

Response to reviewer #1

April 13, 2026

We would like first to thank the reviewer for his/her careful reading and constructive criticism. The suggested modifications have been a decisive help to revise and improve our manuscript. Specific comments are addressed one by one hereafter. Reviewer comments are recalled below in bold font, while text extractions from the revised paper are in italic font.

1 Major Comments

1) You refer to non-Gaussian methods but fail to mention the work at U. Reading and Colorado State University on this topic.

We thank the reviewer for pointing this out. Indeed, significant work on non-Gaussian data assimilation has been carried out at the University of Reading and Colorado State University. We have included the key reference in the revised manuscript: [van Leeuwen, 2010]. These works illustrate the development of particle filters and their localized variants, which address non-Gaussianity and the challenges of high-dimensional geophysical systems. We have updated the Introduction lines 16-22 to explicitly highlight these contributions and their relevance to our discussion of non-Gaussian methods:

In this study, we focus not on the full state but on specific geophysical structures (GSs), such as cyclones or ocean eddies. When sequential DA is applied in gridded space, linear Gaussian updates can struggle to correctly represent the position or shape of coherent structures, potentially altering their physical properties [Chen and Snyder, 2007]. To address these challenges, the literature includes non-Gaussian methods such as particle filters [van Leeuwen, 2010, Poterjoy, 2016], as well as displacement-aware transformations of the state, including alignment or morphing [Ravela et al., 2007, Beezley and Mandel, 2008, Zhen et al., 2025]. Feature-informed DA has also been proposed, where the characteristics of coherent structures guide the assimilation while the state remains defined in the gridded space [Srivastava et al., 2023].

2) Again referring to non-Gaussian DA but not actually performing any of it needs to be addressed.

We thank the reviewer for this comment. We acknowledge that we do not perform non-Gaussian data assimilation in the present study. These methods are mentioned in the introduction solely to provide context and illustrate the range of approaches that have been developed to address the issue of structural mismatches in gridded state space. The goal of our study is instead to examine a Gaussian and linear data assimilation method applied to a problem that is intrinsically non-Gaussian and nonlinear, namely the estimation of coherent structures with positional and shape mismatches. By doing so, we aim to investigate how a standard, computationally efficient linear Gaussian method behaves in this challenging context, and how it compares to assimilation in a reduced parametric space.

To clarify this in the manuscript, we have revised the relevant paragraph in the Introduction as follows:

The goal of our study is instead to examine a Gaussian and linear data assimilation method applied to a problem that is intrinsically non-Gaussian and nonlinear, namely the estimation of coherent structures with positional and shape mismatches. By focusing on this scenario, we aim to isolate the effects of performing assimilation directly on the structural parameters and to understand the limitations of classical linear DA

methods without implementing more computationally intensive particle filters or sophisticated displacement-aware corrections.

3) Present the cost function for the two different approaches so that we can relate how the parameters are solved for.

To clarify how the state is updated in both approaches, we present the following cost function J in the Methodology lines 96-99:

The data assimilation is performed in both spaces, indicated with the superscript param and grid , using the following cost function J

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}^b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}^b) + \frac{1}{2}(\mathbf{y} - \mathbf{x})^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{x}),$$

Minimizing this function provides the analysis both in the parametric space and the gridded space.

4) It is very confusing how you are calculating the observations for both set of experiments.

A similar issue has been raised by Reviewer #2, hence we added the following sentences lines 51-56:

We consider a simplified one-dimensional cross-shore section of an ocean eddy, described by three parameters: the peak amplitude a , the peak position p , and the maximal velocity radius r such that $\mathbf{x}^{\text{param}} = (a, p, r)$. In practice, these parameters can be estimated using standard eddy detection and fitting algorithms, such as local maxima identification combined with Gaussian profile fitting [e.g., Chelton et al., 2011]. For our idealized experiments, however, the parameters are prescribed manually, rather than obtained from a detection algorithm, in order to control the uncertainties and systematically study the assimilation behavior.

5) Figure 1 make no sense to me I cannot understand how each component are related to each other

We modified the figure to clarify the different steps of our methodology and rewrote the caption: *Schematic of the data assimilation workflow comparing parametric (purple) and gridded (red) spaces. For each approach, the sequence background (green box, panel 1) \rightarrow observation (blue box, panel 2) \rightarrow analysis (panels 3) is shown. Panels labeled “a” correspond to parametric space, “b” to gridded space. Background and observation parameters, respectively $\mathbf{x}^{\text{param}}$ and $\mathbf{y}^{\text{param}}$, are mapped to physical profiles, respectively \mathbf{x}^{grid} and \mathbf{y}^{grid} , via the function h . Observations $\mathbf{y}^{\text{param}}$, \mathbf{x}^{grid} may differ from the background. The parametric analysis can be mapped back to the gridded space (panel 4a), and profile fitting in the gridded space allows retrieval of parameter estimates (panel 4b). This figure illustrates how uncertainty in one parameter (among a , p , r) propagates through the assimilation step in each representation.*

6) The remaining figures also do not make sense. The first thing is that the true solution should be plotted or the true errors. These plots do not tell us anything if we do not know that they are supposed to be approximating. Finally I do not understand why you have the standard deviations and what are they supposed to be telling us?

