Response to reviewer #2

April 13, 2026

We greatly appreciate the reviewer's time and constructive effort in reviewing our manuscript. The suggested modifications and recommendations have been a decisive help to revise and strengthen our methodology and results. Specific comments are addressed one by one hereafter. Reviewer comments are recalled below in bold font, while text extractions from the revised paper are in italic font.

1 Major Comments

1. The central methodological comparison appears asymmetric in what is treated as "observed". In Parameter-space DA the experiments effectively assume access to observations in parameter space, i.e. a noisy (a, p, r) with simple Gaussian errors (diagonal, equal-variance structure). Under these assumptions, the parameter update is close to a textbook low-dimensional Gaussian analysis. In that regime the analysis can indeed be interpreted as the Bayesian posterior mean (under linear-Gaussian assumptions).

At the same time in Grid-space DA the gridded state distribution is induced by pushing parameter uncertainty through the nonlinear map $\theta \mapsto h(;\theta)$. For displacement (p) or scale (r) uncertainty, this induced distribution over h is nonlinear and thus inherently non-Gaussian (mixture-like / multi-modal). A Kalman/EnOI-style update uses only mean/covariance information, i.e. implicitly "Gaussianizes" the non-Gaussian problem, which is known to produce smearing for displaced coherent structures.

It reads to me like the presented experiments primarily demonstrate that the Gaussian analysis behaves better in coordinates where uncertainty is closer to Gaussian, rather than providing an end-to-end fair comparison between DA schemes. In a practical setting there appears to be no reason to assume the parameters, predicted and observed, would be Gaussian, while the grid prediction and observations are not.

The assumption that (a, p, r) are so neatly Gaussian, needs justification, if even possible. Please clarify the intended practical workflow:

- **(a) Are parameter observations (a, p, r) assumed to be provided externally by a detection/fitting pipeline with known uncertainty?**
- **(b) If yes, how is this uncertainty quantified in practice (correlations, state dependence, missed detections, multi-feature association)?**
- **(c) If no, how are parameters inferred from raw observations within the DA system?**

Thank you for this relevant comment. We agreed that we should clarify how uncertainties on the eddy parameters are handled. In this study, the parameters are not obtained through an automated detection or fitting procedure. Instead, we prescribe the associated errors manually. In particular, parameters such as the eddy shape and position are subject to fitting errors, since real oceanic vortices are not strictly elliptical and may exhibit significant surface ageostrophic contributions, as well as interactions with neighboring structures. This defines a first source of uncertainty (case 1), corresponding to representation and fitting errors. A second source of uncertainty (case 2) arises from errors in the underlying dynamical model, which can typically be quantified using ensemble-based approaches. In the present work, we explicitly address case 2 by assuming access to an ensemble that allows the estimation of the background error covariance matrix $\boldsymbol{B}$. In contrast, for the observed eddy, only case 1 is considered, combining fitting uncertainties with additional mapping errors associated with the reconstruction of the surface fields. We revised the manuscript lines 65-71: *In practice, parameters estimated from observations are subject to fitting and detection errors, as real oceanic eddies are not strictly Gaussian and may interact with neighboring structures. In our experiments, we mimic these uncertainties by prescribing errors manually. Two sources of uncertainty are considered: (1) representation and fitting errors, corresponding to deviations from the ideal Gaussian shape, and (2) background errors due to dynamical model uncertainties. We explicitly account for the second source through an ensemble that allows estimation of the background error covariance* $\boldsymbol{B}$*, while the first source is applied to the observed eddy to combine fitting and mapping uncertainties through* $\boldsymbol{R}$*. This controlled setup allows us to examine the behavior of Gaussian linear DA in a scenario that is intrinsically nonlinear and non-Gaussian.*

2. To strengthen the claims and make the comparison more symmetric, I recommend adding at least one experiment in which both methods start from the same noisy gridded observations:

$$y_h(x) = h(x; \theta^\star) + \epsilon(x)$$

with specified sampling and noise model (potentially sparse/irregular).

For Grid DA assimilate y_h directly (as currently). For Param DA derive $\hat{\theta} = g(y_h)$ via the stated fitting/detection method and estimate an observation error covariance R_θ for $\hat{\theta}$ (likely non-diagonal and state dependent; e.g. a–r coupling). Then assimilate $\hat{\theta}$ in parameter space.

This would test whether the parametric approach remains advantageous once it "pays" for the inverse nonlinearity that is currently idealized away. This likely also mirrors a real practical setting more closely, and reflects the real computational cost more realistically then the current setting.

Thank you for this new experiment suggestion. It helped us to strongly support the results of the previous experiments. We added several paragraphs related to the experiment with different eddy positions in the Methodology, Experiments and Results parts.

