Supporting Information for

Characteristics of potential evapotranspiration and its estimation from hydrological observation in the Budyko framework

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Introduction

This document provides supplementary table, figures, and detailed derivations of the partial derivative $\frac{\partial \mathrm{E}}{\partial \mathrm{E_P}}$, derived respectively from the non-parametric and parametric Budyko equations.

Table S1. Summary of current E_P estimation approaches used in the Budyko-related studies.

Type	E_P estimation approaches	Reference
temperature-based methods	Hargreaves-Samani equation	(Hwang and Devineni, 2022; Zheng et al., 2021; Wu et al., 2017)
	Hamon equation	(Maurer et al., 2022; Todhunter et al., 2020; Zhou et al., 2015)
radiation-based methods	Priestley-Taylor equation	(Zhang et al., 2004; Ning et al., 2019; Li et al., 2017; Milly et al., 2018; Bharat and Mishra, 2021; Ballarin et al., 2022; Luan et al., 2021)
	net radiation	(Arora, 2002; Roderick et al., 2014; Nijzink and Schymanski, 2022; Gudmundsson et al., 2016; Koppa et al., 2021; Chen et al., 2022; Renner et al., 2012)
combination methods	Penman equation	(Yang et al., 2007; Yang et al., 2014; Liang et al., 2015; Donohue et al., 2011; Zhang et al., 2019; Cheng et al., 2023; Yang et al., 2008)
	Penman-Monteith equation	(Shen et al., 2017; Wamucii et al., 2021; Fu et al., 2023; Wang et al., 2016; Greve et al., 2020; Coron et al., 2015)
pan evaporation	~	(Roderick and Farquhar, 2011; Zhang et al., 2011; Zhou et al., 2016; Mcvicar et al., 2007)

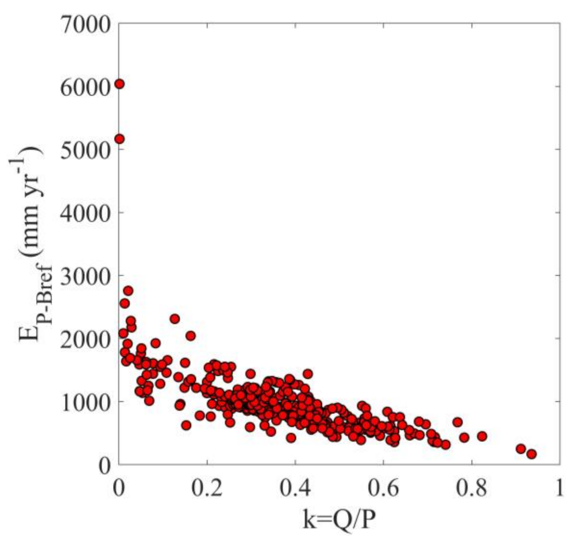

Figure S1. Relationship between $E_{P\text{-}Bref}$ and the mean annual runoff coefficient (k=Q/P) for the MOPEX catchments.

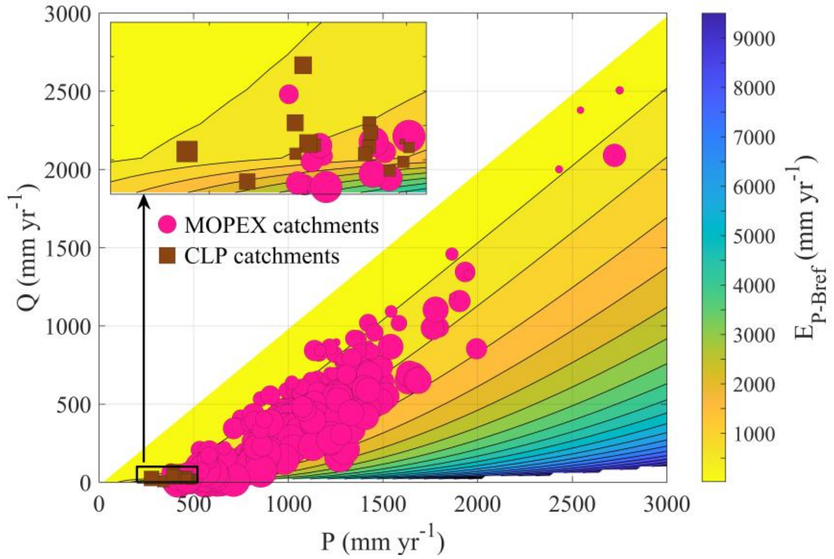

Figure S2. Comparison of $E_{P\text{-}Pen}$ and $E_{P\text{-}Bref}$ in P-Q space across the MOPEX (magenta circles) and the Chinese Loess Plateau (CLP) (brown squares) catchments. Larger points indicate larger values.