We have reworked the Figures 2, 3 and 4 to improve their reading, making it clear that the left and right panels correspond to the mean and the standard deviation of the surface height, respectively. The formula $\sigma_h = \sqrt{\text{diag}(\mathbf{P}^a)}$ is used to obtain the standard deviation of the height so we can have a common reference for the estimation of a posteriori uncertainties for both approaches. For each experiment, the value of the interval between the observed and the background parameters is statistically related to their standard deviation σ , so the uncertainties are correct and the general true solution is given by the parametric analysis. In our idealized experiments, we intentionally set the same error magnitude for both the background and the observations. This allows us to control the system and verify that the assimilation result lies between the background and the observation, as expected in a Gaussian linear update. By using this controlled setup, we can isolate the effect of performing assimilation in parametric versus gridded space and assess how well the analysis preserves the structural characteristics of the eddy. This choice does not reflect an operational scenario but provides a clear framework for evaluating the methodology.

We revised the manuscript lines 122-125: *In these idealized experiments, the background and observation errors are set to the same magnitude. This controlled setup ensures that the analysis is expected to lie between the background and the observation, allowing a clear evaluation of the assimilation performance in parametric versus gridded space.*

2 Minor comments

1) Line 7: There should be a however at the start of the sentence.

It has been added.

2) Line 8: remove the

The correction has been done.

3) Line 14: Do not use the phrase we are not interested! It should be something like in this study we shall not be considering the full state but ...

The sentence has been modified.

4) Line 33: expectation and covariance with respect to what random variable and which PDF?

Thank you for pointing this out. We clarify that the mean and covariance refer to the ensemble of parameter realizations generated within the assimilation framework. We made it explicit in the Introduction lines 38-40: *within the chosen assimilation framework* and in the Conclusion lines 217-219: *This approach could furthermore be extended to other assimilation methods involving dynamic integration, such as the Kalman filter, while preserving the intrinsic properties of the geophysical structure.*

5) Paragraph at line 34: Rephrase.

We rephrased the paragraph lines 44-47: *We present a set of idealized numerical experiments designed to compare the analysis step performed in a conventional gridded space with that carried out in this reduced parametric (object-based) space. The eddy representation and the assimilation methodology are detailed in Sect. 2, the experimental setup in Sect. 3, and the results are presented in Sect. 4. Conclusions and perspectives are discussed in Sect. 5.*

6) Line 40: define what you mean by standard detection algorithms

We added a few words about the detection algorithms used for eddy detection lines 53-54: *standard eddy detection and fitting algorithms, such as local maxima identification combined with Gaussian profile fitting [e.g., Chelton et al., 2011].*

7) Line 45: This equation is confusing for a couple of different reasons: 1) what is lon? Is it a vector of the values of longitude? 2) Is this used as an observation operator or is it the numerical model? lower case h is usually reserved for the nonlinear observation operator.

Thank you for pointing this out. We have clarified in the manuscript that **lon** is the vector of longitudinal positions along the 1D cross-section of the eddy. The function $h(\mathbf{lon}; a, p, r)$ is a mapping from the parametric space to the physical gridded space, and is not an observation operator or the numerical model. It simply defines the physical profile corresponding to a given set of eddy parameters. We have also noted that while lowercase h is commonly used for nonlinear observation operators in DA literature, here it is used as a generic mapping function for clarity, see lines 59-61:

Here, h serves as a mapping function from the parametric space $\mathbf{x}^{\text{param}}$ to the physical gridded space \mathbf{x}^{grid} . It is not an observation operator nor a numerical dynamical model, but simply defines the physical profile corresponding to a given set of parameters.

8) Lines 52 and 54: Define the error covariance matrices \mathbf{B} and \mathbf{R} and use their correct names, also state which PDF type they are associated with.

We added the correct names of the error covariance matrix and indicated that the error was assumed to be Gaussian-distributed line 75: *with a stochastic error $\boldsymbol{\eta}^b \sim \mathcal{N}(\mathbf{0}, \mathbf{B})$ assumed to be Gaussian distributed with the background covariance matrix \mathbf{B}* and lines 78-79: *where \mathbf{y} is the observation vector, \mathcal{H} is the observation operator, and $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the stochastic observation error assumed to be Gaussian distributed with the observation covariance matrix \mathbf{R} .*

9) Line 57: Again what is meant by detection algorithm?

See response to Minor comment 6.

10) Line 68 Define what P_a is.

The definition has been added line 93: *analysis covariance \mathbf{P}^a .*

11) Line 72: This paragraph is not very well written. We do not use the phrase perfectly known. I believe that you are referring to the true values which is the correct way to describe what is happening here.

