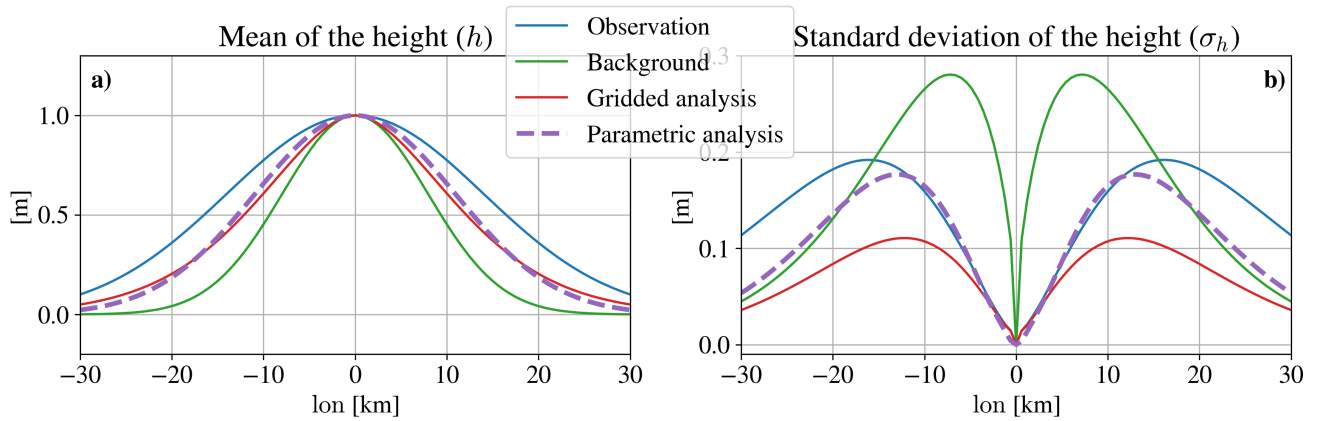


Figure 4. Results for the radius experiment. Left: **Mean of the** surface height of observed (blue) and background (green) eddy, and resulting height from gridded assimilation (red) and parametric assimilation (dashed purple) along the longitude. Right: same label for the standard deviation $\sigma_h = \sqrt{\text{diag}(\mathbf{P}^a)}$ of surface height along the longitude.

Quantifying the EKE provides a robust measure of their intensity and is key to assessing their contribution to lateral transport, air-sea interactions, and the mesoscale energy budget of the ocean. The value of the total EKE is reduced by 67% compared to the value obtained with the reconstructed profile from the `param` space analyzed parameters. In Experiment #3, the decrease in the profile slope leads to smaller height gradients, hence to smaller rotational velocities and underestimated EKE. The value of the total EKE is reduced by 11% compared to the `param` value.

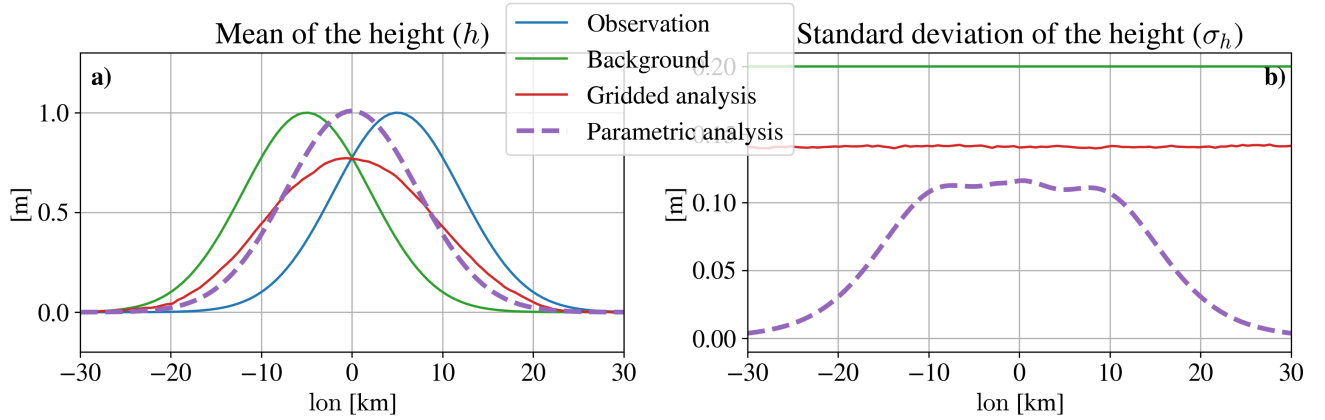


Figure 5. Results for the position perturbed experiment after parametric eddy fitting. Left: Mean of the surface height of observed (blue) and background (green) eddy, and resulting height from gridded assimilation (red) and parametric assimilation (dashed purple) along the longitude. Right: same label for the standard deviation $\sigma_h = \sqrt{\text{diag}(\mathbf{P}^a)}$ of surface height along the longitude.