Text S1. Deviation of the partial derivative of E with respect to E_P in the original non-parametric Budyko equation

The original non-parametric Budyko equation (Budyko, 1958, 1974) (Equation 1) can be rewritten as:

$$E = \sqrt{PE_P \tanh\left(\frac{P}{E_P}\right)\left[1 - \exp\left(-\frac{E_P}{P}\right)\right]} \quad \text{(S1)}$$

Let $f_1 = PE_P$, $f_2 = \tanh\left(\frac{P}{E_P}\right)$, $f_3 = \left[1 - \exp\left(-\frac{E_P}{P}\right)\right]$, and $f = f_1 f_2 f_3$, then:

$$E = \sqrt{f} = \sqrt{f_1 f_2 f_3} \quad \text{(S2)}$$

According to the chain rule, we have:

$$\frac{\partial E}{\partial E_P} = \frac{\partial E}{\partial f}\frac{\partial f}{\partial E_P} = \frac{1}{2\sqrt{PE_P \tanh\left(\frac{P}{E_P}\right)\left[1 - \exp\left(-\frac{E_P}{P}\right)\right]}}\frac{\partial f}{\partial E_P} \quad \text{(S3)}$$

According to the product rule, the partial derivative of f with respect to E_P is:

$$\frac{\partial f}{\partial E_P} = f_2 f_3 \frac{\partial f_1}{\partial E_P} + f_1 f_3 \frac{\partial f_2}{\partial E_P} + f_1 f_2 \frac{\partial f_3}{\partial E_P} \quad \text{(S4)}$$

The three partial derivatives in Equation B4 are:

$$\frac{\partial f_1}{\partial E_P} = P + \frac{\partial P}{\partial E_P} E_P \quad \text{(S5)}$$

$$\frac{\partial f_2}{\partial E_P} = \frac{P - \frac{\partial P}{\partial E_P} E_P}{{E_P}^2} \left[\tanh^2\left(\frac{P}{E_P}\right) - 1\right] \tag{S6}$$

$$\frac{\partial f_3}{\partial E_P} = \frac{P - \frac{\partial P}{\partial E_P} E_P}{P^2} \exp\left(-\frac{E_P}{P}\right) \tag{S7}$$

Substituting Equations S5-S7 into Equation S4, and then substituting Equation S4 into Equation S3, we obtain the partial derivative of E with respect to E_P under the non-parametric Budyko equation:

$$\begin{aligned}\frac{\partial E}{\partial E_P} = & \frac{1}{2\sqrt{PE_P \tanh\left(\frac{P}{E_P}\right)\left[1 - \exp\left(-\frac{E_P}{P}\right)\right]}} \\ & \cdot \left\{\tanh\left(\frac{P}{E_P}\right)\left[1 - \exp\left(-\frac{E_P}{P}\right)\right](P + E_P \frac{\partial P}{\partial E_P}) + P[1 - \tanh^2\left(\frac{P}{E_P}\right)[1 - \exp\left(-\frac{E_P}{P}\right)]](\frac{\partial P}{\partial E_P} - \frac{P}{E_P})] \right. \\ & \left. + E_P \tanh\left(\frac{P}{E_P}\right)\exp\left(-\frac{E_P}{P}\right)(1 - \frac{E_P}{P}\frac{\partial P}{\partial E_P})\right\}\end{aligned} \tag{S8}$$

Assuming that P and E_p are independent (i.e., $\frac{\partial P}{\partial E_P} = 0$), the above expression simplifies to:

$$\frac{\partial E}{\partial E_P} = \frac{\left\{\tanh\left(\frac{P}{E_P}\right) + \frac{P}{E_P}\left[\tanh^2\left(\frac{P}{E_P}\right) - 1\right]\right\}\sqrt{1 - \exp\left(-\frac{E_P}{P}\right)}}{2\sqrt{\frac{E_P}{P}\tanh\left(\frac{P}{E_P}\right)}} + \frac{\exp\left(-\frac{E_P}{P}\right)\sqrt{\frac{E_P}{P}\tanh\left(\frac{P}{E_P}\right)}}{2\sqrt{1 - \exp\left(-\frac{E_P}{P}\right)}} \tag{S9}$$

Text S2: Deviation of the partial derivative of E with respect to E_P in the Budyko-MCY equation

The Budyko-MCY can be rewritten as:

$$E = P \cdot [1 + (\frac{E_P}{P})^{-n}]^{-1/n} \tag{S10}$$

Let $f_4 = [1 + (\frac{E_P}{P})^{-n}]^{-1/n}$, then：

$$E = P \cdot f_4 \tag{S11}$$

According to the product rule, we have:

$$\frac{\partial E}{\partial E_P} = P\frac{\partial f_4}{\partial E_P} + f_4 \frac{\partial P}{\partial E_P} \tag{S12}$$

According to the chain rule, the partial derivative of f_4 with respect to E_P is:

$$\frac{\partial f_4}{\partial E_P} = [1 + (\frac{E_P}{P})^{-n}]^{-1/n-1} \cdot \left(\frac{E_P}{P}\right)^{-n-1} \cdot \frac{1}{P}(1 - \frac{E_P}{P}\frac{\partial P}{\partial E_P}) \tag{S13}$$

Substituting Equation S13 into Equation S12, we obtain the partial derivative of E with respect to E_P under the Budyko-MCY equation:

$$\frac{\partial E}{\partial E_P} = [1 + (\frac{E_P}{P})^{-n}]^{-1/n-1} \cdot \left(\frac{E_P}{P}\right)^{-n-1} \cdot (1 - \frac{E_P}{P}\frac{\partial P}{\partial E_P}) + \frac{\partial P}{\partial E_P}[1 + (\frac{E_P}{P})^{-n}]^{-1/} \quad \text{(S14)}$$

Assuming that P and E_p are independent (i.e., $\frac{\partial P}{\partial E_P} = 0$), the above expression simplifies to (Yang et al., 2008):

$$\frac{\partial E}{\partial E_P} = \left[1 + (\frac{E_P}{P})^{n}\right]^{-\frac{1}{n}-1} \quad \text{(S15)}$$

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