Thank you for this comment. We have revised the manuscript to replace the phrase “perfectly known” with “set to their true values”, which more accurately reflects the experimental setup. We also clarified that in each experiment, one parameter is uncertain while the other two are fixed at their true values, allowing us to isolate the effect of perturbing a single parameter on the assimilation results, as described lines 118-125: *The comparison of DA in both spaces is tested with a first test of three experiments, each focusing on a single uncertain parameter, while the other two parameters are set to their true values. In each experiment, the uncertain parameter differs between the observation and the background, whereas the other two parameters are identical in both. Specifically, the uncertain parameter in the three experiments is respectively: amplitude a (Experiment #1), position p (Experiment #2), and radius r (Experiment #3). This setup allows us to isolate the effect of perturbing a single parameter on the assimilation results. In these idealized experiments, the background and observation errors are set to the same magnitude. This controlled setup ensures that the analysis is expected to lie between the background and the observation, allowing a clear evaluation of the assimilation performance in parametric versus gridded space.*

12) Line 87: Should be Figs. are you are referring to more that one figure.

Correction done.

13) Line 108: geostrophic current field is not defined.

We added the following sentences lines 160-161 for the definition of the geostrophic current: *the geostrophic current field, an oceanic current in which the pressure gradient given by the surface height horizontal gradient is balanced by the Coriolis effect.*

14) Line 109: Eddy Kinetic Energy is not defined mathematically of even explained why it is important or how it is calculated.

We added the following sentences lines 162-164: *proportional to the square of the geostrophic current over the eddy area. Quantifying the EKE provides a robust measure of their intensity and is key to assessing their contribution to lateral transport, air-sea interactions, and the mesoscale energy budget of the ocean.*

15) Line 120: This phrasing is incorrect and has misleading statements, almost to the point where I would recommend rejection.

We have revised the manuscript to clarify that the improved performance of DA in the parametric space is demonstrated in our simplified 1D test case. We replaced overly categorical statements such as “perfect estimation” with more precise phrasing that reflects the limited scope of our experiments. The relevant

sentence now reads in Introduction lines 40-43:

While DA in the parametric space can considerably reduce computational cost in this idealized setting due to the small dimensionality of the parameter vector, practical applications require additional computations for parameter detection, fitting, and multi-feature association, which may offset some of the computational savings.

and in the Conclusion lines 195-196: *In contrast, DA in the parametric space provides an improved estimation of the parameters in this simplified test case, with realistic values of a posteriori errors.*

16) Line 125: This statement is also incorrect and is too sweeping. You have only shown this for the very simple test case shown.

Completing the above remark, we revised the Conclusion lines 198-200: *These results suggest that DA in param space may be particularly useful for well-defined geophysical structures, such as mesoscale ocean eddies, and potentially others, including cyclones, algal blooms, wildfires, or rainfronts.* These changes ensure that the conclusions accurately reflect the scope of our results while maintaining a constructive outlook for future applications.

17) Line 127: Why is a machine learning algorithm required? Justify this!

Thank you for this comment. In the standard gridded approach, the time evolution of the system is governed by physical dynamical equations, such as the Navier-Stokes equations. In contrast, in the parametric approach, there is no explicit physical model describing the temporal evolution of the eddy parameters (amplitude, position, radius). Therefore, a machine learning algorithm is proposed to learn the dynamical evolution of the parameters from data, allowing the parametric DA scheme to propagate the state forward in time in an operational forecasting framework. We have clarified this point in the revised manuscript lines 209-211: *While standard gridded DA relies on physical dynamical models such as the Navier-Stokes equations, the parametric approach lacks an explicit model for the temporal evolution of eddy parameters. Consequently, a machine learning model is required to learn these dynamics from data.*

References

- J. D. Beezley and J. Mandel. Morphing ensemble Kalman filters. *Tellus*, 60(1), 2008.
- D. B. Chelton, M. G. Schlax, and R. M. Samelson. Global observations of nonlinear mesoscale eddies. *Progress in Oceanography*, 91(2):167–216, 2011. doi: 10.1016/j.pocean.2011.01.002.
- Y. Chen and C. Snyder. Assimilating Vortex Position with an Ensemble Kalman Filter. *Monthly Weather Review*, 135(5):1828–1845, 2007. doi: 10.1175/MWR3351.1.
- J. Poterjoy. A Localized Particle Filter for High-Dimensional Nonlinear Systems. *Monthly Weather Review*, 144(1):59–76, 2016. doi: 10.1175/MWR-D-15-0163.1.
- S. Ravela, K. Emanuel, and D. McLaughlin. Data assimilation by field alignment. *Physica D: Nonlinear Phenomena*, 230(1-2):127–145, 2007. doi: 10.1016/j.physd.2006.09.035.
- A. Srivastava, W. Kang, and D. M. Tartakovsky. Feature-informed data assimilation. *Journal of Computational Physics*, 494:112499, 2023. doi: 10.1016/j.jcp.2023.112499.
- P. J. van Leeuwen. Nonlinear data assimilation in geosciences: an extremely efficient particle filter. *Quarterly Journal of the Royal Meteorological Society*, 136(653):1991–1999, 2010. doi: <https://doi.org/10.1002/qj.699>.
- Y. Zhen, V. Resseguier, and B. Chapron. Alignment of geophysical fields: A differential geometry perspective. *Physica D: Nonlinear Phenomena*, 483:134997, 2025. doi: <https://doi.org/10.1016/j.physd.2025.134997>.