At the end of the Methodology, we added the following paragraphs and equations lines 106-116:

In a more realistic scenario, one often starts from gridded observations rather than direct parameter values. Let

$$\boldsymbol{y}^{\mathtt{grid}} = h(\mathbf{lon}; \boldsymbol{y}^{\mathtt{param},\star}) + \boldsymbol{\varepsilon}^{\mathtt{grid}} \tag{7}$$

be the observed surface height along the cross-section, where $\boldsymbol{y}^{\mathtt{param},\star}$ *are the single observed eddy parameters. Similarly, the background in gridded space is*

$$\boldsymbol{x}^{\mathrm{b},\mathtt{grid}} = h(\mathbf{lon}; \boldsymbol{x}^{\mathrm{b},\mathtt{param},\star}) + \boldsymbol{\eta}^{b,\mathtt{grid}}, \tag{8}$$

with $\boldsymbol{x}^{\mathrm{b},\mathtt{param},\star}$ *the background parameter.*

For gridded DA, $\boldsymbol{y}^{\mathtt{grid}}$ *is assimilated directly using the exponentially correlated error matrices:*

$$\boldsymbol{B}^{\mathtt{grid}}_{ij} = \boldsymbol{R}^{\mathtt{grid}}_{ij} = \sigma_h^2 \exp\left(-\frac{|\mathrm{lon}_i - \mathrm{lon}_j|}{\lambda}\right), \tag{9}$$

where σ_h^2 *is the surface height variance and* λ *the correlation length. In the gridded space,* $\boldsymbol{y}^{\mathtt{grid}}$ *is directly assimilated.*

For parametric DA, the parameters $\boldsymbol{y}^{\mathtt{param}}$ *and* $\boldsymbol{x}^{b,\mathtt{param}}$ *are first estimated from* $\boldsymbol{y}^{\mathtt{grid}}$ *and* $\boldsymbol{x}^{\mathrm{b},\mathtt{grid}}$ *via a fitting procedure of the function h, and the covariance matrices* $\boldsymbol{R}^{\mathtt{param}}$ *and* $\boldsymbol{B}^{\mathtt{param}}$ *are derived from the fitting process.* $\boldsymbol{y}^{\mathtt{param}}$ *is then assimilated in the parametric space. This workflow tests whether the parametric approach remains advantageous once the inverse nonlinear mapping from gridded observations is accounted for.*

and similarly at the end of the Experiments lines 129-132: *A second set of one experiment is conducted to test the sensitivity of the parametric DA scheme to realistic operational cycles, see Sect. 2.2. Based on the*

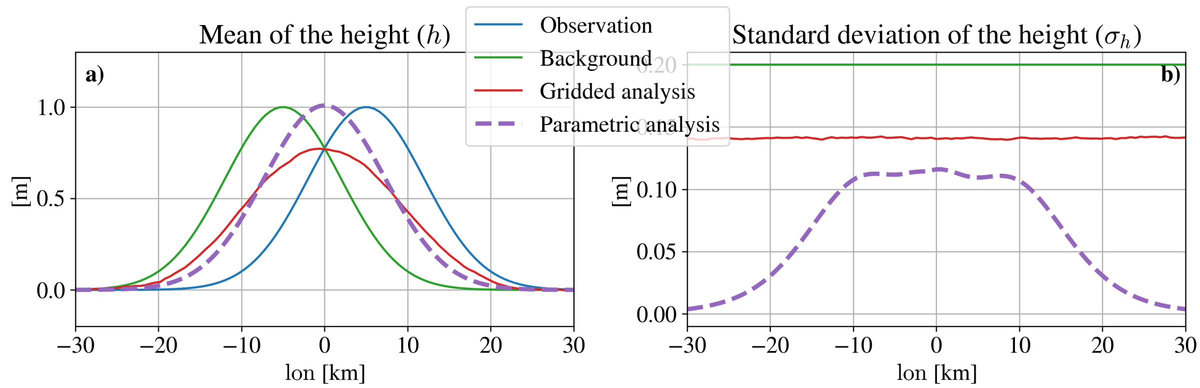

Figure 5: Results for the position perturbation experiment. Left: Mean of the surface height of observed (blue) and background (green) eddy, and resulting height from gridded assimilation (red) and parametric assimilation (dashed purple) along the longitude. Right: same label for the standard deviation $\sigma_h = \sqrt{\mathrm{diag}(\boldsymbol{P}^{\mathrm{a}})}$ of surface height along the longitude.

Experiment #2 parametrization for $\boldsymbol{y}^{\texttt{param},\star}$ *and* $\boldsymbol{x}^{b,\texttt{param},\star}$*, the associated surface heights for both observed and background eddies given by Eqs. 7 and 8 in the* **grid** *space were perturbed with the Gaussian kernel given Eq. 9 with* $\sigma_h = 0.2$ *m and* $\lambda = r/2$*. This experiment is called Experiment #4 and correspond to a position-perturbed experiment after parametric eddy fitting.*

We made an experiment starting with the values of parameters set for the position experiment #2 and σ_h=0.2m. Figure 5 in the revised manuscript displays the results for this experiment.