170 4.2 Realistic case study

Figure 5 shows the results for Experiment #4, the position-perturbed experiment after parametric eddy fitting. It shows similar mean profiles of the eddy surface heights compared to the Experiment #2 with an underestimation of the a posteriori amplitude for DA in the `grid` space and a good estimation for DA in the `param` space. The differences observed in the standard deviation of the height compared to Experiment #2 are explained by the different choices in the covariance settings between the transition from the `param` space to the `grid` one (Experiment #2) and the transition from the `grid` space to the `param` one. In Figure 5b), the standard deviation along the longitude is unchanging and has same value (around 0.2 m) for both observed and background eddies and the a posteriori estimation in the `grid` space is around $0.2/\sqrt{2}$ m and also spatially uniform. In contrast, DA in the `param` space produces a space-varying standard deviation that directly reflects uncertainty in the parameters controlling the eddy, here the peak position, around the range of observed and background positions where the eddy is likely to be found.

This behavior is consistent with the structure of the covariance matrices in the `param` space. The covariance matrices exhibits significant variances for the parameters a , p and r as estimated through the fitting procedure applied to h as well as non-negligible cross-correlations between these parameters, indicating that the covariance matrix is no longer diagonal. These correlations reflect coupled uncertainties, for instance between position and amplitude or spatial extent, which further contribute to shaping the spatial distribution of variance. After assimilation, the posterior covariance matrix $\mathbf{P}^{a,\text{param}}$ shows a substantial reduction in uncertainty, with variances approximately halved compared to their prior values. This reduction directly translates

into a more confined and structured standard deviation profile in physical space, consistent with the improved localization of the eddy.

5 Conclusions

190 This paper explores DA in the parametric space of a 1D simplified ocean eddy compared to the DA in the classic gridded space with a focus on a single analysis step.

The experiments show that performing DA in the gridded space gives a good estimation of the eddy profile if only the linear parameters are perturbed, here the amplitude a . If nonlinear parameters are perturbed, here the position p and the radius r , the estimated eddy profile is deformed, with modification of the eddy parameters compared to the observed and background
195 inputs. This impacts the quantification of dynamical properties, i.e. the kinetic energy and the geostrophic currents. Moreover, still in the gridded space, the a posteriori error is underestimated.

In contrast, DA in the parametric space provides an improved estimation of the parameters in this simplified test case, with realistic values of a posteriori errors. Performing DA in the `param` space also reduces the dimensionality of the system, which is limited by the number of selected parameters rather than the resolution of the grid. Furthermore, it does not require a large
200 ensemble, whereas DA in the `grid` space requires a minimum of 25 members to avoid degradation of eddy properties. These results suggest that DA in `param` space may be particularly useful for well-defined geophysical structures, such as mesoscale ocean eddies, and potentially others, including cyclones, algal blooms, wildfires, or rainfronts.

One important aspect not explored in depth here but that warrants further investigation is the definition of error matrices in the parametric space. In particular, the impact of correlated parameter errors, should be considered in future work. Sensitivity
205 tests on this coupling and the state dependence of the error covariance matrix R^{param} could provide deeper insights into the behavior of the assimilation system. Understanding the correlation structure between parameters, especially for non-linear systems, would be crucial for refining error estimates and improving the robustness of the DA process. Moreover, quantifiable metrics to compare the quality and parameter dependence of each method would be a significant improvement, providing clearer guidelines for the operational implementation of DA in the parametric space.

210 Future work will focus on applying this DA scheme in the parametric space within an operational framework for mesoscale ocean eddy forecasting with realistic detection and fitting pipelines. While standard gridded DA relies on physical dynamical models such as the Navier-Stokes equations, the parametric approach lacks an explicit model for the temporal evolution of eddy parameters. Consequently, a machine learning model is required to learn these dynamics from data, and satellite observations may provide location and shape parameters of the eddies.

215 The framework also opens perspectives for hybrid approaches: the analysis can either be projected back into the physical gridded space for standard post-processing and visualization, or remain in the object-based parametric space for applications that exploit eddy characteristics directly. Such object-based applications benefit from the accurate inversion of parameters, allowing the DA to feed downstream forecasting or impact models efficiently. This approach could furthermore be extended to

220 other assimilation methods involving dynamic integration, such as the Kalman filter, while preserving the intrinsic properties
of the geophysical structure.

Code availability. The code used for the Experiments is available there: <https://gitlab.imt-atlantique.fr/s24dealb/param-grid-1d-eddy>

Author contributions. SD developed the code and performed the experiments and prepared the manuscript with contributions from co-authors, SR proposed the research topic, provided knowledge and background, performed the seminal experiments, and offered mentorship throughout this research, PT and CGB provided knowledge and assistance during code development, CL and BB helped with the redaction
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