The Results have been updated lines 170-186: *Figure 5 shows the results for Experiment #4, the position perturbed experiment after parametric eddy fitting. It shows similar mean profiles of the eddy surface heights compared to the Experiment #2 with an underestimation of the a posteriori amplitude for DA in the* **grid** *space and a good estimation for DA in the* **param** *space. The differences observed in the standard deviation of the height compared to Experiment #2 are explained by the different choices in the covariance settings between the transition from the* **param** *space to the* **grid** *one (Experiment #2) and the transition from the* **grid** *space to the* **param** *one. In Figure 5b), the standard deviation along the longitude is unchanging and has same value (around 0.2 m) for both observed and background eddies and the a posteriori estimation in the* **grid** *space is around* $0.2/\sqrt{2}$ *m and also spatially uniform. In contrast, DA in the* **param** *space produces a space-varying standard deviation that directly reflects uncertainty in the parameters controlling the eddy, here the peak position, around the range of observed and background positions where the eddy is likely to be found.*

This behavior is consistent with the structure of the covariance matrices in the **param** *space. The covariance matrices exhibits significant variances for the parameters* a*,* p *and* r *as estimated through the fitting procedure applied to* h *as well as non-negligible cross-correlations between these parameters, indicating that the covariance matrix is no longer diagonal. These correlations reflect coupled uncertainties, for instance between position and amplitude or spatial extent, which further contribute to shaping the spatial distribution of variance. After assimilation, the posterior covariance matrix* $\boldsymbol{P}^{a,\texttt{param}}$ *shows a substantial reduction in uncertainty, with variances approximately halved compared to their prior values. This reduction directly translates into a more confined and structured standard deviation profile in physical space, consistent with the improved localization of the eddy.*

The notebook has also been modified there: https://gitlab.imt-atlantique.fr/s24dealb/param-grid-1d-eddy.

3. The abstract states that classic DA in gridded space fails to estimate position and

intensity in the presence of coherent structures. This is plausible for Gaussian/linear updates without displacement correction, but reads too broadly. There exists a literature on displacement-aware approaches (alignment/morphing/registration, feature-based increments, mixture methods, particle filters, etc.). I recommend either narrowing the claim to the class of Gaussian/linear updates without alignment corrections, or including at least one baseline that attempts a minimal displacement-aware correction in grid space (e.g. shift registration pre-processing) to avoid a strawman comparison.

We thank the reviewer for this insightful comment. We agree that there exists a literature on displacement-aware data assimilation, including alignment, morphing, feature-based increments, mixture methods, and particle filters. Our statement in the abstract and introduction regarding the failure of classic DA in gridded space is meant specifically for linear Gaussian updates applied directly in the gridded state space without any alignment or displacement correction.

To clarify this, we have revised and Introduction as follows lines 16-27: *In this study, we don't focus on the full state but on specific geophysical structures (GSs), such as cyclones or ocean eddies. When sequential DA is applied in gridded space, linear Gaussian updates can struggle to correctly represent the position or shape of coherent structures, potentially altering their physical properties [Chen and Snyder, 2007]. To address these challenges, the literature includes non-Gaussian methods such as particle filters [van Leeuwen, 2010, Poterjoy, 2016], as well as displacement-aware transformations of the state, including alignment or morphing [Ravela et al., 2007, Beezley and Mandel, 2008, Zhen et al., 2025]. Feature-informed DA has also been proposed, where the characteristics of coherent structures guide the assimilation while the state remains defined in the gridded space [Srivastava et al., 2023]. The goal of our study is instead to examine a Gaussian and linear data assimilation method applied to a problem that is intrinsically non-Gaussian and nonlinear, namely the estimation of coherent structures with positional and shape mismatches. By focusing on this scenario, we aim to isolate the effects of performing assimilation directly on the structural parameters and to understand the limitations of classical linear DA methods without implementing more computationally intensive particle filters or sophisticated displacement-aware corrections.*

4. It is further claimed that parameter-space DA considerably reduces computational cost. This seems obvious in the discussed synthetic setting given the dimension of the parameter space is lower than that of the grid space, but ignores that in practical settings obtaining the parameters, predicted and observed, along with their covariance comes at a cost as well. The current setup, also uses large ensembles in either case, and no timing/complexity evidence is provided. In realistic applications, parameter DA also presupposes a detection/fitting pipeline and (potentially) multi-feature association, which carry costs.

I suggest either: adding explicit wall-clock timing / complexity scaling (dimension m vs. $n = 3$, ensemble size dependence, cost of fitting/detection), or softening the statement to a conditional claim ("potentially reduces cost when reliable parameter extraction is available and the number of features is small relative to grid dimension").

This important point has also been raised by Reviewer #1. We agree that in realistic applications, obtaining the parameters for both the background and observations, along with their covariances, requires additional computations, including detection, fitting, and potentially multi-feature association. In the present study, which uses an idealized synthetic setting with a single coherent structure and known analytical representation, the computational cost of parameter-space data assimilation is reduced compared to gridded-space assimilation primarily due to the lower dimensionality of the state vector.

We have revised the manuscript to clarify this point. The relevant sentence now reads in Introduction lines 40-43:

While DA in the parametric space can considerably reduce computational cost in this idealized setting due to the small dimensionality of the parameter vector, practical applications require additional computations for parameter detection, fitting, and multi-feature association, which may offset some of the computational savings

and in the Conclusion lines 215-216: *Such object-based applications benefit from the accurate inversion of* *parameters, allowing the DA to feed downstream forecasting or impact models efficiently.*

2 Minor comments

1. The claim that DA in reduced parameter/feature space has not been studied seems overreaching and needs softening. Rather elaborate how this fits in with existing feature- based DA and displacement aware DA.

We thank the reviewer for this comment. We agree that feature-based and displacement-aware data assimilation methods have been studied extensively. To clarify the novelty of our work, we have softened the language in the manuscript and now emphasize the distinction between these approaches and our parametric assimilation. Specifically, feature-based or displacement-aware methods operate in the gridded state space and use features of coherent structures to guide the analysis, whereas in our approach the parameters describing the structure constitute the state itself. This allows a direct estimation of the structure's properties (position, amplitude, radius) and a natural preservation of its shape, which is not guaranteed in the conventional gridded approaches.

This has been included in the same paragraph as the Major comment 3 and we added a few lines in the Conclusion: *the analysis can either be projected back into the physical gridded space for standard post-processing and visualization, or remain in the object-based parametric space for applications that exploit eddy characteristics directly.*

2. State explicitly that the experiments focus on a single analysis step (no forecast–analysis cycling), and clarify what would be required to integrate the method into a full DA cycle.

We stated explicitly in the Abstract, Introduction, Methodology and Conclusion that the experiments were focused on the single analysis step to avoid confusion with a full DA cycle. Also, we added in the Conclusion what would be needed for a full DA cycle in the parametric space lines 209-212: *While standard gridded DA relies on physical dynamical models such as the Navier-Stokes equations, the parametric approach lacks an explicit model for the temporal evolution of eddy parameters. Consequently, a machine learning model is required to learn these dynamics from data, and satellite observations may provide location and shape parameters of the eddies.*

3. If possible, include discussion (or sensitivity tests) showing the impact of correlated parameter errors (notably a–r coupling) and state dependence of R_θ. Quantifiable metrics to compare quality and parameter dependence of each method would be a major improvement.

We thank the reviewer for this insightful comment. An important limitation of the present study is the simplified treatment of parameter uncertainties. In particular, we have not explicitly accounted for correlations between parameters, such as the coupling between amplitude and radius (a–r), nor for potential state dependence of the observation error covariance $\boldsymbol{R}^{\texttt{param}}$. In practice, such correlations naturally arise from the nonlinear fitting procedure and from the geometry of the structures. The suggested experiment above within a simplified framework gave an insight of the effect of parameter correlations. As such, they were not explicitly prescribed but partially captured through the ensemble-based estimation of $\boldsymbol{B}^{\texttt{param}}$ and $\boldsymbol{R}^{\texttt{param}}$ via EnOI. Accounting for these effects would likely impact the performance of parametric DA, especially in more realistic settings. This aspect will be investigated in future work.

We added a paragraph in the Conclusion lines 201-207: *One important aspect not explored in depth here but that warrants further investigation is the definition of error matrices in the parametric space. In particular, the impact of correlated parameter errors, should be considered in future work. Sensitivity tests on this coupling and the state dependence of the error covariance matrix $\boldsymbol{R}^{\texttt{param}}$ could provide deeper insights into the behavior of the assimilation system. Understanding the correlation structure between parameters, especially for non-linear systems, would be crucial for refining error estimates and improving the robustness of the DA process. Moreover, quantifiable metrics to compare the quality and parameter dependence of each method would be a significant improvement, providing clearer guidelines for the operational implementation* *of DA in the parametric space.*

We agree that introducing more systematic and quantitative performance metrics would strengthen the analysis. However, several possibilities exist and it is not entirely clear which direction would best address the reviewer's expectations. At this stage, a full implementation of such metrics would require substantial additional development and is therefore better suited as a perspective for future work. We would nevertheless be happy to incorporate a targeted analysis if the reviewer could provide more specific guidance on the preferred type of metric or evaluation framework.

References

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